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Nonlinear dynamics of deep water subsea lifting operations considering KC-dependent hydrodynamic coefficients

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Abstract

The dynamics of deep water subsea lifting operations considering hydrodynamic coefficients that depend on the Keulegan-Carpenter (KC) number are analysed in this study. Firstly, experimental data from the literature is presented for a typical subsea manifold, relating the added mass and drag coefficients to the amplitude of oscillation, represented by the KC number. Then, the nonlinear non-dimensional equation of motion, that considers the variable hydrodynamic coefficients, is presented. The solution of this equation is obtained via the harmonic balance method and by iterative time domain integration, and the results are compared to those of a conventional model with constant hydrodynamic coefficients. The results obtained via the harmonic balance method are considered almost as accurate as from the time domain integration, but require significantly less computational effort. Also, it is shown that the amplitude-dependent model predicts variations in the natural frequency and damping of the system as a function of the amplitude of the response of the payload. This results in significant differences in the maximum cable tension and the payload depth at which it occurs, compared with the constant hydrodynamic coefficient model. Hence this shows the importance of considering variable hydrodynamic coefficients when analysing subsea lifting systems.

Keywords

marine operations; subsea structure installation; hydrodynamic coefficients; Keulegan-Carpenter number; harmonic balance method; cable forces

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Nomenclature

\( A_p \) \quad \text{Vertical projected area of the payload}
\( C_a \) \quad \text{Added mass coefficient}
\( C_d \) \quad \text{Drag coefficient}
\( D \) \quad \text{Characteristic dimension of the payload}
\( E_A \) \quad \text{Axial rigidity of the cable}
\( F_{\text{hyd}} \) \quad \text{Hydrodynamic force}
\( F_{\text{dyn}} \) \quad \text{Dynamic force on the cable}
\( f \) \quad \text{Dimensionless force on the cable}
\( g \) \quad \text{Gravity acceleration}
\( KC \) \quad \text{Keulegan-Carpenter number}
\( L \) \quad \text{Length of the cable}
\( m \) \quad \text{Mass per unit length of the cable}
\( m_s \) \quad \text{Equivalent submerged mass per unit length of the cable}
\( M \) \quad \text{Mass of the payload}
\( N \) \quad \text{Number of harmonic components in the Harmonic Balance Method}
\( \text{Re} \) \quad \text{Reynolds number}
\( V \) \quad \text{Volume of the payload}
\( \tau \) \quad \text{Time}
\( w \) \quad \text{Displacement of the payload}
\( w_{\text{st}} \) \quad \text{Static displacement of the payload}
\( w_{\text{dyn}} \) \quad \text{Dynamic displacement of the payload}
\( w_0 \) \quad \text{Displacement of the input on the top of the cable}
\( W \) \quad \text{Amplitude of the displacement of the payload}
\( W_0 \) \quad \text{Amplitude of the input displacement on the top of the cable}
\( \gamma \) \quad \text{Dimensionless displacement of the payload}
\( Y \) \quad \text{Amplitude of the non-dimensional displacement of the payload}
\( \alpha \) \quad \text{Dimensionless mass ratio}
\( \beta \) \quad \text{Dimensionless amplitude-dependent drag coefficient}
\( \mu \) \quad \text{Dimensionless amplitude-dependent added mass coefficient}
\( \phi \) \quad \text{Phase of the displacement of the payload}
\( \Lambda \) \quad \text{Frequency ratio}
\( \rho \) \quad \text{Density of sea water}
\( \Omega \) \quad \text{Input frequency}
\( \omega_{n_0} \) \quad \text{Natural frequency of the system when } KC \rightarrow 0
\( \tau \) \quad \text{Dimensionless time}
1. Introduction

The exploration of offshore resources has progressively increased throughout past decades, led by the pioneering efforts of the oil and gas industry, and followed by the renewable energy and deep sea mining sectors. The construction of the subsea infrastructure required to enable these activities is usually done by subsea lifting operations via specialist barges or vessels, and correspond to a major part of the capital expenditure necessary to start the production of the field. Therefore, there is a pursuit in the industry to continuously improve these operations and reduce their costs. According to Job et al. (2018), the costs of subsea manifold installations in the Brazilian shore has dropped by 86% since 2010 due to the improvements on the operational procedures and to the use of new technologies such as passive heave compensation and synthetic ropes. Further, Lacal-Arantégui et al. (2018) reported the reduction of up to 70% on the installation time of offshore wind turbines and their foundations, which is one of the reasons for the overall cost reduction of offshore wind electricity last years. These studies strengthen the importance of new improvements to reduce the costs of subsea lifting operations and the positive impacts these enhancements can have on the viability of offshore exploration.

A common approach to improve these operations is to conceive more accurate models to predict the dynamics of subsea lifting systems. This gives more confidence in predictions of maximum cable tensions to ensure safety of the operation and more accurately define the operational weather window, hence saving costs, which can be extremely high for specialist vessels. In this direction, several works have been published dealing with the dynamics of the system when the payload crosses the wave zone. Li et al. (2020) considered the installation of a spool piece by taking into account the influence of vessel shielding and transient load effects; Jeong et al. (2016) presented a model that accounts for the possibility of collisions between the payload and the vessel during the operation; and Hannan and Bai (2016) studied the nonlinear dynamics of a barge and a submerged payload subjected to constrained pendulum motions. Further, crane flexibility effects during the lift off phase of operations was covered by Park et al. (2011) and Hong et al. (2016). The dynamics of the system when the equipment is lowered into deep waters is also an important phase to be analysed due to the possibility of resonance of the cable-payload system at certain water depths. Classical approaches for this problem can be found in the works by Niedzwicki and Thampi (1991), Huang (1999), Driscoll et al. (2000a, 2000b; 2000) where the dynamics of the system were modelled considering constant length cables. More recently, variable length models have been presented by Tommasini et al. (2020, 2018), Gao et al. (2020), Quan et al. (2020) and Quan and Chang (2020) highlighting the influence of the winch speed in the response of the system. Other examples of recent studies in this field cover the use of synthetic ropes to avoid resonance amplifications by de Araujo Neto et al. (2019); the development of a virtual reality simulation system to analyse the installation of subsea equipment by Zhang et al. (2017); and the analysis of the coupled dynamics of the vessel and the cable-equipment system, both numerically and experimentally, by Nam et al. (2017). Finally, a methodology for the assessment of the operational limits of marine operations was presented by Acero et al. (2016).
A crucial aspect of the subsea lifting modelling is the hydrodynamic forces that are applied to the system due to the relative motion of the payload and the water. The traditional approach to model these forces is to use Morison’s equation (Morison et al., 1950), which is a semi-empirical equation that considers the hydrodynamic force as the combination of an inertial term, proportional to the acceleration; and a drag term, proportional to the squared velocity. The use of this equation requires the knowledge of two coefficients: the added mass and the drag coefficient, which are usually a function of the geometry of the equipment and of dimensionless quantities, such as the Reynolds (Re) or Keulegan-Carpenter (KC) numbers.

Extensive research has been done describing these coefficients for simple geometries, especially for cylinders due to their recurrent use as a structural member in offshore structures. A classical reference on this subject is the book by Sarpkaya (2010), which presented a review of the state-of-the-art on this topic and stressed the dependence of these coefficients on KC and Re in time-dependent flows. The hydrodynamics of flat plates have also been considered in several studies. Molin (2011) presented a summary of his studies on the calculation of added mass and damping coefficients for perforated structures in oscillatory flows by using a theoretical model. His results indicated the dependence of the coefficients on the amplitude of oscillation and on the perforation of the plate, which was represented by a porous Keulegan-Carpenter number ($\overline{KC}$). Further, An and Faltinsen (2013) showed a numerical and experimental study to evaluate the added mass and damping coefficients of perforated rectangular plates, taking into account the effects of perforation ratio, plate submergence, forcing period, and KC. Especially, dependence of the coefficients on KC was found to be higher than on the frequency when the plate was deeply submerged. More recently, Mentzoni and Kristiansen (2020a, 2020b, 2019) presented a series of numerical and experimental studies to evaluate the hydrodynamic coefficients of perforated plates. The results obtained were in agreement with previous references, reinforcing the dependence on the amplitude of oscillation.

Other examples of studies addressing the determination of the hydrodynamics of simple shapes that highlight the influence of KC can be found in (Garrido-Mendoza et al., 2015; Li et al., 2013; Tian et al., 2016; Venugopal et al., 2009).

Subsea structures are usually more complex, and correlation to simple geometries are not always possible when evaluating their hydrodynamic coefficients. Although more scarce, direct evaluation of the coefficients for specific cases can also be found in the literature, for example Fernandes and Mineiro (2007) calculated the translational and rotational hydrodynamic coefficients of subsea manifolds. The hydrodynamic coefficients of an ROV (Remotely Operated Vehicle) were presented by Avila and Adamowski (2011), who showed that the influence of the amplitude of oscillation was higher than that of the period on the coefficients. A similar conclusion was obtained in the study by Mentzoni et al. (2018), where simplified subsea structures were experimentally tested, and by Du et al. (2020), analysing the hydrodynamic coefficients of a subsea manifold. Further, Solaas and Sandvik (2017) presented a series of results for the hydrodynamic coefficients of suction anchors; and Computational Fluid Dynamics (CFD) was used by Holmes et al. (2016) to evaluate the hydrodynamics of Blow-Out Preventers (BOP) as a function of KC.
Finally, a summary of hydrodynamic coefficients for subsea structures was presented by Oristland (1989). According to these references, there is strong dependence of the hydrodynamic coefficients on the amplitude of the oscillation, which is commonly represented by the KC number. Despite this conclusion, the majority of studies dealing with the dynamics of deep water subsea lifting operations considers constant added mass and drag coefficients in their models (F R Driscoll et al., 2000a, 2000b; F. R. Driscoll et al., 2000; Gao et al., 2020; Huang, 1999; Niedzwecki and Thampi, 1991; Quan and Chang, 2020; Tommasini et al., 2020, 2018). To the best knowledge of the authors, the only studies considering the influence of the amplitude of oscillation on the dynamics of the system are due to Ireland et al. (2007) and Pestana et al. (2021). In the study by Ireland et al. (2007), only the hydrodynamic damping was considered variable and the solution of the equation of motion was obtained iteratively. While in Pestana et al. (2021), the dynamics of an example operation were assessed by running independent simulations considering all the pairs of added mass and drag coefficients obtained experimentally for different KC numbers. Therefore, the objective of this work is to extend the study about the dynamics of subsea lifting operations in deep waters considering the effects of KC-dependent hydrodynamic coefficients, focusing on (1) the deduction of a non-dimensional nonlinear equation of motion for the system; (2) solution of the equation of motion by the harmonic balance method and by an iterative time domain integration algorithm; and (3) illustration of the dynamical features of the system when the hydrodynamic coefficients are KC-dependent.

The sequence of this paper is as follows: an overview of the hydrodynamics of oscillating bodies including the experimental results from the literature of the KC-dependent hydrodynamic coefficients of a typical subsea manifold in Section 2, the deduction of the non-dimensional nonlinear equation of motion in Section 3, the solution of the equation of motion via the harmonic balance method and via the iterative time domain integration in Section 4, the results obtained in Section 5, and the conclusions in Section 6.

2. **KC-dependent hydrodynamics of a subsea manifold**

An object lifted in deep waters by typical crane vessels experiences an oscillatory motion as a result of the movement of the vessel being excited by the ocean waves. This oscillatory flow introduces hydrodynamic forces on the payload that must be assessed in order to predict the dynamics of the system during the operation. According to Sarpkaya (2010), in time-dependent flows such as this, the direction of the wake changes from upstream to downstream as the velocity of the payload changes sign, the flow may alter from laminar to turbulent regimes, and the equipment may oscillate inside its own wake. Due to the complexity of this flow and to the non-trivial geometry of real equipment, no closed solution is available to represent the hydrodynamic forces that are generated. The representation of these forces must rely on empirical or semi-empirical relations calibrated via experiments or Computational Fluid Dynamics (CFD).

Historically, Morison’s equation (Morison et al., 1950) has been used to approximate these forces. In this formulation, the hydrodynamic load is split into a term proportional to the acceleration and a term...
proportional to the squared velocity. Therefore, for a body oscillating in a stationary fluid, the hydrodynamic force is given by:

\[
F_{\text{hyd}} = -\rho V C_a \ddot{w} - \frac{1}{2} \rho C_d A_p |\dot{w}| 
\]

(1)

where \( \rho \) is the density of the sea water, \( V \) is the volume of the structure, \( A_p \) is the vertical projected area, \( \dot{w} \) is the velocity, \( \ddot{w} \) is the acceleration, \( C_a \) is the added mass coefficient, and \( C_d \) is the drag coefficient. These two coefficients must be determined to represent the forces on the system.

Experimental data for KC-dependent added mass and drag coefficients of typical subsea manifolds were presented by Pestana et al. (2021). In their study, 1:35 scale models were used to perform oscillatory tests in a calm fluid for various \( \text{KC} = \frac{2\pi W}{D} \), where \( W \) is the amplitude of the oscillation and \( D \) is the characteristic length of the body (i.e. its width). The influence of the frequency was not considered, relying on several references (An and Faltinsen, 2013; Avila and Adamowski, 2011; Mentzoni et al., 2018) that indicate the preponderance of the amplitude of oscillation in comparison to the frequency. The frequency of the tests was equal to 0.59 Hz, which corresponds to a period of 10 s in the true scale. The test water depth was 2.5 m and oscillations were performed horizontally. Each test consisted of twenty cycles that were repeated three times for each amplitude and the coefficients were obtained via a least squares algorithm. The base plate of the manifold presents some small perforations, but they are less than 5% of the total vertical projected area. Further details of the experimental set-up and methodology can be found in Pestana et al. (2021).

One piece of equipment analysed in Pestana et al. (2021) is considered as an example payload to evaluate the dynamics of the deep water subsea lifting operation. The geometric data for this subsea manifold is presented in Table 1 and a photograph of the scale model is presented in Figure 1.

Table 1: Geometric data for the manifold used as the example in this study (extracted from Pestana et al. (2021)).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ((M))</td>
<td>170 tonnes</td>
</tr>
<tr>
<td>Volume ((V))</td>
<td>21.8 m³</td>
</tr>
<tr>
<td>Projected area ((A_p))</td>
<td>144.5 m²</td>
</tr>
<tr>
<td>Length</td>
<td>14.9 m</td>
</tr>
<tr>
<td>Width</td>
<td>9.7 m</td>
</tr>
<tr>
<td>Height</td>
<td>3.8 m</td>
</tr>
</tbody>
</table>
The experimental results obtained in Pestana et al. (2021) for the added mass and drag coefficients as a function of KC are presented in Figure 2, together with the fitted curves obtained by using the least squares method. Note that, for consistency with the dynamic models used in this study, the inertia coefficient based on the envelope of the manifold (as in Pestana et al. (2021)) is replaced by the added mass coefficient based on the actual volume of the manifold. For the range of KC considered, a linear trend in the added mass coefficient and a shifted reciprocal behaviour for the drag coefficient as KC increases are observed. This general behaviour is in agreement with the data for other pieces of equipment tested by Pestana et al. (2021) and other results in the literature (Li et al., 2013; Mentzoni and Kristiansen, 2019; Tian et al., 2016).

![Figure 1: 1:35 scale model manifold considered in this study and analysed by Pestana et al. (2021) (picture courtesy of Petrobras).](image)

The general form of the equations for the hydrodynamic coefficients as a function of KC are given by:

\[ C_a = C_{a_0} + C_{a_1}KC \]  
\[ C_d = C_{d_0} + C_{d_1}(KC)^{-1} \]  

The coefficients obtained experimentally for this manifold are \( C_{a_0} = 44.1 \), \( C_{a_1} = 21.2 \), \( C_{d_0} = 4.5 \), and \( C_{d_1} = 1.6 \). Constant added mass and drag coefficients are also evaluated in order to compare the results.
obtained from the KC-dependent model to the results obtained by using constant coefficients. In this case, the constant coefficients are chosen assuming a KC number of unity, such that $C_a = 65.3$ and $C_d = 6.1$. A summary of the coefficients considered in this study is presented in Table 2.

<table>
<thead>
<tr>
<th>Model</th>
<th>Added mass ($C_a$)</th>
<th>Drag ($C_d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KC-dependent coefficients</td>
<td>44.1 + 21.2KC</td>
<td>4.5 + 1.6(KC)$^{-1}$</td>
</tr>
<tr>
<td>Constant coefficients</td>
<td>65.3</td>
<td>6.1</td>
</tr>
</tbody>
</table>

3. The dynamics of deep water subsea lifting operations

Section 2 presented the hydrodynamic modelling of the subsea manifold under steady state oscillatory motion in a calm fluid. It is now necessary to construct a model to predict the dynamics of the system that accounts for the KC dependence of the hydrodynamic coefficients (Figure 3).

The first assumption in this direction is to consider only the deep water phase of the lowering operation, hence the influence of the ocean waves on the payload is not considered in this study. This is in agreement with the experimental set-up that considered the object oscillating in a calm fluid.

Next, Driscoll et al. (2000a) showed, by analysing measurements from a real operation, that unidimensional models were sufficient to predict the dynamics of the payload due to the vessel excitation. Also, Tommasini et al. (2018) showed that single degree-of-freedom and discretized models (e.g. the software Orcaflex) predict similar responses in typical subsea lifting scenarios. Therefore, a single degree-of-freedom model is considered in this study, which is also a common approach in the literature (Huang, 1999; Jordan and Bustamante, 2007; Niedzwecki and Thampi, 1991; Tommasini et al., 2020, 2018).

Another recurrent assumption in the literature (F. R. Driscoll et al., 2000; Niedzwecki and Thampi, 1991; Quan et al., 2020; Tommasini et al., 2018) is to consider the dynamics of the vessel to be independent of the dynamics of the cable-equipment system, due to a large difference in mass. Thus, the vertical displacement of the lifting point is prescribed as an imposed displacement to the top of the cable.

Further, snap loads are not represented in this formulation, as in (F R Driscoll et al., 2000b; Quan and Chang, 2020; Tommasini et al., 2018). This is in agreement with the recommended practice DNVGL-RP-N103 (2017) that requires the cable to be always taut during subsea lifting operations. Scenarios that lead to slack conditions should then be classified as unsafe during the planning of the operation.

Finally, the cable is assumed to have constant unloaded length during the analysis, such that evaluation of the full lowering into deep waters is conducted by running several independent simulations considering different depths for the payload. This is especially important in operations that require constant interruptions of the lowering process, such as in subsea load transfers (Costa and Lima, 2017) or when successive slings are added to the lifting line (Roveri et al., 1996).
The equation of motion for this system, considering the assumptions presented above, was obtained by Tommasini et al. (2018). In their study, the kinetic and potential energies were calculated for the cable and payload, and the hydrodynamic forces were assumed to be a non-conservative generalized force. Using Lagrange’s equation, the equation of motion obtained was:

\[
(M + \rho V C_a + \frac{1}{3} mL) \ddot{w} + \frac{1}{2} \rho A_p C_d |\dot{w}| \dot{w} + \frac{EA}{L} w = \frac{EA}{L} w_0 - \frac{mL}{6} \ddot{w}_0 - \left( M - \rho V + \frac{m_sL}{2} \right) g
\]  

(4)

where \( m \) is the mass per unit length of the cable, \( L \) is the suspended length of the cable, \( EA \) is the rigidity of the cable, \( w \) is the displacement of the structure, \( w_0 \) is the displacement of the lifting point, \( \ddot{w}_0 \) is the acceleration of the lifting point, \( m_s \) is the equivalent submerged mass of the cable, and \( g \) is the gravitational acceleration.

Substituting Eqs. (2) and (3) into Eq. (4) and considering \( KC = 2\pi W/D \), the equation of motion becomes:

\[
(M + \rho V C_{a0} + \frac{1}{3} mL) \ddot{w} + \frac{2\pi\rho V C_{a1} W}{D} \ddot{w} + \frac{1}{2} \rho A_p C_{d0} |\dot{w}| \dot{w} + \frac{\rho A_p C_{d1} D}{4\pi W} |\dot{w}| \dot{w} + \frac{EA}{L} w

= \frac{EA}{L} w_0 - \frac{mL}{6} \ddot{w}_0 - \left( M - \rho V + \frac{m_sL}{2} \right) g
\]  

(5)

This equation presents two new nonlinear terms in addition to the traditional added mass and quadratic drag terms. The first one is an amplitude-dependent term proportional to the acceleration and the second term is inversely dependent on the amplitude and proportional to the square of the velocity.

The solution of this equation can be split into static and dynamic parts \((w = w_{st} + w_{dyn})\) and the solution of Eq. (5) can be obtained by solving the following independent equations:

\[
\frac{EA}{L} w_{st} = - \left( M - \rho V + \frac{m_sL}{2} \right) g
\]  

(6)

Figure 3: Representation of the single degree-of-freedom model for the evaluation of deep water subsea lifting operations.
\[
\begin{align*}
(M + \rho V C_{a_0} + \frac{1}{3} m L) \ddot{w}_{\text{dyn}} + \left(\frac{2\pi \rho V C_{a_1} W}{D}\right) \dot{w}_{\text{dyn}} + \frac{1}{2} \rho A_p C_{d_0} |w_{\text{dyn}}| \dot{w}_{\text{dyn}} \\
+ \frac{\rho A_p C_{d_1} D}{4\pi W} |w_{\text{dyn}}| \dot{w}_{\text{dyn}} + \frac{EA}{L} w_{\text{dyn}} = \frac{EA}{L} w_0 - \frac{mL}{6} \ddot{w}_0
\end{align*}
\]  
(7)

Next, the static force acting on the top of the cable can be directly obtained by the submerged weight of the cable and the payload. Meanwhile, the dynamic force acting on the cable is given by:

\[F_{\text{dyn}} = \frac{EA}{L} (w_0 - w_{\text{dyn}})\]  
(8)

The static displacement of the equipment can be directly obtained from Eq. (6), but evaluation of the dynamic response is more challenging, since it requires the solution of a nonlinear differential equation. In order to reduce the number of independent variables, Eq. (7) can be written in non-dimensional form. So, assuming a harmonic input for the displacement at the top of the cable:

\[w_0 = W_0 \cos(\Omega t)\]  
(9)

and introducing the following dimensionless variables:

\[
\begin{align*}
\tau &= \omega_{n0} t \\
\Lambda &= \frac{\Omega}{\omega_{n0}} \\
y &= \frac{w_{\text{dyn}}}{W_0}
\end{align*}
\]  
(10-12)

where \(\omega_{n0}\) is the natural frequency of the system when \(KC \to 0\):

\[\omega_{n0} = \sqrt{\frac{EA}{L \left( M + \rho V C_{a_0} + \frac{1}{3} m L \right)}}\]  
(13)

it is possible to re-write Eq. (7) in the non-dimensional form:

\[ (1 + \mu Y)y'' + \left( \gamma + \frac{\beta}{Y} \right) |y'| y' + y = (1 + \alpha \Lambda^2) \cos(\Lambda \tau) \]  
(14)

In this case, \(y'\) and \(y''\) denote the single and double differentiation of \(y\) with respect to \(\tau\), respectively, \(Y\) is the amplitude of \(y\), and the new parameters of the system are defined as:

\[\alpha = \frac{mL}{6 \left( M + \rho V C_{a_0} + \frac{1}{3} m L \right)}\]  
(15)

\[\gamma = \frac{\rho A_p C_{d_0} W_0}{2 \left( M + \rho V C_{a_0} + \frac{1}{3} m L \right)}\]  
(16)
\[ \mu = \frac{2\pi \rho V C_{a_1} W_0}{D \left( M + \rho V C_{a_0} + \frac{1}{3} mL \right)} \]  
(17)

\[ \beta = \frac{\rho A_p C_{d_1} D}{4\pi \left( M + \rho V C_{a_0} + \frac{1}{3} mL \right)} \]  
(18)

The mass ratio (\( \alpha \)) and the dimensionless damping (\( \gamma \)) are related to the constant terms of the hydrodynamic coefficients (\( C_{a_0} \) and \( C_{d_0} \)). On the other hand, the dimensionless amplitude-dependent added mass coefficient (\( \mu \)) and the dimensionless amplitude-dependent drag coefficient (\( \beta \)) are new parameters introduced in the system, and they are related to the KC-dependent terms of the hydrodynamic coefficients (\( C_{a_1} \) and \( C_{d_1} \)). Finally, the non-dimensional form of the dynamic force on the cable is given by:

\[ f = \frac{F_{dyn}}{W_0 \left( M + \rho V C_{a_0} + \frac{1}{3} mL \right) \Omega^2} \]  
(19)

which can be presented in non-dimensional form as:

\[ f = \frac{1}{\Lambda^2} (y_0 - y) \]  
(20)

where \( y_0 = \cos(\Lambda t) \).

4. Solution of the nonlinear equation of motion

After presenting the non-dimensional equation of motion considering amplitude-dependent hydrodynamic coefficients, it is necessary to present the methods to solve this equation. To this end, this study considers two different approaches: an iterative procedure for the time domain integration, presented in Section 4.1; and an analytical approach based on the Harmonic Balance Method (HBM), presented in Section 4.2. Aiming to compare the results obtained by using KC-dependent hydrodynamic coefficients, the dynamics of the system with constant hydrodynamic coefficients is obtained directly from time domain integration of Eq. (14) considering \( \mu = \beta = 0 \) and substituting the constant coefficients from Table 2 for \( C_{a_0} \) and \( C_{d_0} \). A Runge-Kutta solver is used via the Matlab function ‘ode45’.

4.1 Iterative time domain integration

The equation of motion presented in Eq. (14) is nonlinear, including the term \( \gamma |y'|y' \) and the amplitude-dependent terms \( \mu Y y'' \) and \( \beta Y^{-1} |y'|y' \). Due to the presence of these two amplitude-dependent terms, time domain integration of Eq. (14) is not straightforward, as knowledge of the amplitude of the response is not available a priori. A simple way to deal with this fact is to use an iterative procedure, as presented in Table 3, where \( \epsilon \) is the tolerance for the convergence of the algorithm.
Similarly to the constant coefficient case, time domain integration of Eq. (14) has been obtained by using a Runge-Kutta solver, implemented via the ‘ode45’ function in Matlab.

Table 3: Algorithm for the iterative time domain integration of the equation of motion.

1. Set an initial value for $Y$
2. Solve Eq. (14) in the time domain
3. Obtain the steady state amplitude of $y$: $Y_{\text{new}} = \left[ \max(y) - \min(y) \right]/2$
4. Check convergence of $Y$ using $|Y_{\text{new}} - Y| < \epsilon Y$
5. If convergence has been achieved, stop; otherwise, return to 2 using $Y = Y_{\text{new}}$

4.2 Harmonic Balance Method

The steady state response of Eq. (14) can also be obtained by using analytical procedures, which can provide deeper understanding of the dynamics of the system and faster results. In this study, the Harmonic Balance Method (Chen et al., 2019; Kim et al., 2005; Li and Du, 2020) is used to solve the equation of motion analytically. The non-dimensional displacement is assumed to be represented by a truncated Fourier series:

$$y = \sum_{k=1}^{N} Y_k \cos(k\Lambda \tau + \phi_k)$$

The quadratic term $|\sin(\Lambda \tau + \phi_1)| \sin(\Lambda \tau + \phi_1)$ can be approximated by a Fourier series keeping only the first term, such that:

$$|\sin(\Lambda \tau + \phi_1)| \sin(\Lambda \tau + \phi_1) \approx \frac{8}{3\pi} \sin(\Lambda \tau + \phi_1)$$

Substituting Eq. (24) into Eq. (23) leads to:

$$- (1 + \mu Y_1) Y_1 \lambda^2 \cos(\Lambda \tau + \phi_1) - (\gamma Y_1 + \beta) Y_1 \lambda^2 |\sin(\Lambda \tau + \phi_1)| \sin(\Lambda \tau + \phi_1) + Y_1 \cos(\Lambda \tau + \phi_1)$$

$$= (1 + \alpha \lambda^2) \cos(\Lambda \tau)$$

4.2.1 Solution considering the first harmonic component

A first approximation for the response of the system can be obtained by taking only the first harmonic as the response of the system:

$$y = Y_1 \cos(\Lambda \tau + \phi_1)$$

Substituting Eq. (22) into Eq. (14) leads to:

$$-(1 + \mu Y_1) Y_1 \lambda^2 \cos(\Lambda \tau + \phi_1) - (\gamma Y_1 + \beta) Y_1 \lambda^2 |\sin(\Lambda \tau + \phi_1)| \sin(\Lambda \tau + \phi_1) + Y_1 \cos(\Lambda \tau + \phi_1)$$

$$= (1 + \alpha \lambda^2) \cos(\Lambda \tau)$$
Making the variable substitution $\Lambda \tau + \phi_1 = \Lambda \hat{t}$ and expanding the trigonometric function:

\[
(1 - \Lambda^2 - \mu Y_1 \Lambda^2) Y_1 \cos(\Lambda \hat{t}) - \frac{8}{3\pi} (\gamma Y_1 + \beta) Y_1 \Lambda^2 \sin(\Lambda \hat{t}) = (1 + \alpha \Lambda^2) \cos(\Lambda \hat{t}) \cos(\phi_1) + (1 + \alpha \Lambda^2) \sin(\Lambda \hat{t}) \sin(\phi_1)
\]  

(26)

Equating the coefficients of $\cos(\Lambda \hat{t})$ and $\sin(\Lambda \hat{t})$ on both sides of the Eq. (26) results in:

\[
(1 - \Lambda^2 - \mu Y_1 \Lambda^2) Y_1 = (1 + \alpha \Lambda^2) \cos \phi_1 \\
- \frac{8}{3\pi} (\gamma Y_1 + \beta) \Lambda^2 Y_1 = (1 + \alpha \Lambda^2) \sin \phi_1
\]  

(27a) (27b)

Adding the squares of these two equations leads to the following quartic equation for the amplitude of the non-dimensional displacement:

\[
\left(\frac{8 \gamma \Lambda^2}{3\pi}\right)^2 + (\mu \Lambda^2)^2 \right) Y_1^4 + \left[2 \mu \Lambda^4 - 2 \mu \Lambda^2 + \frac{128 \gamma \beta \Lambda^4}{9 \pi^2}\right] Y_1^3 + \left(1 - \Lambda^2\right)^2 + \left(\frac{8 \beta \Lambda}{3 \pi}\right)^2 Y_1^2 \\
- (1 + \alpha \Lambda^2)^2 = 0
\]  

(28)

Further, the phase of the response can be obtained by:

\[
\phi_1 = \arctan\left(\frac{- (\gamma Y_1 + \beta) 8 \Lambda^2}{3 \pi (1 - \Lambda^2 - \mu Y_1 \Lambda^2)}\right)
\]  

(29)

The amplitude of the non-dimensional displacement ($Y_1$) is obtained by solving Eq. (28) (via the Matlab function ‘roots’ in this study). This equation leads to four possible solutions; however, only positive real values are physically significant. The stability of each solution is assessed by substituting it into Eq. (14) as an initial condition and checking if the response of the system maintains the calculated response or diverges to a different condition.

The amplitude of the dimensionless dynamic force on the cable can then be obtained from Eq. (20) by considering the difference of two sine waves with the same frequency:

\[
f_{\text{max}} = \frac{\sqrt{1 + Y_1^2 - 2 Y_1 \cos \phi_1}}{\Lambda^2}
\]  

(30)

Finally, the maximum tension at the top and the maximum and minimum tension at the bottom of the cable can be obtained by combining the static weight of the system and the amplitude of the dynamic force (via Eq. (19)):

\[
F_{\text{top max}} = (M - \rho V + m_s L) g + \frac{E A W_0}{L} \sqrt{1 + Y_1^2 - 2 Y_1 \cos \phi_1}
\]  

(31a)

\[
F_{\text{bot max}} = (M - \rho V) g + \frac{E A W_0}{L} \sqrt{1 + Y_1^2 - 2 Y_1 \cos \phi_1}
\]  

(31b)
\[ F_{\text{bot,min}} = (M - \rho V)g - \frac{EAW_0}{L} \sqrt{1 + Y_1^2 - 2Y_1 \cos \phi_1} \] (31c)

The maximum forces must then be compared to the structural limits of the cable and payload and the minimum force should always be positive to comply with the safety requirements during real operations.

### 4.2.2 Solution considering the first and third harmonic components

The quadratic term \(|y'|y'\) is an anti-symmetric nonlinear function that introduces odd harmonics in the response of the system when a sinusoidal input is considered. In order to increase the accuracy for the response of the system, and also to be able to represent super-harmonic resonances (as presented in (Tommasini et al., 2018)), these higher harmonics can be included in the assumed solution for \(y\).

Considering the first and third harmonics, the non-dimensional displacement is represented by:

\[ y = Y_1 \cos(\Lambda \tau + \phi_1) + Y_3 \cos(3\Lambda \tau + \phi_3) \] (32)

Substitution of Eq. (32) into Eq. (14) results in difficulties in representing the quadratic nonlinear term as a Fourier series, as presented in Eq. (24). This difficulty can be avoided by approximating the quadratic term by a cubic polynomial:

\[ y'|y'| \approx A_1 y' + A_3 y'^3 \] (33)

As the function \(|y'|y'|\) does not have smooth derivatives, the coefficients \(A_i\) cannot be calculated directly via a Taylor series. Alternatively, these coefficients may be found via a least squares fit of the nonlinear function. Considering the amplitude of the first harmonic as the maximum non-dimensional velocity to make the fit, the coefficients \(A_i\) can be obtained by the following minimization problem:

\[ \min: \int_0^{\Lambda Y_1} (y'^2 - A_1 y' - A_3 y'^3)^2 \, dy' \] (34)

where only the positive non-dimensional velocities are needed in the evaluation of the integral due to the anti-symmetry of \(|y'|y'|\).

Furthermore, the amplitude of the response of the non-dimensional displacement is assumed to be equal to the amplitude of only the first harmonic term \((Y = Y_1)\), relying on the fact that, whilst higher harmonics contribute to the total force on the cable, they have a negligible contribution to the total displacement of the payload. Therefore, the non-dimensional equation of motion becomes:

\[ (1 + \mu Y_1) y'' + \left( Y + \frac{\beta}{Y_1} \right) \left( \frac{5\Lambda Y_1}{16} y' + \frac{35}{48\Lambda Y_1} y'^3 \right) + y = (1 + \alpha \Lambda^2) \cos(\Lambda \tau) \] (35)

Substituting Eq. (32) into Eq. (35) and applying a similar procedure as presented above for the solution with only one harmonic component, it is possible to obtain a nonlinear algebraic system of equations for the amplitudes and phases of each harmonic component.
\[ 64(1 - \lambda^2 - \mu \lambda^2 Y_1)Y_1 - 105(\gamma Y_1 + \beta)\lambda Y_3 \sin(3\phi_1 - \phi_3) - 64(1 + \alpha \lambda^2) \cos(\phi_1) = 0 \]  
(36a)

\[ 55(\gamma Y_1 + \beta)\lambda^2 Y_1^2 + 630(\gamma Y_1 + \beta)\lambda^2 Y_3^2 - 105(\gamma Y_1 + \beta)\lambda^2 Y_1 Y_3 \cos(3\phi_1 - \phi_3) \]
\[ + 64(1 + \alpha \lambda^2) \sin(\phi_1) = 0 \]  
(36b)

\[ 192(1 - 9\lambda^2 - 9\mu \lambda^2 Y_1)Y_3 + 35(\gamma Y_1 + \beta)\lambda^2 Y_1 \sin(3\phi_1 - \phi_3) = 0 \]  
(36c)

\[ 162Y_1^2 Y_3 + 567Y_3^3 - 7Y_1^3 \cos(3\phi_1 - \phi_3) = 0 \]  
(36d)

This nonlinear system of equations in \( Y_1, \phi_1, Y_3 \) and \( \phi_3 \) is solved numerically by using the Matlab function \( \text{fsolve} \), which is based on the interior trust region method (Coleman and Li, 1996). The maximum dynamic cable force is then found from Eqs. (19) and (20).

5. Numerical results and discussion

Aiming to evaluate the influence of the variable hydrodynamic coefficients on the dynamics of the lifting system, the subsea manifold presented in Section 2 is considered in combination with a 8.25 inch Braid Optimized for Bending (BOB®) 12x12 strand rope (Cortland, n.d.), as an example, as presented in Table 4. The choice of a synthetic rope construction in cases such as this is because of its lower submerged weight in comparison to traditional steel wire cables, thus allowing greater structural capacity to withstand dynamic loads in ultra-deep water operations.

Table 4: Geometric data for the cable used in this study (Cortland, n.d.).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass per unit length (m)</td>
<td>36.3 kg/m</td>
</tr>
<tr>
<td>Equivalent submerged mass per unit length (m&lt;sub&gt;e&lt;/sub&gt;)</td>
<td>6.5 kg/m</td>
</tr>
<tr>
<td>Axial rigidity (EA)</td>
<td>584 MN</td>
</tr>
<tr>
<td>Safe working load</td>
<td>7300 kN</td>
</tr>
</tbody>
</table>

Also, the results presented in the sequence consider a maximum water depth of 2200 m, which is a typical condition in the Pre-Salt fields, in Brazil, and cover most of the worldwide scenarios. Finally, the solutions obtained by the Harmonic Balance Method considering the first harmonic are referred to as HBM 1, while the results obtained by considering the first and third harmonics are referred to as HBM 1,3.

5.1 Overview of the dynamic response of the system

The dynamic response of the system, as a function of the water depth, for the different solution procedures and considering an input amplitude equal to 1.0 m and input period equal to 0.5 rad/s is presented in Figure 4. The maximum dynamic force in the steady state regime is presented in Figure 4(a). The results obtained by the KC-dependent hydrodynamic coefficient models clearly deviate from the results obtained by the constant coefficient model. The peak dynamic load in the system is higher for the KC-dependent models (1344 kN for the iterative Runge–Kutta, 1333 kN for the HBM 1,3, and 1284 kN for the HBM 1 compared to 1047 kN for the constant coefficients model) and it occurs in shallower depths (1020 m for the variable
coefficients compared to 1115 m for the constant coefficients). On the other hand, the dynamic loads in the system in shallow waters and in the 1:3 super-harmonic resonance zone (around 175 m depth) are higher for the constant coefficients model in comparison to the variable coefficients models. Further, the results obtained by the different solutions of the variable hydrodynamic coefficients model are in agreement with each other (error of the peak dynamic force compared to the Iterative Runge-Kutta equal to −0.82 % for HBM 1.3 and −4.46 % for HBM 1), except for HBM 1 in the super-harmonic resonance, since this solution is not able to predict the amplifications of loads in this zone. For comparison, the static force due to the submerged weight of the payload is 1449 kN and the total submerged weight of the cable in 2200 m depth is 141 kN. Further, the general behaviour presented above is also observed in Figure 4(b) and (c), where the maximum dynamic displacement and the maximum velocity are presented.

Reasons for this behaviour can be addressed by analysing the graphs of the added mass and drag coefficient as a function of the depth, as presented respectively in Figure 4(d) and Figure 4(e). The added mass coefficient is linearly proportional to the amplitude of the response, so when the system reaches resonance, \( W \) (and consequently \( C_a \)) reach a maximum. This higher added mass coefficient, increases the effective mass of the system, and thus brings the resonance to shallower depths, as seen previously in Figure 4(a).

Additionally, as the drag coefficient is inversely proportional to the amplitude of the response, \( C_d \) reaches a minimum in the resonance zone. Thus, the damping of the system is reduced and the maximum displacement and dynamic cable force in the resonance zone are higher in comparison to the constant coefficient model, as presented in Figure 4(a). This is not the only factor that leads to higher loads for the variable coefficients model. According to Eq. (8), the dynamic load on the cable depends on the stiffness and elongation of the cable. So, when the resonance occurs at shallower depth (due to an increase in the added mass), the stiffness of the cable is higher due to the shorter suspended length of cable, leading to higher forces on the cable even for the same displacement. This effect can be observed by comparing the higher deviation between the variable and constant coefficients models in the dynamic force (Figure 4(a)) than in the dynamic displacement (Figure 4(b)), which is only due to the lower drag coefficient.

Also, the variation of the added mass coefficient not only affects the depth at which the resonance occurs but also creates a slight bending to the left in the dynamic response of the system at the resonance, which is a typical feature of a softening nonlinear system. In contrast to the more common case, where the stiffness is responsible for the nonlinear features of the system, in this problem, it is the inertial term that presents nonlinear behaviour, increasing its value as the amplitude of the response increases.
Figure 4: Dynamics of the subsea lifting for different solutions considering $W_0 = 1.0 \text{ m}$ and $\Omega = 0.5 \text{ rad/s}$: (a) maximum dynamic load in the cable, (b) maximum dynamic displacement ($\dot{w} = W_0 y$), (c) maximum velocity ($\ddot{w} = \omega n_0 W_0 y'$), (d) added mass coefficient, and (e) drag coefficient.
5.2 Influence of the input amplitude and frequency

The influence of the amplitude of the input on the dynamic forces in the cable is presented in Figure 5, where values of 0.1, 0.5, 1.0, and 1.5 m are used. The frequency of the input is kept constant at 0.5 rad/s. For clarity of the plots, in this Section, only the results obtained via the iterative Runge-Kutta method are presented for the variable coefficients model, since the solutions obtained by using the harmonic balance method deviate by less than 5% from the results from the iterative Runge-Kutta method (as presented in Section 5.1).

![Figure 5: Maximum dynamic force in the cable as a function of depth for different input amplitudes. Results obtained considering $\Omega = 0.5$ rad/s and (a) $W_0 = 0.1$ m, (b) $W_0 = 0.5$ m, (c) $W_0 = 1.0$ m, (d) $W_0 = 1.5$ m.](image)

Considering the variable hydrodynamic coefficients models, as the input amplitude increases, the maximum dynamic loads increase relative to the constant coefficients model results and they occur at shallower depths. The increase in the input amplitude increases the amplitude of the dynamic displacement of the payload, which reduces the drag coefficient and increases the added mass coefficient, as described in Section 5.1. Also, as presented previously, not only the reduction of the damping coefficient, but also the increased stiffness of the cable in shallower depths (when the cable is shorter), contribute to the higher dynamic loads when the resonance peak occurs in shallower depths.

The influence of the frequency of the input is highlighted in Figure 6 considering the frequencies 0.4, 0.6,
0.8, and 1.0 rad/s, while the amplitude is equal to 1.0 m. In this case, the increase of the input frequency only brings the resonance to shallower depths in a similar way for both the variable and the constant coefficient models. The ratio of the maximum dynamic force at the resonance for the variable coefficients models and for the constant coefficient model are kept nearly constant when the input frequency is varied.

Another feature of the variable hydrodynamic coefficients model is the prediction of undamped resonance period curves that are dependent on the input, differently from the traditional approach considering constant hydrodynamic coefficients. The undamped resonance period of the cable-equipment system considering the KC-dependent hydrodynamic coefficient is given by:

$$T_{\text{resonance}} = 2\pi \sqrt{\frac{\left(M + \rho V \left(C_{a0} + \frac{2\pi C_{a1} W}{D} \right) + \frac{1}{3} mL \right)}{EA}}$$

and is presented in Figure 7 considering various input frequencies and amplitudes.
Figure 7: Undamped resonance period as a function of the depth considering various input amplitudes and (a) $\Omega = 0.5$ rad/s and (b) $\Omega = 1.0$ rad/s.

In this case, higher input amplitudes lead to higher values of $W$ and thus higher values for the resonance period. The influence of the frequency is presented in the form of a further increase in the values of the resonance period in the depths when the system matches the input frequency. From a different perspective, for a given input frequency, the resonance occurs at shallower depths when the input amplitude increases, in accordance with Figure 5. Furthermore, if the input frequency is higher, the resonance occurs in shallower depths, in accordance with Figure 6.

Differently from the constant coefficients model, the curves for the resonance period of the system can only be obtained after actually solving the equation of motion in the variable coefficients case, since the amplitude of the response is necessary in this approach.

5.3 Dynamics of the system under general conditions

Sections 5.1 and 5.2 presented the results for the dynamics of the system in specific conditions for the input. The aim of this section is to extend the results for more general scenarios, so deeper insight about the behaviour of the system can be obtained. To this end, the non-dimensional response of the system is presented as a function of the non-dimensional variables: $\alpha$, $\beta$, $\gamma$, $\mu$, and $\Lambda$. However, due to the high number of independent variables, some assumptions are taken to simplify the study.

The first one is to consider the product $\alpha \Lambda^2$ (equal to $mL^2 \Omega^2 / 6EA$) negligible in Eq. (14). In the example case, the term $\alpha \Lambda^2$ is less than 0.1 when $L < 2200$ m and $\Omega < 1.4$ rad/s (which is representative of most of scenarios) and, thus, the term $(1 + \alpha \Lambda^2)$ can be approximated to unity in Eq. (14). The second assumption is to consider $\mu$ as a function of $\gamma$, which can be obtained by dividing Eq. (17) by Eq. (16):

$$\mu = \left(\frac{4\pi V C_{d_1}}{A_p D C_{d_0}}\right) \gamma$$

Noting that the term inside the brackets in Eq. (38) is constant for a given payload, $\mu$ is linearly proportional to $\gamma$ (i.e., $\mu = 0.9208 \gamma$ for the example manifold). Finally, $\beta$ varies from 0.1547 to 0.1583 when 100 ≤
$L \leq 2200$ for the example considered. So, a constant value of $\beta = 0.1565$ is taken as a simplification.

Based on these assumptions, only $\Lambda$ and $\gamma$ are considered as independent variables when evaluating the non-dimensional response of this particular system.

The non-dimensional force as a function of $\Lambda$ and $\gamma$ obtained via the iterative Runge-Kutta solution is presented in Figure 8. For higher values of $\gamma$ (i.e. higher input amplitude, according to Eq. (16)), the peak value of the non-dimensional force moves to lower values of the frequency ratio, which translates to shallower depths as per Eq. (11). This is in agreement with the discussion presented in Section 5.2. Further, a ridge is observed for $0.5 < \Lambda < 1.0$ and a saddle occurs at $\Lambda = 0.77$ and $\gamma = 0.18$. This behaviour is due to the combined influence of two effects. The first one is the reduction of the maximum non-dimensional force as $\gamma$ increases, since $\gamma$ controls the damping level of the system according to Eq. (14). This effect governs the dynamics of the system when $\gamma < 0.1$. On the other hand, the increase of $\gamma$ brings the resonance to lower $\Lambda$, which increases the non-dimensional force as it is inversely proportional to $\Lambda^2$, according to Eq. (20). This effect governs the dynamics of the system when $\gamma > 0.2$. These opposing effects on the non-dimensional force as $\gamma$ increases leads to the saddle. Super-harmonic resonance is also evident in Figure 8 when $\Lambda \approx 1/3$ and at other integer fractions shown by downward inflexions of the contour lines. These super-harmonic resonances present amplifications in the non-dimensional force that are not preponderant in comparison to the forces obtained in the fundamental resonance of the system, as illustrated also in Figure 4.

Figure 8: Maximum dimensionless force ($f$) obtained via the iterative Runge-Kutta solution as a function of the dimensionless damping ($\gamma$) and frequency ratio ($\Lambda$). Assuming $\beta = 0.1565$, $\mu = 0.9208\gamma$ and $\alpha\Lambda^2 = 0$.

The non-dimensional force obtained from the constant coefficients model and for the variable coefficients model via the Harmonic Balance Method are presented in Figure 9, along with their percentage deviation in comparison to the results from the variable coefficients model using the iterative Runge-Kutta method. The results obtained by the constant coefficient model (Figure 9(a-b)) show clear deviations from the results for the variable coefficients model obtained via the iterative Runge-Kutta method, which is highlighted by non-dimensional forces under-predicted by up to 69% when $\Lambda < 1$ or over-predicted by up to 94% when $\Lambda > 1$. 
The non-dimensional force obtained via HBM 1 (Figure 9(c-d)) shows very small errors in comparison to the iterative Runge-Kutta solution, except in the super-harmonic resonance zones ($\Lambda \approx 1/3$) where the force can be up to 27% lower. The results obtained by the HBM 1,3 (Figure 9(e-f)) improves the results obtained by the HBM 1 by predicting the dynamic amplifications in the 1:3 super-harmonic resonance ($\Lambda \approx 1/3$), but higher super-harmonic resonances are still not represented, which translates in errors of the order of 9% at...
the 1:5 super-harmonic resonance ($\Lambda \cong 1/5$). Although deviations occur in the super-harmonic resonance zones for the harmonic balance method, the forces in these zones are not preponderant when the system is expected to cross the fundamental resonance zone. In this case, the HBM 1 solution can be used for accurate estimation of the dynamic loads in the system and, due to its analytical form, results are directly obtained with little computational effort. If 1:3 super-harmonic resonances are expected to be important in the definition of the viability of the operation, HBM 1,3 can be considered as an accurate and fast method (up to 50 times faster than the iterative Runge-Kutta method considering 50 periods of simulation and a tolerance of $10^{-3}$ for $Y$).

5.4 Influence of $\mu$ and $\beta$ on the dynamics of the system

Given the high number of independent variables for the system, some of them were kept constant in the above section and the results presented were only valid for the particular manifold presented in Section 2. The aim of this section is then to vary the variables that were kept constant previously in order to understand their influence in the dynamics of the system in different operational scenarios.

The first analysis presents the non-dimensional dynamic force as a function of $\Lambda$ and $\gamma$ by considering different values of the dimensionless amplitude-dependent drag coefficient ($\beta$) and a constant value of $\mu = 0.9208\gamma$ and $\alpha\Lambda^2 = 0$ (the same as in Section 5.3). This study is presented in Figure 10, which assumes $\beta = 0.0782$ (50% less compared to Figure 8) in Figure 10(a) and $\beta = 0.2347$ (50% higher compared to Figure 8) in Figure 10(b).

![Figure 10](image)

Figure 10: Non-dimensional dynamic force obtained by the iterative Runge-Kutta solution. Assuming $\mu = 0.9208\gamma$, $\alpha\Lambda^2 = 0$ and (a) $\beta = 0.0782$, and (b) $\beta = 0.2347$.

The maximum deviation between the results presented in Figure 8 and Figure 10 is less than 10%, except when $\gamma < 0.1$ and $0.8 < \Lambda < 1.2$, where the dimensionless force is increased by up to 38% in Figure 10(a) and reduced by up to 23% in Figure 10(b) in comparison to Figure 8. This is in agreement with the conclusions presented in Section 5.3, where it was shown that the influence of the damping level governed the response of the system near the resonance zone when $\gamma < 0.1$. In addition, the saddle moves to $\gamma = 0.23$...
and $\Lambda = 0.74$ in Figure 10(a) and to $\gamma = 0.12$ and $\Lambda = 0.81$ in Figure 10(b).

The second analysis considers the influence of the dimensionless amplitude-dependent added mass coefficient ($\mu$) on the dimensionless force. The results are obtained by assuming $\beta = 0.1565, \alpha \Lambda^2 = 0$ (the same as in Section 5.3) and (a) $\mu = 0.4604\gamma$ (50% less compared to Figure 8) and (b) $\mu = 1.3812\gamma$ (50% higher compared to Figure 8). In this case, reducing the value of $\mu$ reduces the values of the dimensionless forces, especially on the ridge, where it can be up to 45% less in comparison to the results observed in Figure 8. Also, the ridge is not so steep as in Figure 8 and the saddle moves to $\gamma = 0.41$ and $\Lambda = 0.73$. On the other hand, increasing the value of $\mu$ tends to bring closer the contour lines to the left of the ridge, which is reflected in a steep increase in the non-dimensional forces when the system starts to cross the resonance zone from lower to higher values of $\Lambda$. Furthermore, the saddle moves to $\gamma = 0.09$ and $\Lambda = 0.80$ and the dimensionless force is majorly affected at the resonance zone, where it can be increased by up to 108% in comparison to the results presented in Figure 8.

![Figure 11: Non-dimensional dynamic force obtained by the iterative Runge-Kutta solution. Assuming $\beta = 0.1565, \alpha \Lambda^2 = 0$ and (a) $\mu = 0.4604\gamma$, and (b) $\mu = 1.3812\gamma$.](image)

If even larger values of $\mu$ are considered, the non-dimensional force presents a fold to the left in the resonance zone, as presented in Figure 12. The results in this scenario consider $\beta = 0.1565, \alpha \Lambda^2 = 0, \gamma = 0.3, \mu = 3.6832\gamma$ and are obtained by HBM 1, since the unstable branch of the solution can be directly obtained by the analytical solution of Eq. (28), without the need for more sophisticated methods. According to this figure, when $0.38 < \Lambda < 0.41$, the system has three possible solutions: the stable branches A-B and C-D, and the unstable branch B-C. Points A-B-C-D are the solutions of the system at $\Lambda = 0.38$ and $\Lambda = 0.41$, when the discriminant of Eq. (28) is null (adding one real solution for the system). The stability of each branch is obtained by substituting the solutions of Eq. (28) as initial conditions into Eq. (14) and checking if the response of the system maintains the calculated response or diverges to a different condition, depending on the domain of attraction of each stable solution. In this case, if the system slowly progresses forward in the frequency ratio (e.g. during lowering operations), the non-dimensional force will follow the curve A-B and then jump directly to D. On the other hand, if the system slowly progresses backward in frequency ratio (e.g. during recovering operations), the non-dimensional force will follow curve D-C and
then drop directly to A. This is a typical hysteretic behaviour observed in nonlinear systems. Further details on this behaviour might be found in classical nonlinear dynamics literature, such as Nayfeh and Mook (1995).

Figure 12: Maximum non-dimensional force as a function of the frequency ratio obtained via the HBM 1. Assuming $\beta = 0.1565, \alpha \Lambda^2 = 0, \gamma = 0.3$ and $\mu = 3.6832 \gamma$.

Based on the results presented in Section 5, it seems clear the importance of considering variable hydrodynamic coefficients to model the dynamics of subsea lifting operations. The inclusion of the amplitude-dependent coefficients in the equation of motion leads to some phenomena that are not observed when considering constant coefficient models. In particular, variations in the natural frequency and damping of the system as functions of the amplitude of the response of the payload are apparent. This leads to shifts in the magnitude and corresponding depths of the maximum cable loads, and may even lead to multiple solutions in certain scenarios, such as presented in Figure 12. These outcomes reinforce the need to accurately predict the hydrodynamic coefficients of subsea structures as a function of the Keulegan-Carpenter number to safely install or recover subsea structures.

Finally, the conclusions obtained in this study were based on a linear law for the added mass coefficient and a reciprocal relation for the drag coefficient as a function of the KC number. In scenarios where the hydrodynamic coefficients follow different trends, deviations in the dynamics of the system might occur. For example, if the added mass decreases with the KC number, the system will act as a hardening nonlinear system and the resonant peak will increase in frequency as amplitude increases. More complex laws for the hydrodynamic coefficients might need specific studies to understand the full dynamical behaviour of the system. Furthermore, comparison of the proposed models with data from real operations or experiments considering the coupled system (vessel, cable and payload) would be of great value for the analysis of the dynamics of the subsea lifting system. This might be considered in the scope of future works.
6. Conclusions

The dynamics of deep water subsea lifting operations considering KC-dependent hydrodynamic coefficients were analysed in this study. Initially, experimental data from the literature for the hydrodynamics coefficients of a typical subsea manifold was presented, indicating a linear trend in the added mass and a shifted reciprocal behaviour in the drag coefficient as a function of the amplitude of oscillation of the body, which was represented by the Keulegan-Carpenter number. Then, the non-dimensional equation of motion for a single degree-of-freedom system representing deep water subsea lifting operations was presented. This equation included two amplitude-dependent terms that, in addition to the traditional quadratic drag term, are responsible for the nonlinear behaviour of the system.

The solution of the nonlinear equation of motion was obtained by two different approaches. The first approach was based on an iterative time domain integration, while the second relied on an analytical technique known as the Harmonic Balance Method. The results obtained for the model with variable hydrodynamic coefficients were then compared with the results found for an equivalent model with constant coefficients.

The first example presented the maximum dynamic force on the cable as a function of the depth. It was shown that the model with variable coefficients predicted a shift in the magnitude and depth where the peak forces occurred, which were due to the variation of the damping and natural frequency of the system as a function of the amplitude of oscillation of the payload. In this case, for a higher amplitude of oscillation of the payload, the drag decreased and the added mass (and natural period) increased, resulting in resonances at shallower depths with higher magnitude. Additionally, it was found that the reduction of the damping and the higher stiffness (due to shorter suspended length of cable) of the system led to increased loads in the resonance zone, when the resonance occurred at shallower depths.

The general behaviour of the system was assessed by analysing the non-dimensional force on the cable as a function of the frequency ratio and non-dimensional damping coefficient. It was noted that the natural resonance zone occurred at lower frequency ratio when the non-dimensional damping was increased, creating a ridge in the dimensionless force map. Furthermore, the magnitude of the dimensionless force in the ridge was found to be influenced in two opposing ways as the dimensionless damping coefficient was increased, which led to the presence of a saddle point in the dimensionless force map. Variations of the nonlinear damping and added mass coefficients could change the location and magnitude of the dimensionless force in the ridge and saddle. Additionally, a fold of the map could occur, leading to multiple solutions in certain zones.

Further, the accuracy of the Harmonic Balance Method solution and of the constant coefficient model in comparison to the iterative time domain integration method was presented. It was found that the use of the constant coefficient model could lead to overestimation of the dimensionless force by up to 94% when the frequency ratio was greater than unity or underestimated by up to 69% when the frequency ratio was lower.
than unity. The Harmonic Balance Method presented negligible error in comparison to the iterative time
domain integration, except in the super-harmonic resonance zones, where maximum errors reached 27%
when considering only the first harmonic component, or 9% when considering both the first and third
harmonic components. However, the Harmonic Balance Method, particularly considering only the first
harmonic, requires much less computation effort than the iterative time domain integration due to its
analytical form, which represents a significant advantage of this method when super-harmonic resonances
are not the critical zones of the operation.

These results have demonstrated the importance of using amplitude-dependent hydrodynamic coefficients to
safely plan and execute deep water subsea lifting operations, since several phenomena introduced by this
nonlinear behaviour are not represented by constant coefficient models.

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