Wake-aerofoil interaction noise control with trailing-edge serrations

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Abstract

The present study experimentally investigates the effective reduction of wake-aerofoil interaction noise in a tandem aerofoil configuration using trailing-edge sawtooth serrations. The far-field noise results clearly show that at the ‘head-on’ wake impingement, the use of sawtooth serrations on the trailing-edge of the front aerofoil can lead to substantial reduction of the wake-aerofoil interaction noise of approximately 10 dB without any noticeable increase at other frequencies and changes to directivity pattern. Subsequently, detailed near-field measurements were carried out to investigate further the flow dynamics associated with the reduction of the interaction noise. The flow field and unsteady loading results suggest that the wake-aerofoil interaction primarily takes place within the first 30% of the chord close to the leading-edge area, while towards the trailing-edge, the flow is influenced more heavily by the boundary layer development. The velocity and wall pressure fluctuation spectra show significant decrease of the energy frequency content of the flow structures over the wake-aerofoil interaction frequency range after the wake impingement, due to the modification of the wake turbulence by trailing-edge serrations. This leads to notable reduction in the unsteady aerodynamic loading, particularly in the lift direction. Further coupled pressure–velocity analyses reveal that the sawtooth serrations modify the energy frequency content associated with the dominant turbulence eddies by spreading it over a wider frequency range. Near-field to far-field pressure coherence also reveals the loss of coherence, and thus abatement of the radiated far-field noise. The experimental results are relevant to practical applications in rotor-stator and outlet guide vanes.

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1. Introduction

Wake-aerofoil interaction noise is generated when the turbulent wake produced from the upstream aerofoil impinges directly upon the downstream aerofoil, and is considered as the primary source of noise for rotating blades, omnipresent in turbines and compressors [1]. With increasing bypass-ratio turbofan engines for improved energy efficiency, the broadband turbulence interaction noise becomes the dominant noise source. Together with the growingly stringent regulatory framework in noise emission level, mitigating turbulence interaction noise, such as wake-aerofoil interaction noise, becomes highly desirable and of great research interests. In the ‘Flightpath 2050’ [2], the European commission envisioned a reduction of 65% noise emission from the present level by 2050. From the perspective of reducing wake-aerofoil interaction noise, extensive research efforts are needed to provide an improved understanding on the flow dynamics and the associated noise generation mechanisms and subsequently, investigate and device effective and practical noise mitigation strategies for engineering applications.

One of the pioneering work in the study of wake-aerofoil interaction noise was performed by Hanson [1], in which he identified that the interaction between the tip of the rear-blade and the front-blade wake constituted the major noise source in counter-rotating propellers. A decade later, Tuncer and Platzer [3] performed computations on the effects of unsteady interference between the front and rear aerofoil in a tandem configuration to explore the rudimentary effects of the relative motion between the two aerofoils. They concluded that altering the relative position of the two aerofoils in tandem led directly to the changes in which the rear aerofoil interacted with the front aerofoil wake and hence, the aerodynamic noise generated during the dynamic interaction. Later, a number of studies have focused on several aspects of the wake-aerofoil interaction phenomenon, such as its aerodynamic performance at low to moderate Reynolds numbers [4, 5, 6], the interaction noise characteristics and effects of aerofoil geometries [7, 8, 9, 10, 11, 12] and the reduction of interaction noise using passive control methods [13, 14, 15, 16, 17], just to name a few.

From these studies, the turbulence interaction noise was found to be broadband in nature and dominant in the lower frequency range than the
aerofoil self-noise. Yet, majority of the previous work have focused on a single aerofoil being subjected to a turbulent free-stream, where the turbulence is artificially generated prior to the interaction. Also, wavy leading-edge configuration, *i.e.* leading-edge serrated aerofoil, has been heavily investigated as a passive mean to reduce the turbulence interaction noise. As Lyu and Azarpeyvand [18] showed from their analytical model, the primary noise reduction mechanism was attributed to the destructive interference of the scattered pressure field, induced by those leading-edge serrations. They have also noted that the optimised noise reduction was closely related to the geometry of the serration and the hydrodynamic characteristics of impinging turbulence. Indeed, in a seminal analytical work by Amiet [19], it was shown that the sound radiated from an aerofoil immersed in a turbulent stream was proportional to the integral length scale of the incoming turbulence, the power spectral density of the velocity fluctuations and the resulting correlation length on the unsteady aerofoil loading. As such, it can be inferred from their analyses that modifying the impinging turbulence, for instance minimizing the turbulence intensity of the flow, could potentially lead to significant reduction in the overall wake-aerofoil interaction noise. Therefore, in the case of wake-aerofoil interaction noise produced from a tandem aerofoil configuration, it is natural to look into the methods in modifying the wake turbulence originated from the front aerofoil.

Among different methods in manipulating the aerofoil wake flow [20, 21, 22, 23, 24, 25], the application of passive trailing-edge serrations on an aerofoil has shown promising effects in altering the turbulence characteristics in the wake. One of the very first attempts to understand the effect of trailing-edge serration on the aerofoil flow was carried out by Gruber et al. [20], who performed hot-wire anemometry measurements at the trailing-edge of a single-sawtooth serrated aerofoil, and reported that the boundary layer along the tip plane was approximately 15% thicker than that along both the root plane and the unserrated aerofoil. The notable disparity between the tip and root plane of the trailing-edge serrated aerofoil would lead to a highly three-dimensional wake flow behind it and a significantly different turbulence length scale and intensity with respect to the unserrated aerofoil. This was later confirmed by the high fidelity direct numerical simulation study on the trailing-edge serration [21]. By comparing the vortical results with and without the trailing-edge serrations for a NACA 0012 aerofoil, they showed that the spanwise-coherent turbulent structures existed upstream of the trailing-edge serrations were essentially destroyed by the serrations and
reorganized themselves into horseshoe vortices downstream into the wake. Similar observations have also been discussed from the tomographic particle image velocimetry (PIV) measurements conducted by Avallone et al. \cite{22} and León et al. \cite{26} on the trailing-edge serrations. Avallone et al. \cite{22} applied a sharp sawtooth with small wavelength-to-chord ratio serrations to a NACA 0018 aerofoil, and captured pairs of counter-rotating streamwise vortices developed close to the side edge of the serrations. They attributed the emergence and development of the three-dimensional vortical structures to the balancing of pressure fields between the suction and pressure sides through the gap of the side edges, which resulted in the vortex roll-ups. Moreover, the locally induced streamwise vortical structures were found to distort the mean flow into the aerofoil wake. León et al. \cite{26} showed further that these streamwise vortical structures existed predominantly at low to moderate aerofoil angles of attack. Meanwhile, Chong et al. \cite{27} examined in details the turbulence energy content in frequency domain from the wall pressure measurements of a highly instrumented flat plate fitted with a serrated trailing-edge and found that high energy frequency content appeared on the sides of the serrations. However, since most studies examining the application of trailing-edge serrations were motivated by its ability and potential to reduce aerofoil self-noise, which is a significant noise source in many other applications, the experimental and numerical analyses were very much limited to regions in vicinity to the trailing-edge of the aerofoil.

More recently, Liu et al. \cite{23} performed extensive hot-wire anemometry measurements in the wake up to two-chord length downstream of both a NACA 0012 and a NACA 65-(12)10 aerofoil. At a free-stream velocity of 30\,m/s and two angles of attack of 5° and 10°, it was observed that the wake deficit reduced substantially along the tip and root plane in the near-wake region until approximately one-chord downstream. With the increased flow velocity in the wake, the turbulent kinetic energy experienced a drastic drop by almost 50% as compared to the unserrated aerofoil. Inspired by these studies on the trailing-edge serrated aerofoils and their associated modification on the aerofoil wake, utilizing a front aerofoil with serrated trailing-edge to achieve mitigation on the wake-aerofoil interaction noise in a tandem aerofoil configuration warrants more research efforts. In fact, Gruber et al. \cite{28} have earlier conducted an experimental investigation to examine the overall noise reduction of a tandem aerofoil configuration with a combined application of trailing-edge serration on the front aerofoil and leading-edge serration on the rear aerofoil. With both aerofoils at an incidence angle of 5° and sepa-
rated by 0.67 chord length, they studied primarily the effects of the Reynolds number on the extent of noise reduction using this combined serration approach and observed up to 6dB and 8.5dB reduction in the overall aerofoil noise at 20m/s and 80m/s free-stream velocity, respectively. Moreover, they reported briefly that the geometric profile of the trailing-edge serration influenced considerably its ability to mitigate noise. Nevertheless, the relative vertical position between the two aerofoils in their study was determined by tracking the centreline of the wake deficit behind a single, i.e. isolated front, aerofoil. According to the PIV experiments from Liu et al. [29] later, the presence of a second aerofoil in the tandem aerofoil configuration tends to induce a greater wake deflection angle than the case for a single aerofoil.

To authors’ knowledge, there has not been any elaborated experimental or numerical studies exploring the performance of different trailing-edge serrations in reducing the wake-aerofoil interaction noise and the underlying physical mechanisms through extensive near-field velocity and wall pressure measurements. Therefore, the present study aims to provide a better and more complete understanding of the wake-aerofoil noise control characteristics by applying trailing-edge serrations to the front aerofoil in the tandem aerofoil configuration. The study first carries out a series of experiments to locate and identify the relative position between the two aerofoils, where the wake-aerofoil interaction is fully captured. Then extensive near-field pressure and velocity measurements are conducted to shed more light on the flow dynamics and physical phenomenon leading to the changes in the far-field noise. The remainder of the paper is organised as follows: in Section 2 details on the experimental set-up including the tandem aerofoil arrangement, the trailing-edge serration geometries and the measurement approaches will be introduced; in Section 3 the far-field noise spectra and subsequently, the associated wall pressure and velocity measurements elucidating the dynamics of the wake-aerofoil interaction will be discussed; lastly in Section 4, the primary findings from the experiments will be summarised.

2. Experimental Set-up

2.1. Wind tunnel facility and tandem aerofoil setup

The experiments were carried out in the closed-loop, open-jet aeroacoustic wind tunnel facility at University of Bristol, which is fully anechoic above 160Hz. Measuring 775mm ($H$)× 500mm ($W$) at the exit, the contraction
nozzle is capable of delivering a uniform flow up to 40m/s with a turbulence intensity of approximately 0.2%. More details on the flow and acoustic characteristics of the wind tunnel facility can be found in Mayer et al. [30]. As shown in Fig. 1, the two aerofoils were mounted in a tandem configuration, i.e. one behind another with reference to the streamwise direction, on two side-plates, which were attached securely to the nozzle exit. Both aerofoils have a chord length of $c=150\text{mm}$ for the baseline trailing-edge configuration, i.e. with straight trailing-edge. The leading-edge of the front aerofoil is approximately $1.3c$ downstream of the nozzle exit, where the incoming free-stream flow remains uniform [30]. The free-stream velocity was maintained at $U_\infty=25\text{m/s}$ with a constant temperature of $20^\circ\text{C}$, corresponding to a Reynolds number of $Re_c=250,000$. The front aerofoil was tripped at both side at 15% chord from the leading-edge, to ensure turbulent boundary layers over the trailing-edge. In order to achieve a ‘head-on’ impingement of the wake upon the rear aerofoil and to identify the location of maximum interaction noise, the rear aerofoil, a NACA 65-710, was mounted on a separate pair of adjustable side-plates with linear screws to accurately control the vertical movement, while the front aerofoil, a NACA 65-(12)10 was fixed with the quarter-chord aligned with the center of the nozzle exit (see Fig. 1). This is inspired by the previous study from the authors (see [23]), which they have shown that the deflection of wake profile is modified by the presence of a second aerofoil in tandem.
Both aerofoils were positioned at an angle of incidence, $\alpha = 10^\circ$, which $\alpha$ is defined as the magnitude of the incidence angle between the aerofoil chord line and the free-stream velocity direction for both the front and rear aerofoils, as shown in Fig. 2(b). It was found from the previous study [29] that at this angle of incidence, the aerofoil operates close to its maximum aerodynamic performance with a relatively strong wake, which facilitates the present investigation. The far-field directivity angle ($\theta$), also shown in Fig. 2(b), is defined with respect to streamwise direction and the leading-edge of the rear aerofoil such that $\theta = 90^\circ$ denotes the far-field microphone directly above the leading-edge of the rear aerofoil, perpendicular to the free-stream direction. Furthermore, the relative horizontal and vertical distances between the trailing-edge of the front aerofoil and the leading-edge of the rear aerofoil are denoted as $x_g$ and $y_g$, see Fig. 2(b). To facilitate the subsequent discussion, two sets of coordinate systems ($x, y$ and $x', y'$) were employed. The ($x, y$) coordinate system positioned at the leading-edge of the rear aerofoil, see Fig. 2(a), will be used to locate the position on the rear aerofoil, and the ($x', y'$) coordinate system, positioned at the trailing-edge of the front aerofoil, see Fig. 2(b), will be used to denote specifically the wake locations in between the aerofoil gaps. In the present study, two horizontal gap distances, $x_g/c = 0.3$ and 0.5, were investigated, similar to those used in Gruber et al. [28], where turbulence interaction noise had been clearly observed. To fully capture the effect of rear aerofoil interaction with different parts of the front aerofoil wake flow field, the rear aerofoil was traversed vertically over a vertical gap range of $y_g$ from $0c$ to $0.32c$ with a
step size of 0.013c.

2.2. Aerofoil trailing-edge serrations

As identified by several studies [27, 31, 32, 33], two serration geometric parameters, namely the $h/\delta$ and $h/\lambda$, are critical in reducing trailing-edge noise effectively, where $\delta$ is trailing-edge boundary layer thickness, $h$ and $\lambda$ represent the half amplitude and wavelength of the serration profiles, respectively, as shown in Fig. 3. According to Howe [31] and Gruber et al. [32], noise reduction was observed when $h/\lambda \leq 2$ and $h/\delta \geq 0.5$. Nevertheless, majority of these studies have concerned with the trailing-edge noise from only an isolated aerofoil. To examine the effect of different serration geometries on the wake interaction noise of the tandem aerofoil configuration, two serrated profiles were chosen in addition to the baseline (i.e. straight trailing-edge) in the present study. As can be seen in Fig. 3, both sawtooth profiles have an identical amplitude of $2h=30\text{mm}$, which corresponds to an $h/\delta \approx 1.25$ (the trailing-edge boundary layer thickness was estimated to be approximately 12mm based on Liu et al. [29]). The sharp sawtooth has a wavelength of $\lambda=9\text{mm}$ and hence, a serration edge angle of $\alpha_s=8.53^\circ$ ($h/\lambda=1.67$), similar to that used in Gruber et al. [32], while the wide sawtooth is of a wavelength $\lambda=24.8\text{mm}$ with a corresponding serration edge angle of $\alpha_s=22.5^\circ$ ($h/\lambda=0.6$).

The baseline and serrations were designed as add-on inserts into the $15\times 0.8\text{mm}$ slot of the front aerofoil along the aerofoil span. Thus, the baseline is a flat plat with half of the serration amplitude ($h$) such that the total area as well as the mean chord length (chord length averaged over one serration wavelength) between the baseline and the serrated cases are kept the same.
Table 1: Pressure measurement locations relative to the chord x/c on both the suction (left) and the pressure (right) sides of the rear aerofoil.

<table>
<thead>
<tr>
<th>Suction side</th>
<th>Pressure side</th>
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<tbody>
<tr>
<td>port number</td>
<td>x/c</td>
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<tr>
<td>s1</td>
<td>0.013</td>
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<tr>
<td>s2</td>
<td>0.033</td>
</tr>
<tr>
<td>s3</td>
<td>0.053</td>
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<tr>
<td>s4</td>
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<td>s5</td>
<td>0.113</td>
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<tr>
<td>s6</td>
<td>0.153</td>
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<tr>
<td>s7</td>
<td>0.193</td>
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<tr>
<td>s8</td>
<td>0.233</td>
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Figure 4: Schematic of the remote sensor configuration used in the present experiment, with its essential components annotated.
2.3. Pressure and velocity measurements

Far-field wake-aerofoil interaction noise was measured with an overhead microphone array (see Fig. 1) consisting of 20 G.R.A.S. 40PL free-field microphones that span from $\theta=40^\circ$ to $135^\circ$ at $5^\circ$ interval at an arc distance of 1.7m. These 40PL microphones have a flat sensitivity of 10 mV/Pa between 20Hz to 20kHz with. Each microphone had been calibrated with a G.R.A.S. 42A pistophone prior to the measurements. To obtain simultaneous measurements of the wall pressure fluctuations and far-field noise, a total of twenty remote sensing microphones (Panasonic WM61A condenser microphones) were connected to the 0.4mm diameter pressure ports located on both the suction and pressure sides of the rear aerofoil, as illustrated in Fig. 2(a). Table 1 lists the pressure measurement locations relative to the chord, where the remote sensors are connected to. As shown in the schematic of the remote sensor configuration in Fig. 4, the remote sensing microphones register the wall pressure fluctuations transmitted through the thin brass tube with a 0.4mm diameter pinhole to minimize attenuations at high frequencies [34]. Note that the suction side was populated with more pressure measurement ports to better capture the highly dynamic flow characteristics in the wake-aerofoil interactions. Having a dynamic frequency range from 50Hz to 15kHz, these remote sensing microphones were calibrated according to the procedures described by Mish [35] and Elsahhar et al. [36]. Similar remote sensing technique and the calibration procedures have been used to study the aerofoil trailing-edge noise in static and dynamic conditions [37, 38]. The synchronous measurements were acquired with National Instruments PXIe-4499 data acquisition modules at a sampling rate of $2^{16}$ Hz for 16 seconds and subsequently subjected to Welch function for post-processing to yield the power spectral density (PSD) and sound pressure level (SPL) of the near-field pressure fluctuations and far-field noise, respectively. With a window size of $2^{13}$ and 50% window overlap, the frequency resolution, i.e. bin size, was kept constant at 8Hz for the present experiments. The uncertainty of the power spectral density determined from the wall pressure measurements was estimated to be $\pm 1.5$dB/Hz at 95% confidence interval.

The wake and boundary layer velocity measurements were carried out using Hot-Wire Anemometry (HWA) coupled with Dantec Streamline Pro system. 1D Miniature Wire Sensors (Dantec 55P14 90-degree single wire probe: 5$\mu$m diameter and 1.25mm long platinum-plated tungsten wires) were used for the front aerofoil wake in the tandem aerofoil arrangement due to space constraints. In addition, a Dantec 55P15 built-in boundary layer wire
probe was used to map the boundary layer velocity profiles at the suction side of the rear aerofoil close to the leading-edge. Figure 5 shows the schematics of the hot-wire measurement set-up for between the gap of the tandem aerofoils (Fig. 5(a)) and the boundary layer at the leading-edge of the rear aerofoil (Fig. 5(b)). All the probes were operated by CTA91C10 modules and calibrated by Dantec 54H10 calibrator prior to the measurements. The signals from the hot-wire probes were low-pass filtered by the Streamline Pro acquisition module at 30kHz, and the frequency response was subjected to the standard square-wave test before carrying out any calibration and measurement procedures. At a sampling rate of $2^{16}$ Hz, similar to that of the wall pressure measurements, the resulting velocity was obtained through a fourth-order polynomial and had an estimated uncertainty of ±1.1%, ±3.2% and ±5.0% for the boundary layer, single aerofoil wake and tandem aerofoil wake, respectively [39]. Note that the higher uncertainty in the tandem aerofoil wake measurements is due to a relatively large blockage immediately behind the wire sensors. Lastly, the hot-wire probes were mounted onto a two-axis ThorLabs traverse system covering a length of 300mm in each axis, to accurately traverse through the desired locations in the wake and boundary layer, with a positioning accuracy of ±5µm. Readers are advised to refer to Showkat Ali et al. [24] for more details on the wall pressure and velocity measurements.
3. Results and Discussion

In this section, the results from the far-field noise, near-field unsteady wall pressure and boundary layer flow characteristics will be presented and discussed as follows: firstly, the far-field noise spectra will be used to identify the rear aerofoil location of a full wake-aerofoil interaction and examine the noise-reduction capability when the trailing-edge serrations are applied to the front aerofoil. Secondly, the velocity measurements from the wake and the boundary layer profiles of the tandem aerofoils as well as the wall pressure fluctuations and the decomposed unsteady aerodynamic loading will be presented to provide direct and comprehensive flow field information of the wake-aerofoil interaction. Thirdly, the relation between the near-field unsteady wall pressure and far-field radiated noise will be established through calculating the near- and far-field coherence. Lastly, spectral analyses such as coupled pressure–velocity coherence and cross-correlation will be determined and discussed to develop further insights into the changes of turbulence characteristics in the wake-aerofoil interaction when the trailing-edge serrations are applied. As such, the far-field results can be linked closely to the flow dynamics and physical events taking place in the near-field and a comprehensive understanding on the noise-reduction mechanisms with the use of different trailing-edge serrations on the front aerofoil of the tandem aerofoil configuration can be elucidated.

3.1. Far-field interaction noise and directivity

As the presence of a rear aerofoil will lead to significant changes to the topology of the front aerofoil wake [29], it is thus essential to allow variations in the vertical alignments, $y_g$, with respect to the front aerofoil in order to fully capture the wake. The so-called ‘wake-locking’ motion was achieved through adjusting side-plates with linear screws and conducting a set of preliminary experiments on the far-field noise over a range of vertical gap distances from $y_g/c=0$ to 0.32. Note that $y_g/c=0$ corresponds to the tandem arrangement where the trailing-edge of the front baseline aerofoil aligns horizontally with the leading-edge of the rear aerofoil. Consequently, the rear aerofoil, initially outside of the front aerofoil wake, traversed into the wake region and then out of the wake again. At a fine step of $\Delta y_g/c=0.013$, the ‘head-on’ wake impingement location can be accurately identified and used as the starting point for the subsequent simultaneous near- and far-field analyses as well as velocity measurements to allow insights into the underlying
flow dynamics and noise reduction mechanisms.

To begin with, Fig. 6 shows the magnitude difference in the power spectral density (PSD) of the far-field noise levels measured from the $\theta=90^\circ$ far-field microphone, when the baseline (straight trailing-edge) and the sawtooth serrations are applied to the front aerofoil of the tandem aerofoil arrangement, i.e. $\Delta PSD=PSD_{serrated}-PSD_{baseline}$. Note that while the horizontal gap distance is fixed at either $x_g/c=0.3$ or 0.5, respectively, the rear aerofoil sweeps through the vertical gap distance of $0 \leq y_g/c \leq 0.32$. Here, the far-field noise PSD is defined as:

$$PSD = 10\log_{10}(\phi_{pp}/p_0^2),$$

(1)

and

$$\phi_{pp}(f) = \int_{-\infty}^{\infty} R_{pp}(\tau)e^{-2i\pi f\tau}d\tau,$$

(2)

where $R_{pp}(\tau)$ is the calibrated time sequence of the pressure fluctuations and $p_0=2 \times 10^{-5}$ Pa. Thus, a negative difference indicates noise reduction and vice versa. In general, some extent of noise reduction can be observed from frequencies of approximately $f=200$Hz to 1000Hz, regardless of the horizontal and vertical gap distances. This corresponds to the non-dimensional Helmholtz number range of $k c =0.55$ to 2.75, where $k =2\pi f c/c_0$ with $c_0$ referring to the speed of sound at the present experimental conditions. Note that to facilitate discussion and comparison with the existing literature, the non-dimensional Helmholtz number ($k c$) will be used instead of frequency ($f$) in the subsequent discussion. In addition, for brevity, the terms ‘baseline’, ‘wide sawtooth serration’ and ‘sharp sawtooth serration’ in the discussion refer to the application of straight (unserrated), wide sawtooth and sharp sawtooth serrated trailing-edges on the front aerofoil in the tandem aerofoil arrangement.

As the rear aerofoil traverses gradually into the wake region from $y_g/c=0$, the noise reduction becomes more prominent in both the magnitude and the range of frequencies. For instance, for $x_g/c=0.3$ and $y_g/c=0.13$, as shown in Figs. 6(a) and (b), a maximum noise reduction of above 10dB can be observed over $0.55 \leq k c \leq 2.75$, i.e. $200$Hz $\leq f \leq 1000$Hz. This is encouraging since Gruber et al. [28] earlier identified the $k c$ range between 0.5 to 5 as the region dominated by turbulence interaction noise and $k c>5.5$ by trailing-edge self-noise for tandem aerofoils at a similar Reynolds number. Moreover, comparing between the wide (Figs. 6(a) and (c)) and sharp (Figs. 6(b) and (d)) sawtooth serrations with both $x_g/c=0.3$ and 0.5 aerofoil gap distances, the...
Figure 6: Contour maps of the $\Delta$PSD comparison of the far-field noise between the baseline and (a) wide sawtooth for $x_g/c=0.3$ and $y_g/c=0.13$, (b) sharp sawtooth for $x_g/c=0.3$ and $y_g/c=0.13$, (c) wide sawtooth for $x_g/c=0.5$ and $y_g/c=0.17$ and sharp sawtooth for $x_g/c=0.5$ and $y_g/c=0.17$, respectively. Note that the dashed line indicate vertical location of maximum overall noise reduction.
use of sharp sawtooth serrations on the front aerofoil produces greater noise reduction over a more extended $kc$ range, suggesting a better mitigation performance in wake-aerofoil interaction noise. It will be shown from the wake velocity measurements later (Section 3.2) that the maximum wake deficit location approximately coincides with the maximum overall noise reduction across the range of frequencies investigated. This is also in agreement with the analytical solution from Amiet [19] that the energy frequency content of the incoming turbulence is one of the parameters in determining the radiated turbulence interaction noise of an aerofoil. For the present experiments, these locations are determined to be $y_g/c=0.13$ and $0.17$ for the aerofoil gap distances of $x_g/c=0.3$ and 0.5, respectively, as marked by the dashed line in Fig. 6. Nevertheless, it should be reminded that the ‘head-on’ impingement location was determined from the maximum noise reduction location, which approximates the aerodynamic ‘head-on’ impingement location. Moreover, in a rotor-stator arrangement, the blades move relative to each other, so that it is important to quantify the wake-aerofoil interaction noise over the range of relative positions. When the rear aerofoil traverses further up outside of the front aerofoil wake, the sharp sawtooth serration show consistently a 2 dB to 4 dB noise reduction across the entire frequency range, as opposed to the wide sawtooth serration with negligible reduction or even amplification of the noise level at similar locations. Note that the dotted lines indicate an estimate of wake boundaries, determined from the wake velocity profiles shown in Fig. 11. Within the front aerofoil wake, the minor noise increase is likely due to the modification of the wake turbulence energy content by the application of trailing-edge serrations, as will also be seen later from the velocity fluctuation spectra in Fig. 14.

Figure 7 shows the comparison of the far-field noise spectra for the tandem aerofoils with the baseline, wide sawtooth and sharp sawtooth serrations determined from the $\theta=90^\circ$ far-field microphone directly above the leading-edge of the rear aerofoil, for the aerofoil gap distances of $x_g/c=0.3$, $y_g/c=0.13$ and $x_g/c=0.5$, $y_g/c=0.17$, respectively. Clearly, the noise spectra for the baseline case is primarily characterised by a broadband hump spanning approximately from $kc=0.44$ to 4.4 (i.e. $160Hz \leq f \leq 1900Hz$), followed by a linear decay at higher frequencies. Such a broadband hump, similar to those observed by Gruber et al. [28] and Paruchuri et al. [15, 17] in their study of turbulence/gust-aerofoil interactions, represents the wake-aerofoil interaction noise. Comparing between the two gap distances of $x_g/c=0.3$ and 0.5, the elevated noise level present at $kc<1.0$ for the smaller gap distance becomes
Figure 7: (Colour line) Far-field noise spectra with aerofoil gap distance of (a) \( x_g/c = 0.3 \), \( y_g/c = 0.13 \) and (b) \( x_g/c = 0.5 \), \( y_g/c = 0.17 \), at the maximum noise reduction location, as indicated in Fig. 6. Baseline (---); wide sawtooth (--); sharp sawtooth (-----); background (····).

Figure 8: (Colour line) Overall sound pressure levels and directivity of the interaction noise with aerofoil gap distance of (a) \( x_g/c = 0.3 \), \( y_g/c = 0.13 \) and (b) \( x_g/c = 0.5 \), \( y_g/c = 0.17 \), at the maximum noise reduction location, as indicated in Fig. 6. Baseline (---); wide sawtooth (--); sharp sawtooth (-----).
Figure 9: (Colour line) One-third octave band sound pressure levels and directivity of the interaction noise with aerofoil gap distance of $x_g/c=0.3$ and $y_g/c=0.13$ at $kc=0.7, 1.4, 2.5$ and $4.4$ respectively, as marked in Fig. 6. Baseline (—); wide sawtooth (-----); sharp sawtooth (———).

385 diminished for the larger gap distance. This is expected as turbulent kinetic energy associated with the wake flow has decayed further with the rear aerofoil moving further downstream, which will be shown later from the velocity measurements in Section 3.2. Nevertheless, the use of trailing-edge serrations on the front aerofoil leads to significant reduction in the wake-aerofoil interaction noise for both $x_g/c=0.3$ and 0.5, up to $kc=2.2$ and $kc=3.4$ for the wide and sharp sawtooth serrations, respectively. Moreover, a closer examination of the two serrations reveals that the sharp sawtooth is able to achieve a greater interaction noise reduction than the wide sawtooth, especially beyond $kc=1.0$, as shown in Fig. 7.

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Figure 10: (Colour line) One-third octave band sound pressure levels and directivity of the interaction noise with aerofoil gap distance of $x_g/c=0.5$ and $y_g/c=0.17$ at $kc=0.7$, 1.4, 2.5 and 4.4 respectively, as marked in Fig. 6. Baseline (- - -); wide sawtooth (——); sharp sawtooth (-----).

In addition to the individual noise spectra, the overall sound pressure level (OASPL) and its directivity are determined from all far-field microphones and shown in Fig. 8. The OASPL can be calculated as:

$$OASPL = 10\log_{10}\left(\frac{\int_{f_1}^{f_2} \phi_{pp}(f)df}{p_0^2}\right),$$

where the integration was performed over a frequency range from 100Hz to 8000Hz using trapezoidal rule. Consistent with the individual spectra, substantial overall noise reduction can be obtained with the use of trailing-edge serrations, and the sharp sawtooth, with approximately 10dB noise reduction at all polar angles, outperforms the wide sawtooth serrations with 7dB. At this point, it is useful to note that the trailing-edge serrations appear to be most effective in mitigating interaction noise when the rear aerofoil is being placed at the center, i.e. ‘head-on’, of the front aerofoil wake. Nevertheless, the directivity patterns remain comparable between the baseline and serration cases, of which they do not exhibit much variations with respect
to the polar angles. In order to understand and better differentiate the directivity pattern at different $kc$ numbers, the 1/3 octave band directivity at four different $kc$ numbers for both $x_g/c=0.3$ and 0.5 are presented in Figs. 9 and 10, respectively. These $kc$ numbers are representatives of the central region ($kc=1.4$ and 2.5) as well as the lower and upper limits ($kc=0.7$ and 4.4) of the interaction noise frequency range. Although the axis for the sound pressure level is zoomed in here to clearly show the small variations of noise magnitude in the polar angles, the directivity pattern essentially exhibits a dipolar pattern for all four selected $kc$ numbers [13]. Nevertheless, there exists marked difference between the directivity at low and high frequencies. At $kc=0.7$ and 1.4, the directivity pattern is tilted in the downstream direction ($\theta=180^\circ$) whereas at $kc=2.5$ and 4.4, it is tilted oppositely towards the upstream direction ($\theta=0^\circ$), indicating a upstream shift of the noise propagation direction, possibly due to the noise scattering at high frequencies becoming weaker downstream [18]. Moreover, while the directivity pattern mostly resembles each other between the baseline and trailing-edge serrated tandem aerofoils, the directivity pattern at $kc=0.7$ for the sharp sawtooth serration deviates noticeably from either the baseline or the wide sawtooth serration.

3.2. Characteristics of the wake and boundary layer profiles

Having determined the ‘head-on’ wake impingement location and its associated far-field noise spectra, it is now appropriate to conduct detailed near-field measurements on the flow field to better understand the noise reduction mechanisms from the flow dynamics perspective. First of all, the wake characteristics in between the tandem aerofoils as well as the velocity profiles of the boundary layers at the leading-edge of the rear aerofoil will be presented and discussed. Figure 11 shows the development of the wake velocity and turbulent kinetic energy profiles for the baseline and the sawtooth serration aerofoils, with $x_g/c=0.3$ and 0.5, respectively. Note that for $x_g/c=0.3$, the velocity was measured at two downstream locations of $x'/c=0.12$ and 0.22, while for $x_g/c=0.5$, it was taken at $x'/c=0.32$ and 0.42. This is due to the nature of the 90° hot-wire probe having to extend into the wake from the leading-edge of the rear aerofoil, as can be seen in Fig. 5(a). Moreover, it should be reminded that the both the wake and boundary layer velocity results are obtained from single probe measurements, which were aligned parallel to the free-stream to yield primarily the velocity information in the x-direction, and that $x'$ and $y'$ are the local coordinates designated to the gap region of the tandem aerofoils (see Fig. 2(b)). Expectedly, the wake
flow deflects upward due to the angle of incidence of the front aerofoil, as can be seen from Figs. 11(a) and (b). Comparing the deflection between the single and tandem aerofoil configurations, it is observed that the presence of a rear aerofoil shortly downstream of the front aerofoil leads to noticeably larger deflection of the wake flow profiles (note that the wake measurement of a single aerofoil was not shown here for the sake of brevity). The increase in the upward deflection of the wake profiles in the tandem aerofoils continues to develop downstream until the wake impinges directly upon the rear aerofoil regardless of the horizontal gap distances. This corroborates with the observation from the particle-image velocimetry measurements by Liu et al. [29] that positioning the two aerofoils according to the wake development of a single aerofoil does not necessarily achieve a ‘head-on’ wake impingement in the tandem aerofoil configuration, thus requiring the preliminary measurements considered in the present study covering a range of $y_g/c$ positions. Moreover, comparing the cases of the tandem aerofoils between the baseline and sawtooth serrations, the deficit peak of the wake flow profiles is located at $y'/c=0.062$ for the front aerofoil with baseline trailing-edge (see Fig. 11(a)), whereas it is at $y'/c=0.028$ and 0.004, respectively for the wide and sharp sawtooth serrations.

Returning to the actual wake profiles of the tandem aerofoils, a typical wake profile with velocity deficit peak along the wake centreline can be observed for both the baseline and sawtooth serration aerofoils. As shown in Figs. 11(a) and (b), the wide sawtooth serration experiences the smallest centreline velocity deficit of approximately $U/U_\infty=0.16$ in magnitude immediately downstream of the front aerofoil trailing-edge, followed by 0.20 for the sharp sawtooth serration and 0.38 for the baseline, respectively. Not surprisingly, a smaller velocity deficit, meaning a weaker wake profile, translates to a lower turbulent kinetic energy (TKE), i.e. $\frac{1}{2}(u_t'u_t')/U_\infty^2$, level of the wake flow, which is apparent by comparing the TKE levels between the baseline and the serrated aerofoil cases in Figs. 11(c) and (d). Here, $u_t'$ denotes the measured velocity fluctuations. A closer examination on the two serrations reveals that although the differences in velocity deficit are quantifiable with the wide sawtooth serration having a comparatively smaller deficit at all downstream locations measured, the differences in the TKE levels remain mostly indistinguishable. Even more interesting is the fact that the wake TKE associated with the wide sawtooth serration shows only a single peak at all downstream locations, in contrast to the double-peak shape exhibited by both the baseline and sharp sawtooth serration. Therefore, the two
serration geometries possibly influence the wake flow dynamics in different manners, yet achieving a comparable reduction in the TKE levels prior to impingement upon the rear aerofoil.

Following the wake impingement, a complex flow field develops over the suction side of the rear aerofoil. Figures 12 and 13 show the boundary layer velocity profiles developed at the leading-edge of the rear aerofoils and their corresponding turbulent kinetic energy profiles for the two aerofoil gap distances of \( x_g/c = 0.3 \) and 0.5, respectively. The boundary layers were measured at \( x/c = 0.013 \) and 0.113 chordwise locations on the suction side of the rear aerofoil starting from \( y/c = 0.007 \) away from the aerofoil surface. These measurement locations correspond to the first (s1) and fifth (s5) remote sensors such that it allows pressure–velocity correlation and coherence analyses, which will be discussed later in Section 3.5. Close to the leading-edge, as seen in Figs. 12(a) and (c), the boundary layers for the baseline and serrated aerofoil cases all experience a rapid acceleration and velocity overshoot close to the aerofoil surface before gradually decreasing back to the freestream velocity \([40]\). For instance, with \( x_g/c = 0.3 \), the maximum velocity overshoot is approximately \( U/U_\infty = 1.80 \) at \( y/c = 0.008 \) for the wide sawtooth while those for the sharp sawtooth and the baseline are \( U/U_\infty = 1.72 \) and 1.60 at \( y/c = 0.012 \) and 0.013, respectively. Moreover, increasing the horizontal gap distance from \( x_g/c = 0.3 \) to 0.5 does not alter the velocity profile trend, instead the overshoots reduce slightly for all cases, as their wake strength decays over the downstream distance. More importantly, the differences in the initial development of the boundary layers between the baseline and the sharp and wide sawtooth serrations are consistent with the wake velocity measurements, where the wide sawtooth exhibits the smallest wake velocity deficit with the least upward deflection, as compared to the baseline and sharp sawtooth cases.

Moving further downstream to \( x/c = 0.113 \), the development of the boundary layer profiles becomes even more complex. As can be seen from Fig. 12(c), a region resembling an additional, small boundary layer emerges from the aerofoil surface, extending up to approximately \( y/c = 0.02 \), for both the baseline and serrated aerofoil cases. The experiments by Squire \([40]\) on the wake-boundary-layer interactions under various wake characteristics and relative aerofoil positions confirmed the emergence of a ‘naturally’ developed boundary layer on the aerofoil surface, underneath the wake-influenced profile, as the wake strength diminished and subsequently the actual boundary layer began to interact with the wake flow over the downstream distance,
Figure 11: (Colour line) Normalised wake velocity profiles ($U/U_\infty$) of tandem aerofoils with aerofoil gap distance of (a) $x_g/c=0.3$, $y_g/c=0.13$ and (b) $x_g/c=0.5$, $y_g/c=0.17$ and normalised turbulent kinetic energy ($TKE/U_\infty^2$) with aerofoil gap distance of (c) $x_g/c=0.3$, $y_g/c=0.13$ and (d) $x_g/c=0.5$, $y_g/c=0.17$. Baseline (---); wide sawtooth (---); sharp sawtooth (-----).
Figure 12: (Colour line) Normalised velocity profiles ($U/U_\infty$) of the boundary layer at the leading-edge of the rear aerofoil at (a) $x/c=0.013$ and (b) $x/c=0.113$ with aerofoil gap distance of $x_g/c=0.3$, $y_g/c=0.13$ and (c) $x/c=0.013$ and (d) $x/c=0.113$ with aerofoil gap distance of $x_g/c=0.5$, $y_g/c=0.17$, respectively. Baseline (-----); wide sawtooth (- - -); sharp sawtooth (.....).
Figure 13: (Colour line) Normalised turbulent kinetic energy profiles ($TKE/U_\infty^2$) of the boundary layer at the leading-edge of the rear aerofoil at (a) $x/c=0.013$ and (b) $x/c=0.113$ with aerofoil gap distance of $x_g/c=0.3$, $y_g/c=0.13$ and (c) $x/c=0.013$ and (d) $x/c=0.113$ with aerofoil gap distance of $x_g/c=0.5$, $y_g/c=0.17$, respectively. Baseline (---); wide sawtooth (--.--); sharp sawtooth (-----).
which is in good agreement with the behaviour of the boundary layer velocity profiles observed in the present experiments. Note that the term ‘naturally developed boundary layer’ here is used to differentiate between the profile induced by the wake flow and that originated from the leading-edge of the aerofoil, rather than to suggest that the boundary layer is developed without the presence of wake flow. As a result of the small ‘natural’ boundary layer observed in Fig. 12(c), the boundary layer profile indicates that a transition from a wake-dominant to a flow-developed boundary layer has already been taking place at approximately 10% of the chord. In another word, the source for turbulence interaction noise is expected to concentrate around the leading-edge area of the rear aerofoil where the wake flow remains sufficiently influential in the present experiments. Although similar behaviour in the boundary layer velocity profile was not captured for the aerofoil gap distance of $x_g/c=0.5$, as shown in Fig. 12(d), it is clear that the velocity profile is evolving into the shape of a standard boundary layer as the effect from the wake flow wears off.

In terms of the turbulent kinetic energy levels of the flow over the suction side of the rear aerofoil, it is worthwhile to highlight that firstly for $x_g/c=0.3$, shown in Figs. 13(a) and (b), both the wide and sharp sawtooth serrations attain significant reduction in TKE levels compared to that of the baseline aerofoil case in the area close the leading-edge location, i.e, $x/c=0.013$. In particular, the sharp sawtooth serration is at one third and the wide sawtooth serration half of the baseline TKE level, suggesting possibly a notable reduction in the wall pressure fluctuations, since the previous studies by Showkat Ali et al. [24] and Afshari et al. [25] have shown that the velocity fluctuations within the boundary layer are closely related to that of the wall pressure fluctuations. However, as boundary layer flow convects downstream, the TKE levels grow to be comparable for all cases. Secondly, for $x_g/c=0.5$ shown in Figs. 13(c) and (d), while the sharp sawtooth serration maintains its ability to reduce the TKE level in comparison to the baseline aerofoil, the wide sawtooth serration shows a drastic change by exceeding the TKE level of the baseline by approximately 12%. Nevertheless, this is still within expectation as the ‘wake-induced’ boundary layer flow experiences substantially greater acceleration at the leading-edge area of the rear aerofoil for the wide sawtooth serration than that for the baseline and the sharp sawtooth serration aerofoils, as observed from Fig. 12(c), which then leads to a greater TKE level for the wide sawtooth serration when it is normalised with the free-stream velocity, $U_\infty$. 


Figure 14: Contour maps of the differences in the energy frequency content, $\Delta PSD_u$, of the velocity fluctuations between the baseline aerofoil and the wide and sharp sawtooth serration aerofoils for (a) the wide sawtooth and (b) the sharp sawtooth with $x_g/c=0.3$, $y_g/c=0.13$ and (c) the wide sawtooth and (d) the sharp sawtooth with $x_g/c=0.5$, $y_g/c=0.17$. 
The turbulent kinetic energy level provides information on the overall turbulence energy contents of the boundary layer flow. To gain further insight into the changes caused to the energy frequency content of the flow structures, the differences in the energy frequency content ($\Delta PSD_u$) between the two sawtooth serrations and the baseline aerofoil cases are calculated and presented in Fig. 14. The $\Delta PSD_u$ results are presented for the measurement location immediately downstream of the leading-edge of the rear aerofoil at $x/c=0.013$, and the gap distances of $x_g/c=0.3$ and 0.5. Note that similar to the far-field noise spectra, $\Delta PSD_u = PSD_{u,serrated} - PSD_{u,baseline}$, where $PSD_u = 10\log_{10}(\phi_{uu}/U_\infty^2)$ and $\phi_{uu}(f) = \int_{-\infty}^{\infty} R_{uu}(\tau)e^{-2\pi f\tau}d\tau$. Thus, a negative value indicates reduction in the energy content. As can be seen clearly from Figs. 14(a) and (b), the use of both sawtooth serrations on the trailing-edge of the front aerofoil can lead to a reduction of the energy content of the velocity fluctuations by more than 15dB/Hz over the entire frequency range investigated below wall distance of $y/c=0.016$. The reduction extends up to approximately $y/c=0.03$ over the frequency range of $0.6<kc<3.4$, which matches generally well with the broadband hump frequency range observed in the far-field noise spectra in Fig. 7. Meanwhile, a significant increase in the energy frequency content of the velocity fluctuations can be observed at higher frequencies beyond $kc=3.5$ from $y/c=0.016$ to 0.045. Nevertheless, the overall increase in the velocity fluctuation PSD is confined to the wall distance of 0.018<$y/c<0.045$, and is considerably smaller than its reduction in the wake-induced boundary layer. For the larger gap distance of $x_g/c=0.5$, the reduction in the velocity fluctuation PSD is observed almost everywhere within the wake-induced boundary layer for both the wide and sharp sawtooth serration aerofoils. It should be mentioned that results from the wake and boundary layer energy frequency content will be constantly referred to in the following section to help better relate wall pressure fluctuations to the velocity fields.

3.3. Aerodynamic loading on the rear aerofoil

With an in-depth knowledge on the wake and boundary layer flow development in tandem aerofoil configuration, it is now useful to examine the wall pressure fluctuations, in order to understand the aerodynamic loading on the aerofoil and correlate it with the flow development and far-field noise results. Figures 15 and 16 depict the wall pressure fluctuation PSD along both the suction and pressure sides of the rear aerofoil for the aerofoil gap distances of $x_g/c=0.3$ and 0.5, respectively. Note that for clarity, the figures are arranged...
Figure 15: (Colour line) Power spectral density of the wall pressure fluctuations with aerofoil gap distance of \( x_g/c = 0.3 \) and \( y_g/c = 0.13 \). Locations of the wall pressure transducers are indicated for ease of reference. Baseline (---); wide sawtooth (- - -); sharp sawtooth (......).
in a way that it starts from the remote sensor location furthest away from the leading-edge at the pressure side of the rear aerofoil and approaches the leading-edge before subsequently moving downstream on the suction side, i.e. in a clockwise sense of rotation with reference to the quarter chord.

At a first glance, the use of both the sharp and wide sawtooth serrations on the front aerofoil in the tandem aerofoil configuration appears to be effective in reducing the wall pressure fluctuation PSD around the leading-edge of the rear aerofoil for $x_g/c=0.3$, as shown in Fig. 15. The distinct broadband hump in the non-dimensional frequency range of approximately $0.6 \leq kc \leq 2.75$, which is prominent in the baseline wall pressure results, is mostly subsided for the sawtooth serration aerofoils. For instance, at the pressure measurement
locations closest to the leading-edge on both the suction and pressure sides, as illustrated in Figs. 16(d) and (e), both the wide and sharp sawtooth serrations attain substantial wall pressure fluctuation PSD reduction over the mentioned kc range, in contrast to the baseline aerofoil case. The maximum wall pressure fluctuation PSD reduction magnitude is about 6dB/Hz and 10dB/Hz for the wide and sharp sawtooth serrations, respectively. Furthermore, a closer examination of the two sawtooth serrations reveals that the sharp sawtooth serration clearly achieves a better reduction in the wall pressure fluctuations than the wide sawtooth serration for all measurement locations over the frequency range. It is useful to recall that in the far-field noise spectra, shown in Fig. 7 earlier, the most significant noise reduction arising from the use of either the wide or sharp sawtooth serration comes from the ‘flattening’ of the wake-aerofoil interaction region (i.e. the distinct broadband hump), matching very well with the 0.55≤kc≤2.75 range identified from the wall pressure results. The matching broadband hump between the near- and far-field results reaffirms that the hump in the near- and far-field spectra indeed originates from the wake-aerofoil interaction [28].

To link the unsteady loading experienced by the rear aerofoil with the wake-induced boundary layer, the wall pressure fluctuation closest to the leading-edge (x/c=0.013) in Fig.15(e), with the tandem aerofoil gap distance of xg/c=0.3 and yg/c=0.13, can be compared with the turbulence kinetic energy of the boundary layer flow measured directly above this location. For both the wide and sharp sawtooth serrations, the wall pressure fluctuations lie below that of the baseline aerofoil case throughout the entire range of kc, and similarly do their TKE levels compared to the baseline case. In particular, the sharp sawtooth serration, which possesses the lowest TKE level, also produces the lowest wall pressure fluctuation PSD. On the other hand, when the aerofoil gap distance increases to xg/c=0.5, the TKE level of the wide sawtooth serration becomes greater than that of the baseline case, as seen from Fig. 13(c). Meanwhile, it can be partly seen in the wall pressure results that the wide sawtooth wall pressure fluctuation PSD is observed to be slightly higher than the baseline at kc<0.55 (see Fig. 16(e)). At this larger gap distance, the sharp sawtooth continues to outperform the wide sawtooth in the suppression of wall pressure fluctuations, except at relatively low kc after x/c=0.23 chord, e.g. Figs. 16(j) and (k), where the effect of the ‘wake-induced’ boundary layer flow over the aerofoil begins to attenuate.

In order to confirm that the wake-aerofoil interaction noise essentially emits from the leading-edge area of the rear aerofoil, an attempt was made
to decompose the unsteady aerodynamic loading from the wall pressure measurements into the lift and drag directions, i.e. x- and y-directions, to better compare the magnitudes and distinguish the changes. Similar decomposition approach has been adopted by Maryami et al. \cite{41} to differentiate the unsteady aerodynamic loading along the lift and drag directions of a cylinder immersed in a turbulent flow. The unsteady aerodynamic loading in the lift and drag directions can be calculated straightforwardly as:

\[
L'(f) = 10 \log_{10} \left( \int_{\xi_i}^{\xi_{i+1}} \phi_pp(f) \cos(\beta) \cdot d\xi / p_0^2 \right), \quad i = 1, 2, \ldots, 15,
\]

(4)

and,

\[
D'(f) = 10 \log_{10} \left( \int_{\xi_i}^{\xi_{i+1}} \phi_pp(f) \sin(\beta) \cdot d\xi / p_0^2 \right), \quad i = 1, 2, \ldots, 15,
\]

(5)

where \(L'(f)\) and \(D'(f)\) are the unsteady loading decomposed in the lift and drag directions in frequency domain with prime (‘) indicating the nature of the loading being unsteady, \(\beta\) denotes the local angle between the surface normal and the vertical axis (y) and segment length is determined between two adjacent unsteady pressure transducers (\(\xi_i\) and \(\xi_{i+1}\)) along the aerofoil surface. Note that here the unsteady aerodynamic loading is also expressed in dB/Hz (i.e., \(\log\) is applied to the wall pressure spectra) to keep the magnitude consistent with the wall pressure fluctuation spectra.

Figures \[17\] and \[18\] depict the unsteady lift and drag loading, i.e. y- and x-direction respectively, on the rear aerofoil at four selected \(kc\) of 0.7, 1.4, 2.5 and 4.4, for the aerofoil gap distance of \(x_g/c=0.5\). These different \(kc\) values correspond to the lower, central and upper bands of the far-field wake-aerofoil interaction noise hump. Note that the leading-edge is aligned at \(x/c=0\) with the positive \(x/c\) running along the suction side and negative \(x/c\) along the pressure side. For brevity, results from the aerofoil gap distance of \(x_g/c=0.3\) are not shown as similar observations can be made. As shown in Fig. \[17\] the unsteady lift loading peaks very close to the leading-edge area at \(x/c=0.05\) for all selected \(kc\) values and subsequently decreases exponentially along the chordwise direction (i.e. linearly as shown in the log-scaled Fig. \[17\]). Moreover, there exists rather significant differences in terms of the lift loading magnitude between the pressure and suction sides, especially so around the leading-edge area where the unsteady lift loading on the suction side is over 15dB/Hz higher than on the pressure side. It is worth mentioning that unsteady loading in the lift and drag directions is obtained from the fluctuating
Figure 17: (Color line) Unsteady aerodynamic loading along the pressure and suction sides of the aerofoil decomposed in the lift direction, i.e. \( y \)-direction, with the aerofoil gap distance of \( x_g/c = 0.5 \) and \( y_g/c = 0.17 \) at \( kc = 0.7, 1.4, 2.5 \) and 4.4 respectively, as marked in Fig. 6. Baseline (---); wide sawtooth (---); sharp sawtooth (-----).

Figure 18: (Color line) Unsteady aerodynamic loading along the pressure and suction sides of the aerofoil decomposed in the drag direction, i.e. \( x \)-direction, with the aerofoil gap distance of \( x_g/c = 0.5 \) and \( y_g/c = 0.17 \) at \( kc = 0.7, 1.4, 2.5 \) and 4.4 respectively, as marked in Fig. 6. Baseline (---); wide sawtooth (---); sharp sawtooth (-----).
(time-dependent) component of the wall pressure. Therefore, the magnitude of the unsteady loading does not contribute to the mean (time-averaged) aerodynamic performance of the aerofoil (i.e. the mean lift and drag forces). When comparing the unsteady lift loading between the baseline and the two trailing-edge serrated aerofoils, the differences are equally noteworthy. For instance, at \( kc=1.4 \), the baseline loading is approximately 7dB/Hz higher than both sawtooth serrations, with the sharp serrations having a small and yet noticeable edge over the wide serrations. At higher \( kc \) values of 2.5 and 4.4, the use of wide sawtooth serration results in an even larger peak than that from the baseline aerofoil case, suggesting possibly a deterioration in the far-field noise reduction. This is consistent with the far-field noise spectra shown earlier in Fig. 7, which the wide sawtooth ceases to be able to attain noise reduction beyond \( kc=2.5 \).

In contrast to the unsteady lift loading, the unsteady drag loading results for both the baseline and the wide and sharp sawtooth serrations, as seen in Fig. 18 are essentially uneventful and almost collapse onto each other. Both sawtooth serrations cause very minor reduction in the unsteady drag loading on the pressure side at lower frequencies of \( kc=0.7 \) and 1.4. It should be mentioned that the differences are not noticeable due to its much smaller magnitude of the unsteady loading as compared to those of the unsteady loading along the lift direction. Still, there are several noteworthy features from the unsteady drag loading results. Firstly, the unsteady drag loading peaks at the leading-edge of the rear aerofoil, which supports the notion that the wake-aerofoil interaction is the primary source of loading under the present experimental conditions. Secondly, it decreases rapidly from the leading-edge to a minimum at approximately \( x/c=0.2 \) on the suction side, because that the surface normal vector of the local aerofoil curvature is aligned with the lift direction \((y)\) at the angle of incidence of \( \alpha=10^\circ \). Lastly, it gradually increases from the minimum back to a level, relatively significant for the unsteady drag loading over the aerofoil. More importantly, the magnitude of unsteady drag loading are substantially lower than the unsteady lift loading. Comparing the results in Figs. 17 and 18, the unsteady drag loading is generally about 60dB/Hz lower in magnitude than the unsteady lift loading at similar chordwise locations. Hence, it is reasonable to infer from the unsteady lift and drag loading results that the wall pressure fluctuations in the lift direction of the aerofoil suction side contribute primarily to the unsteady loading on the aerofoil. Furthermore, the wake-aerofoil interaction takes place essentially within the first 20% – 30% of the chord of the
rear aerofoil since the unsteady lift and drag loading is substantially higher than that from the downstream locations. Such observation corresponds well with the results from the boundary layer velocity measurements, where the wake-aerofoil interaction is most influential close to the leading-edge of the rear aerofoil.

3.4. Near- and far-field coherence behaviour

It has been shown from the results above that the characteristic broadband hump of the wake-aerofoil interaction noise in the range of $0.55 \leq kc \leq 5.5$ is closely related to the wall pressure fluctuations and the energy frequency content of the wake-induced boundary layer close to the leading-edge of the rear aerofoil. Subsequently, an assessment from the simultaneous near- and far-field measurements can prove to be effective and useful to directly relate the near-field velocity and wall pressure fluctuations to the far-field radiated noise [42]. Therefore, the coherence between the near- and far-field pressure signals is determined and presented in Fig. 19 for the baseline and the wide and sharp sawtooth serrations with two aerofoil gap distances of $x_g/c = 0.3$ and 0.5. The magnitude squared pressure coherence, $\gamma^2$, is calculated as:

$$\gamma^2(f) = \frac{|\phi_{p_n,p_{90^\circ}}(f)|^2}{\phi_{p_n,p_n}(f)\phi_{p_{90^\circ},p_{90^\circ}}(f)}, \quad i.e., \quad n = 1, 2, \ldots 15 \quad (6)$$

between all the near-field pressure measurement locations, $p_n$ and the $90^\circ$ microphone from the far-field arc, $p_{90^\circ}$. The near-field to far-field pressure coherence is hence determined only for the suction side of the rear aerofoil for brevity since it has been observed earlier from the wall pressure fluctuation results that the suction side contributes primarily to the unsteady loading of the rear aerofoil. Also, consistent with the previous studies [43, 44], the low level of coherence is expected between the near-field and far-field pressure fields since the spatially distributed noise sources tend to be incoherent and their coherence with far-field radiated noise often reduces quickly.

There are several important observations to be drawn from the near- and far-field coherence results. Firstly, a region of high coherence level along the suction side can be seen to concentrate around the leading-edge of the aerofoil with very small and limited coherence beyond $x/c = 0.5$ locations for both the baseline and the wide and sharp sawtooth serration aerofoils. Secondly and more importantly, the region of significant coherence level is located in the $kc$ range of $0.55 \leq kc \leq 3$, which corresponds considerably well with the
Figure 19: Contour maps of coherence between wall pressure fluctuations and far-field noise measured from the overhead microphone at 90° for (a) baseline, (b) wide sawtooth and (c) sharp sawtooth, with the aerofoil gap distance of \( x_g/c = 0.3 \), \( y_g/c = 0.13 \) (top) and (d) baseline, (e) wide sawtooth and (f) sharp sawtooth with the aerofoil gap distance of \( x_g/c = 0.5 \), \( y_g/c = 0.17 \) (bottom).
broadband hump frequency range resulting from the wake-aerofoil interaction in both the wall pressure fluctuations and far-field noise spectra. This further shows that the wake-aerofoil noise around the leading-edge of the rear aerofoil is indeed the primary source of the far-field radiated noise in the present experimental condition, and more to it, the wall pressure fluctuations at the leading-edge area are directly related to the generation of wake-aerofoil interaction noise. Moreover, comparing the wide and sharp sawtooth serration aerofoils to the baseline case, there is clearly a drop in the coherence level throughout the aerofoil chord on the suction side. As a matter of fact, there appears to be a significant loss of coherence for the two sawtooth serrations after $x/c=0.1$, except the slightly elevated level of coherence from the leading-edge to $x/c=0.4$ for the sharp sawtooth serration with the larger aerofoil gap distance of $x_g/c=0.5$. The substantial decrease and even loss of the near-field to far-field pressure coherence suggests that there may exist notable changes to the frequency energy content on the leading-edge of the rear aerofoil. Therefore, it is crucial to examine the near-field flow in terms of the energetic and large-scale turbulent structures and their associated energy frequency content in more details.

3.5. Turbulence characteristics in wake-aerofoil interaction

The results so far have shown that an effective reduction of the wake-aerofoil interaction noise can be achieved by applying the sawtooth trailing-edge serrations to the front aerofoil in the tandem aerofoil configuration. The decrease in the energy frequency content of the boundary layer velocity fluctuations, and subsequently the reduction of the unsteady aerodynamic loading close to the leading-edge area over the broadband hump frequency range could play a crucial role in the observed interaction noise reduction, which in turn arises from the reduction of the turbulent kinetic energy of the wake flow field. In order to obtain a complete picture of the mechanisms leading to the wake-aerofoil interaction noise reduction and the correlation between the dynamic near-field turbulence characteristics and the far-field radiated noise, more spectral analyses of the velocity and wall pressure measurements are performed. It aims to provide deeper insights into the fundamental flow development in terms of the turbulent coherent structures and the changes that the wide and sharp sawtooth serrations can make. To authors’ knowledge, the use of coupled pressure–velocity analyses for understanding of noise generation mechanisms in the wake-aerofoil interaction phenomenon has remained very limited in the literature.
With the simultaneous boundary layer velocity and wall pressure fluctuation measurements, it is possible to examine the pressure–velocity \((p–u)\) coherence and correlation, which could relate the energy frequency content of the wall pressure to the turbulence characteristics in the boundary layer. It should be noted that several studies have employed coupled pressure–velocity analyses to examine and better understand the flow dynamics in their experimental studies on aerofoils and flat-plates under different flow conditions, such as \[22, 23, 25, 34, 42, 45, 46, 47\], to name a few. First of all, the magnitude squared \(p–u\) coherence, \(\gamma^2(f)\), is calculated similar to that of the near-field and far-field pressure coherence as:

\[
\gamma^2(f) = \frac{|\phi_{pu}(f)|^2}{\phi_{pp}(f)\phi_{uu}(f)},
\]

where \(\phi_{pu}(f) = \int_{-\infty}^{\infty} R_{pu}(\tau)e^{-2\pi if\tau}d\tau\). The \(p–u\) coherence results of the baseline and the wide and sharp sawtooth serrations at the chord locations of \(x/c=0.013\) immediately downstream of the leading-edge are shown in Fig. 20 for the two different aerofoil gap distances of \(x_g/c=0.3\) and 0.5. For the baseline aerofoil of gap distance \(x_g/c=0.3\), as shown in Fig. 20(a), the most significant level of \(p–u\) coherence is found to be due to the turbulent eddies very close to the aerofoil surface at \(y/c\leq0.016\) within the non-dimensional frequency range of \(0.44\leq kc\leq2.4\). An additional region with notable coherence is caused by the flow above \(y/c=0.02\) from the aerofoil surface, extending across a wider frequency range, from approximately \(kc=0.5\) to 6. Recall from both the wall pressure fluctuations at the same chord location of \(x/c=0.013\) in Fig. 15(e) and the far-field radiated noise of the baseline aerofoil in Fig. 7 that the wake-aerofoil interaction manifests primarily as a broadband hump in the spectra within \(0.44\leq kc\leq5\). Clearly, the heightened level of \(p–u\) coherence corresponds well with the frequency range of the wake-aerofoil interaction, and therefore, suggests that the energy frequency content of the wake-induced boundary layer profile is closely associated with those determined from the wall pressure fluctuations, and as a result the far-field radiated noise.

Comparing the wide and sharp sawtooth serrations to the baseline aerofoil, as seen in Figs. 20(b) and (c), the \(p–u\) coherence behaves quite differently, with the elevated coherence level spreading more evenly and widely across the range of \(kc\) investigated. More specifically, instead of having the most significant coherence level to coincide with the broadband hump of the wake-turbulence interaction, both sawtooth serrations cause a drastic shift of the most significant coherence level to approximately \(3\leq kc\leq10.5\), and also a
moderately increased coherence level below $kc=0.4$. Furthermore, the maximum coherence magnitude is relatively higher at about $\gamma^2=0.83$ and 0.72 for the wide and sharp sawtooth serrations than that of $\gamma^2=0.68$ for the baseline aerofoil. Nevertheless, it remains noteworthy to mention that a higher $p-u$ coherence does not indicate a greater wall pressure fluctuation arising from the velocity field of the wake-induced boundary layers, but rather a higher degree of resemblance in the energy frequency content between the velocity and wall pressure fluctuations.

A closer examination of the coherence contour maps reveals that the contour plots can be divided into two separate regions at approximately $y/c=0.016$, regardless of the baseline or serrated trailing-edge used. Recall from the flow velocity measurements in Section 3.2 that the boundary layer profile downstream of the rear aerofoil leading-edge under heavy wake flow influence, consists of a flow-developed profile very close to aerofoil surface and a wake-induced profile above it. Although this is only evidenced from the boundary layer profile measured at $x/c=0.113$, the $p-u$ coherence results remarkably, as can be seen in Fig. 20, are able to capture the transition closer to the leading-edge from the spectral analysis perspective. It reinforces the notion that an interaction between the wake flow fields and the flow-developed boundary layer takes place immediately after the impingement.

Similarly, such distinct regions of noticeable coherence level can be observed for both the baseline and the wide and sharp sawtooth serrations aerofoils with the larger aerofoil gap distance of $x_g/c=0.5$, as depicted in Figs. 20(d), (e) and (f). However, for the larger gap distance, the elevated $p-u$ coherence level is observed to become more widely distributed, as compared to that of $x_g/c=0.3$. Yet, the high $p-u$ coherence is still confined to $0.6\leq kc\leq4$, i.e. the wake-aerofoil interaction region. The wide and sharp sawtooth serrations show comparable effects on the $p-u$ coherence as they do at the smaller aerofoil gap distance, establishing a more evenly and widely distributed coherence level across the $kc$ range investigated. Between the two sawtooth serrations, the sharp sawtooth serration extends the $p-u$ coherence more uniformly from approximately $kc=0.4$ to 10 than the wide sawtooth, from approximately $kc=0.4$ to 6. With reference to the wall pressure measurements, the sharp sawtooth serration achieves a higher reduction in the wall pressure fluctuation PSD (see Figs. 14(e) and 15(e)). Therefore, it can be inferred from the $p-u$ coherence that the use of sawtooth serrations can lead to spread of the coherence content across a wider frequency range,
Figure 20: Contour maps of coherence between wall pressure fluctuations obtained at $x/c=0.013$ and boundary layer velocity measurements for (a) baseline, (b) wide sawtooth and (c) sharp sawtooth, with aerofoil gap distance of $x_g/c=0.3$, $y_g/c=0.13$ (top) and (d) baseline, (e) wide sawtooth and (f) sharp sawtooth with aerofoil gap distance of $x_g/c=0.5$, $y_g/c=0.17$ (bottom).
which possibly indicates that the turbulence content are spread over a wider frequency range. Yet, the increase in $p-u$ coherence at higher frequencies indicates the emergence of more turbulent structures with smaller length scales.

The insights gained from the pressure–velocity coherence showed a close relation between the boundary layer velocity profiles and the wall pressure fluctuations in the wake-aerofoil interaction region. Furthermore, using the closely-spaced wall pressure measurement ports around the leading-edge area, the cross-correlation spectra between two consecutive measurement ports can be determined. Since they contain rich information on the large-scale turbulent structures convecting across those locations, the results will help illustrate further the correlation between the energetic turbulent eddies and the wall pressure fluctuations. In order to extract these information in the frequency domain, a two-step calculation has been performed with a newly defined parameter, $\alpha_{Rpp}$, to represent the energy contents derived from the cross-correlation spectra of the unsteady wall pressure collected over the surface of the rear aerofoil. Thus, it is useful to briefly outline the calculation procedures and the physical meaning carried with parameter.

Beginning with the cross-correlation spectra, the temporal cross-correlation coefficients between two consecutive pressure measurement ports, $R_{p_ip_{i+1}}(\tau)$, is defined as:

$$R_{p_ip_{i+1}}(\tau) = \frac{p_i'(t)p_{i+1}'(t-\tau)}{p_{i,rms}p_{i+1,rms}^{\prime}},$$

where $i=1, 2, \ldots, 14$ corresponds to the pressure measurement locations listed in Table 1 and subsequently, the correlation frequency spectra, $\alpha_{Rpp}$, can be calculated similar to $\phi_{pp}$:

$$\alpha_{Rpp}(f) = \int_{-\infty}^{\infty} R_{p_ip_{i+1}}(\tau) e^{-2i\pi f \tau} d\tau.$$  (9)

Finally, it is necessary to normalise the correlation frequency spectra by its maximum magnitude, such that the magnitude of the correlation frequency spectra can be consistently compared:

$$\hat{\alpha}_{Rpp}(f) = \frac{\alpha_{Rpp}(f)}{\max(\alpha_{Rpp}(f))}.$$  (10)

Note that for a direct comparison between the baseline and sawtooth serrations aerofoils, the correlation frequency spectra are normalised with
the maximum magnitude among the three aerofoil cases instead, i.e. in the present experiments, $\alpha_{Rpp}$ for the baseline aerofoil obtained at $x/c=0.193$ location. Conventionally, by plotting the cross-correlation coefficients, $R_{p,p_{i+1}}$, in the time domain, the dominant turbulent eddies can be inferred from either the periodicity of cross-correlation or the temporal offset of the peak cross-correlation from time zero (i.e., $\tau=0$). When the correlation coefficients are Fourier transformed to the fluctuations in the frequency domain, the frequencies associated with the most energetic turbulent eddies can be identified, if their correlation is sufficiently strong. In another words, the frequency information embedded in the correlation coefficients can be extracted through this additional Fourier transformation step. Therefore, the normalised correlation frequency spectra, $\hat{\alpha}_{Rpp}$, can be considered as an indicator for the presence of dominant turbulent eddies convecting along the streamwise direction. Figure 21 shows the normalised correlation frequency spectra, $\hat{\alpha}_{Rpp}$, for the baseline and the wide and sharp sawtooth serrations with the aerofoil gap distance of $x_g/c=0.5$. Here, the maximum fluctuation magnitude of the baseline aerofoil is used as the normalisation factor for all aerofoil cases. Therefore, a greater magnitude in the correlation frequency spectra indicates the presence of more energetic large-scale structures in the flow within the interested frequency range. Note that only the correlation frequency spectra from the first ten pressure measurement locations from $x/c=0.013$ to 0.293 are analysed in order to focus on the wake-aerofoil interaction over the leading-edge area. In addition, for clarity of the line plot, the results are separated into two groups with the first group, i.e. the top row of the figure, from positions $x/c=0.013$ to 0.113 and the second group, i.e. the bottom row of the figure, from $x/c=0.153$ to 0.293.

The correlation frequency spectra obtained from the cross-correlation coefficients begin to rise noticeably from approximately $kc=0.55$ and peak around $kc=1.5$ before returning close to zero at $kc=3$, for both the baseline and the sawtooth serration cases, as can be seen from Figs. 21(a), (b) and (c). This essentially validates the present method of analyses, as not only does the peak coincides well with the centre of the broadband hump observed from the near-field pressure fluctuation PSD (see Figs. 15(e) to (h)), but also with that of the wake-aerofoil interaction noise from the far-field noise spectra (see Fig. 7(b)). Thus, it implies that the most energetic structures in the flow, being identified from the correlation spectra, are likely to be responsible for the broadband hump, i.e. the characteristics of the wake-aerofoil interaction. Furthermore, comparing the magnitude of the correlation frequency
spectra between the baseline and the serrated trailing-edge aerofoils, which the corresponding results at \( x/c = 0.013 \) and the extracted far-field spectra are shown as the insets in Fig. 21(b) as a visual aid, the reduced correlation spectra by the sawtooth serrations seem to also correspond directly to the mitigated far-field noise frequency range of \( 0.55 \leq kc \leq 3 \). As the turbulent structures convect downstream with increasing \( x/c \), the correlation frequency spectra, initially growing in magnitude, start to decrease beyond \( x/c = 0.2 \), agreeing well with the previous discussion from the unsteady loading analyses that the dynamic effects of the wake-aerofoil interaction gradually diminishes from 20\% to 30\% of the chord.

From the cross-correlation between two adjacent pressure taps, it is subsequently viable to estimate the flow convective velocity to examine the ‘bulk’ behaviour of the turbulent structures. The calculation of normalised convective velocity, \( U_c/U_\infty \), follows the standard approach described in [48]:

\[
\frac{U_c}{U_\infty} = \frac{\zeta}{\tau_{\text{max}} U_\infty},
\]

where \( \zeta \) denotes the streamwise separation distance between two pressure measurement locations used to determine the cross-correlation and \( \tau_{\text{max}} \) refers to the time lag between the time zero and the time when the maximum cross-correlation is achieved. Figure 22(a) shows the ‘bulk’ convective velocity determined for the baseline and the wide and sharp sawtooth serrations with aerofoil gap distances of \( x_g/c = 0.5 \), from the leading-edge to \( x/c = 0.25 \) of the rear aerofoil. Note that the behaviour of the convective velocity at \( x_g/c = 0.3 \) is similar to that of \( x_g/c = 0.5 \) and the convective velocity downstream of \( x/c = 0.25 \) is not presented here since the pressure taps are separated increasingly further apart moving downstream of the rear aerofoil, which could lead to larger errors in the estimation of convective velocity [49].

As observed by many studies, such as Goldschmidt et al. [48], Del Álamo and Jiménez [50] and Lehew et al. [51], convective velocity in the flow often varies with frequency. Thus, the frequency dependent convective velocity is also determined at single downstream location of \( x/c = 0.033 \), close to the leading-edge of the rear aerofoil, for \( x_g/c = 0.5 \) to provide additional knowledge of the frequency dependent flow behaviour, as shown in Fig. 22(b). The frequency dependent convective velocity, \( U_c(f) \), can be obtained as [27]:

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Figure 21: Normalised correlation frequency spectra, $\hat{\alpha}_{Rpp}$, determined from the cross-correlation coefficients of the wall pressure fluctuations for (a) baseline, (b) wide sawtooth and (c) sharp sawtooth from $x/c=0.013$ to 0.113 and (d) baseline, (e) wide sawtooth and (f) sharp sawtooth from $x/c=0.153$ to 0.293 on the suction side, with the aerofoil gap distances of $x_g/c=0.5$ and $y_g/c=0.17$. Note that two insets are shown in (b) to zoom into the broadband hump frequencies with (b1) the comparison of the $\hat{\alpha}_{Rpp}$ between the baseline and sawtooth serrations aerofoils and (b2) the far-field noise spectra as the reference.
\[ \frac{U_c(f)}{U_\infty} = 2\pi \zeta \frac{\Delta f}{\Delta \phi_{ij}(f)}, \]  

(12)

where \( \Delta \phi_{ij} \) denotes the phase shift calculated from the cross-spectral density of the pressure fluctuations between two adjacent pressure taps.

Looking first at the ‘bulk’ convective velocity obtained for the aerofoil gap distance of \( x_g/c = 0.5 \) in Fig. 22(a), it undergoes noticeable changes immediately downstream of the leading-edge before settling gradually to \( U_c/U_\infty = 0.55 \), regardless of the baseline or serrated trailing-edge used. According to the experimental study by Lowson [52], for turbulent boundary layers, the convective velocity close to the wall surface is around \( 0.6 U_\infty \). Thus, the convective velocity estimated from the cross-correlation spectra from the present measurements validates well with the literature. For the baseline aerofoil, the convective velocity increases from initially \( U_c/U_\infty \approx 0.5 \) to 0.6 as the wake turbulence decays over downstream distance. In contrast, although the wide and sharp sawtooth serrations initially have higher convective velocity than the baseline aerofoil, owing to the fact the flow velocity immediately after the wake-aerofoil impingement is notably faster, as shown in Fig. 12(a), there is little increase in the convective velocity over the downstream distance when for instance, the convective velocity at \( x/c = 0.013 \) and 0.113 is compared. According to Lowson [52], Krogstad et al. [53] and Atkinson et al. [54], the convective velocity is likely to be higher for large-scale low-frequency turbulent structures and lower for the turbulent structures associated with higher frequencies. Therefore, it is likely that the sawtooth serrations see an increase in the energy content at high frequencies with smaller turbulent structures, which are convecting at a lower ‘bulk’ convective velocity. This is also evident from the frequency dependent convective velocity results, which the most energetic turbulent structures, \( i.e. \) being convected with the highest velocity, are at lower frequencies as compared to the sawtooth serrations. It is also worthwhile to note that the shift of peak convective velocity to higher frequencies in both sawtooth serrations, as shown in Fig. 22(b), corroborates quite well with the increase of the \( p-u \) coherence at higher frequencies observed earlier. Therefore, both pressure–velocity coherence and pressure cross-correlation results indicate an increase in the energy content and possibly turbulent structures towards higher frequencies which leads to an overall reduction of the broadband hump of the interaction noise at approximately \( 0.6 \leq kc \leq 3 \).

At this point of time, it is constructive to briefly summarize the impact
Figure 22: (Colour line) Normalised convective velocity along the aerofoil suction side for the aerofoil gap distances of $x_g/c=0.5$ and $y_g/c=0.17$ of (a) $U_c/U_\infty$ determined from cross-correlation and of (b) $U_c(f)/U_\infty$ determined from cross-spectral phase shift at single downstream location of $x/c=0.033$. Baseline (—); wide sawtooth (---); sharp sawtooth (-----).

of sawtooth trailing-edge serrations on the energetic structures. As can be seen from the pressure–velocity coherence results at the leading-edge area of the rear aerofoil, the elevated $p-u$ coherence is less accentuated at the broadband hump frequency range and more evenly distributed over a wider frequency range for the wide and sharp sawtooth serrations, which coincides with that of the near-field to far-field pressure coherence. Although a direct relation cannot be drawn, such observation provides a plausible to the decrease in near-field to far-field pressure coherence over the broadband hump frequency range as the changes in turbulence characteristics of the boundary layers will have a direct impact on the unsteady wall pressure fluctuation and hence, noise generation. Later, the convective velocity results from the cross-correlation spectra further corroborate with the findings, where the convective velocity for the sawtooth serrations appear not to experience noticeable increase over the downstream distance, suggesting the presence of smaller turbulent structures. Moreover, from the correlation frequency spectra, it shows that the sawtooth serrations reduce substantially the energetic turbulent eddies associated with the broadband hump frequencies. Comparing between the two sawtooth serrations, the sharp sawtooth produces more significant modifications to the frequency energy content than the wide sawtooth over the wake-interaction frequency range, thus achieving a bet-
ter reduction in the unsteady aerodynamic loading and radiated interaction noise.

4. Concluding remarks

A thorough experimental investigation of the use of sawtooth serrations to the trailing-edge of the front aerofoil in a tandem aerofoil configuration has been undertaken to examine their effectiveness in mitigating wake-aerofoil interaction noise. Results from two sawtooth serrations with different geometries, referred as the wide and sharp sawtooth serrations, were compared to the baseline case with a straight trailing-edge. From the far-field noise spectra, it is clear that the wake-aerofoil interaction acts as the primary noise source when the wake fully impinges upon the rear aerofoil, which is characterised by the broadband hump. Moreover, the use of sawtooth trailing-edge serrations on the front aerofoil can lead to significant overall noise reduction up to 7 and 10dB for the wide and sharp sawtooth serrations, respectively. The velocity measurements reveal that both sawtooth serrations give rise to a smaller turbulent kinetic energy level of the flow in the front aerofoil wake and the boundary layer over the suction side of the rear aerofoil. A further analysis on the energy frequency content confirms that the power spectral density (PSD) of the velocity fluctuations are notably reduced over the broadband hump frequency range while elevated at relatively higher frequencies close to the aerofoil surface. The unsteady wall pressure measurements also show a clear reduction of approximately 10dB/Hz in the unsteady pressure loading over the wake-aerofoil interaction frequency range, matching well with that observed from the far-field noise spectra. More importantly, the decomposed unsteady lift loading suggests within first 30% of the chord contributes primarily to the loading due to wake-aerofoil interaction.

In-depth spectra analyses from the pressure–velocity coherence, wall pressure cross-correlation frequency spectra and the convective velocity of the turbulent flow over the rear aerofoil are determined to provide further insights into the underlying physical mechanisms. Interestingly, the pressure–velocity coherence results indicate that both the wide and sharp sawtooth serrations yield a more evenly distributed coherence levels across a wider frequency range downstream of the leading-edge of the rear aerofoil. This possibly helps explain the loss of coherence in the near-field to far-field pressure coherence results over the similar frequency range. Furthermore, both the correlation frequency spectra and the convective velocity corroborate
well with the pressure–velocity coherence by establishing a reduced energy content over the wake-aerofoil interaction frequency range and a possible increase of energetic turbulent eddies towards relatively higher frequencies. Consequently, the ability to reduce wake-aerofoil interaction noise from the sawtooth trailing-edge serrations appear to be the combined results from the reduced unsteady loading from the lower turbulence levels and a favourable spread of the energy frequency content in the wake-induced flow close to the aerofoil surface.

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6. Conflicts of Interest

The authors declare no conflicts of interest in the present study.

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