Bootstrap Frequency Equalisation for MIMO Wireless Systems

Robert J. Piechocki, Christos Kasparis, Andrew R. Nix, Paul N. Fletcher and Joe P. McGeehan
University of Bristol, Centre For Communications Research
Woodland Road, MVB, BS8 1UB, Bristol, UK.
E-mail: r.j.piechocki@bristol.ac.uk, Tel. +44 117 9545203, Fax. +44 117 9545206

***Abstract*— This paper reports on a new class of equalisation/detection for Wideband Multiple Input - Multiple Output Communications Systems (MIMO). The proposed scheme is somewhat akin to a multi-carrier MIMO system, and more precisely, builds upon a Single Carrier Frequency Domain Equalised (SCFDE) MIMO System. At the core of our system lies a novel concept of iterative self-reused equalisation/detection (bootstrapping) combined with semi-hard decision making. The bootstrapping concept is derived from a new formulation of Tikhonov Regularisation. The proposed scheme achieves a remarkable performance/complexity trade-off. In particular we show that our system approaches the performance of ML-MIMO-OFDM while being only slightly more complex than MMSE-MIMO-OFDM. The coded version achieves virtually the same performance as Turbo MIMO-OFDM at a fraction of the complexity. The resulting system is termed Zero Tailed Bootstrap Frequency Equalised (ZTBFE) MIMO system.**

***Index Terms*— MIMO, SC-FDE, Bootstrap, Tihkonov Regularisation, Rigde Regression**

I. Introduction

The MIMO system concept first appeared in [1] and it was soon proclaimed by the research community and the industry alike as the only hope to deliver substantial bandwidth efficiency improvements, much needed for wireless systems. However, practical impairments of the wireless channel (in particular the delay spread) penalise the receiver design in term of complexity. The complexity increase is particularly prominent in MIMO systems as compared to more traditional single antenna systems. To give an example: if a GSM-EDGE system was to be extended to a MIMO system and a Viterbi equaliser was to be employed, the receiver would need to search though the trellis comprising of $8^{4\times 4} = 2.8147 \times 10^{14}$ states (assuming 8-PSK modulation, 5 tap channel and $N_T = 4$ transmit antennas). The exponential increase in complexity with N_T is readily evident in this example. The exponential complexity increase will characterise all optimal receivers, since it can be shown that the problem is NP-hard. One of the more popular way to tackle this complexity is to employ an OFDM based system. The MIMO-OFDM systems inherit the same features as standard CP-OFDM. The basic advantage is ”cheap” (reduced complexity) equalisation/detection, where the complexity is independent of the channel delay spread. However, the optimal approaches in MIMO-OFDM will also scale exponentially with N_T. To this end many flavours of sphere decoders have been proposed to reduce this complexity. Another, perhaps more significant drawback of MIMO-OFDM is a high peak-to-mean power ratio, which significantly complicates the design of amplifiers (high linearity required). The OFDM also suffers from poorer performance than single carrier systems, especially when compared to MLSE solutions. The performance discrepancy is due to the fact that OFDM does not utilise frequency diversity (coding and interleaving ameliorates this to a certain degree).

High peak-to-mean power ratio in OFDM is caused by the IFFT pre-coding before transmission. However, it was shown that the channel circularity can also be induced with the IFFT shifted to the receiver, with cyclic prefix all that is needed at the transmitter. The resulting system is termed a Single Carrier Frequency Domain Equalised System (see [2] for a review and further references). Such a system can be extended to a MIMO configuration [3][4].

In this contribution we develop a system that builds upon MIMO-SCFDE. We introduce an iterative frequency bootstrapping concept as an equalisation counterpart to a technique developed in [5] for multiuser detection in CDMA. The resulting system is termed ZTBFE-MIMO system. The proposed solution overcomes the aforementioned peak-to-mean power drawback, since as in SCFDE there is no IFFT pre-coding at the transmitter. Simulation results confirm performances that approach optimal (ML) detectors, as frequency diversity is utilised in a far more dramatic fashion than in MIMO-SCFDE. At the same time the complexity is moderately increased over MMSE based MIMO-SCFDE.

II. System Description

Orthogonal multi-carrier concepts thrive on algebraic properties of circulant matrices. A circulant matrix is defined by the first row only, and each row is obtained by cyclically shifting the previous row to the right. The most important property of circulant matrices is that all share the same set of singular vectors. Those singular vectors form the FFT matrix. In other words, any circulant matrix is diagonalised by the FFT matrix. The remaining trick is to pre- or postprocess (or both) the signal, such that the equivalent channel can be modelled as a circulant matrix. The most common technique is to cyclically repeat the last part of each transmitted block. The result is known as OFDM with Cyclic Prefix (CP-OFDM). In this paper we choose to use an alternative technique - zero tailing (ZT). Zero-tailing (padding) was proposed for OFDM in [6] (see also [7]), and for MIMO-SCFDE in [3].

The i^{th} block of data $\bar{\mathbf{u}}_i$ transmitted from each antenna is given by $\bar{\mathbf{u}}_i = \mathbf{T}_{ZT}\mathbf{u}_i$. The data vector, $\mathbf{u}_i$, is of length K, the size of the ZT insertion matrix $\mathbf{T}_{ZT}$ is $P \times K$, where $P = C + K$, C represents the length of the ZT. For brevity, we omit pulse shaping filters, since they are not essential for the signal model. The above operations are all that is needed at the transmitter. The data is formed into K long packets with silent periods in between (zero tailing). The silent periods have to be longer or equal to the excess length of the channel delay profile. Since there is no IFFT operation, the peak-to-mean power ratio is the same as any other single carrier system.

Consider for the moment a single antenna case. The receiver receives the current transmitted block of data $\bar{\mathbf{u}}_i$, in addition to a fraction of the previous block through the excess length of the channel impulse response. This is described by Toeplitz channel matrices $\mathbf{H}_0$ and $\mathbf{H}_1$, and the received signal block pertaining to $\mathbf{u}_i$ is given by:

$$\bar{\mathbf{x}}_i = \mathbf{H}_0\bar{\mathbf{u}}_i + \mathbf{H}_1\bar{\mathbf{u}}_{i-1} + \bar{\eta}_i \tag{1}$$

Both of the above channel matrices are of size $P \times P$ and are given by: $(h_0, \ldots, h_{L-1}, 0, \ldots, 0)^T$ for the first column and $(h_0, 0, \ldots, 0)$ for the first row of $\mathbf{H}_0$; $(0, \ldots, 0)^T$ for the first column and $(0, \ldots, h_{L-1}, \ldots, h_1)$ for the first row of $\mathbf{H}_1$. As aforementioned, it is assumed that the length of ZT is at least that of the channel: $C \geq L$. This is a general model for all linearly pre-coded orthogonal systems. In our case $\bar{\mathbf{u}}_{i-1}$ can be neglected due to zero-tailing.

The receiver takes advantage of ZT pre-processing and induces circularity onto the equivalent channel model. This is achieved by post-multiplication with a matrix $\mathbf{T}_{CI}$ defined as: $\mathbf{T}_{CI} = \left[\mathbf{I}_{K\times K}, \left[\mathbf{I}_{C\times C}; \mathbf{0}_{(K-C)\times C}\right]\right]$. Hence the input-output relationship can be expressed as:

$$\mathbf{x}_i = \mathbf{T}_{CI}\mathbf{H}_0\mathbf{T}_{ZT}\mathbf{u}_i + \eta_i \tag{2}$$

where η_i represents the ubiquitous additive noise vector. Pre-processing by $\mathbf{T}_{ZT}$ and post-processing by $\mathbf{T}_{CI}$ guaranties that the concatenation $\mathbf{T}_{CI}\mathbf{H}_0\mathbf{T}_{ZT}$ is a circulant matrix, and thus is diagonalised by $\mathbf{F}$ (the FFT matrix):

$$\begin{aligned}&\mathbf{F}\mathbf{T}_{CI}\mathbf{H}_0\mathbf{T}_{ZT}\mathbf{F}^{-1} = \mathbf{F}\mathbf{H}_{Cir}\mathbf{F}^{-1} = \\ &\mathbf{\Lambda} = diag\left\{\lambda^{(0)}, \ldots, \lambda^{(K-1)}\right\}\end{aligned} \tag{3}$$

This result suggests that the simplest zero-forcing (ZF) equalisation can be performed such that the whole system is modelled as:

$$\hat{\mathbf{x}}_i = \underbrace{\mathbf{F}^{-1}\mathbf{\Lambda}^{-1}\mathbf{F}}_{\mathbf{H}_{Cir}^{-1}}\underbrace{\mathbf{T}_{CI}\mathbf{H}_0\mathbf{T}_{ZT}}_{\mathbf{H}_{Cir}}\mathbf{u}_i + \mathbf{F}^{-1}\mathbf{\Lambda}^{-1}\mathbf{F}\eta_i \tag{4}$$

The ZF equalising matrix $\mathbf{\Lambda}^{-1}$ exists as long as the channel frequency response does not have zeros on the FFT grid (a condition equivalent to $\det(\mathbf{\Lambda}) \neq 0$ or $\det(\mathbf{H}_{Cir}) \neq 0$). If some of the λ_k are close to zero, the system is ill-conditioned and significant noise amplification occurs. This problem will be ameliorated by bootstrapping based on Tikhonov Regularisation presented in the next section.

This model is extended to MIMO configuration with N_T transmit and N_R receive antennas as follows. We define $\chi = \mathbf{F}\mathbf{H}_{Cir}\mathbf{u}$ as a vector of post-processed received data to be equalised. We have dropped the subscript "i" since we are no longer concerned with inter-block-interference (IBI). We notice that the equalisation in the SISO case has been performed by a diagonal matrix $\mathbf{\Lambda}^{-1}$. This is equivalent to stating that no inter carrier interference occurs (as long as $C \geq L$). Hence, for a synchronised transmission it can be shown that the MIMO equalisation/detection problem reduces to a set of K linear problems given by

$$\mathbf{y}_k = \mathbf{G}_k\mathbf{x}_k + \mathbf{n}_k \tag{5}$$

with the mixing matrix $\mathbf{G}_k$ defined by

$$\mathbf{G}_k = \begin{bmatrix} \lambda_{1,1}^{(k)} & \lambda_{1,2}^{(k)} & \cdots & \lambda_{1,N_T}^{(k)} \\ \lambda_{2,1}^{(k)} & \lambda_{2,2}^{(k)} & \cdots & \lambda_{2,N_T}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N_R,1}^{(k)} & \lambda_{N_R,2}^{(k)} & \cdots & \lambda_{N_R,N_T}^{(k)} \end{bmatrix} \tag{6}$$

where $\lambda_{m,n}^{(k)}$ represents the frequency response of the channel between the n^{th} transmit and the m^{th} receive antenna at the frequency tone k and $\mathbf{x}_k = \left(\chi_1^{(k)}, \ldots \chi_{N_T}^{(k)}\right)^T$.

The MIMO-ZTFBE receiver (figure 1) is built around an iterative loop comprising a MIMO detector based on Tikhonov Regularisation (Ridge Regression), FFTs and a semi hard decision block. This is not a turbo system since it does not rely on some other external devise to provide prior information about the data (in the turbo principle it is typically a soft-in-soft-out channel decoder). The system reuses its own output information and hence it is termed "bootstrap". Although, seemingly similar to a decision feedback equaliser, it operates using different principles. The system operates on a block by block basis. The use of FFTs in the detection loop introduces inter-relation in the frequency domain, which is utilised to re-capture frequency diversity and significantly improve performance. The semi-hard decision block makes a decision only on the symbols that are deemed to be sufficiently reliable. Symbols that are not reliable are fed back in a soft form. This is a crucial block since the "good symbols" help out to detect "weak symbols" across both the space and the frequency domains.

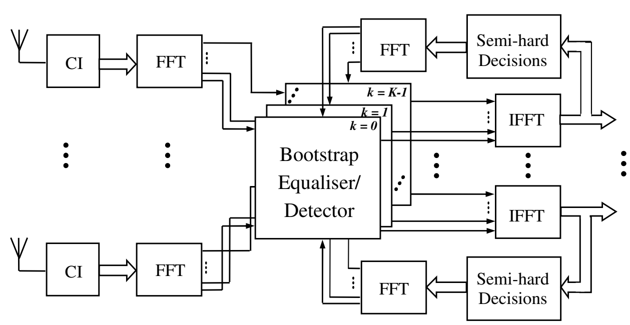

Fig. 1. Block structure of the proposed receiver.

III. Frequency Bootstrapping based on Tikhonov Regularization

Tikhonov Regularization adds to the classical LS constraint an additional regularization constraint:

$$J_{TR}(\mathbf{x}) = \|(\mathbf{y} - \mathbf{G}\mathbf{x})\|_2^2 + \Omega(\mathbf{x}) \tag{7}$$

The side constraint helps to narrow down the set of possible solutions which satisfy the LS constraint, provided the former is consistent with the problem. The side constraint also needs to be a sufficiently simple criterion if an analytical solution to (7) is required. A general choice of $\Omega(\mathbf{x})$, which proves to be meaningful in many problems and is also simple enough to provide an analytical solution is the one proposed by Tikhonov (see [8], [9]):

$$\Omega(\mathbf{x}) = \lambda^2 \|\mathbf{L}\mathbf{x}\|_2^2 \tag{8}$$

$\mathbf{L}$ is some linear operator acting on the solution. λ^2 is a smoothing regularization parameter whose value dictates the smoothnes on the filtering function which is imposed on the spectrum of the design matrix by the Regularization constraint. Clearly, as $\lambda^2 \to 0$ no weighting is imposed on the singular values of $\mathbf{G}$ and the TR solution coincides with the LS one. On the other hand as $\lambda^2 \to \infty$, an exessively smooth function is applied on the spectrum of $\mathbf{G}$ and information about the solution in the observation is lost in the attempt to over supress noise. Optimal selection of λ^2 is not a trivial task in practice and many methods have been proposed in the literature [9].

In the case where knowledge about an initial-default solution $\tilde{\mathbf{x}}$ is known for the problem then (9) can be further generalized as:

$$\Omega(\mathbf{x}) = \lambda^2 \|\mathbf{L}(\mathbf{x} - \tilde{\mathbf{x}})\|_2^2 \tag{9}$$

In this case λ^2 controls the bias in the estimator towards the default solution. As $\lambda^2 \to \infty$ the estimator will coincide with the default solution and no information will be extracted from the observations.

Starting from the general formulation of the TR criterion:

$$J_{TR}(\mathbf{x}) = \|(\mathbf{y} - \mathbf{G}\mathbf{x})\|_2^2 + \lambda^2 \|\mathbf{L}(\mathbf{x} - \tilde{\mathbf{x}})\|_2^2 \tag{10}$$

A solution can be found by setting:

$$\frac{\partial}{\partial x_i^*}\left\{ \begin{array}{l} \lambda^2(\mathbf{x}-\tilde{\mathbf{x}})^H\mathbf{L}^H\mathbf{L}(\mathbf{x}-\tilde{\mathbf{x}})+ \\ (\mathbf{y}-\mathbf{G}\mathbf{x})^H(\mathbf{y}-\mathbf{G}\mathbf{x}) \end{array} \right\} = 0 \tag{11}$$

which leads to the following solution:

$$\begin{array}{l} 2\lambda^2\mathbf{L}^H\mathbf{L}(\mathbf{x}-\tilde{\mathbf{x}}) - 2\mathbf{G}^H(\mathbf{y}-\mathbf{G}\mathbf{x}) = 0 \Rightarrow \\ \hat{\mathbf{x}}_{TR} = (\lambda^2\mathbf{L}^H\mathbf{L} + \mathbf{G}^H\mathbf{G})^{-1}(\lambda^2\mathbf{L}^H\mathbf{L}\tilde{\mathbf{x}} + \mathbf{G}^H\mathbf{y}) \end{array} \tag{12}$$

The TR estimator provides a generalisation of the MMSE estimator in the presence of a prior solution, provided that the parameter is selected optimally (see [10] for more thorough discussion.

The proposed MIMO-ZTBFE is based on the formulation of the TR criterion in which $\mathbf{L} = \mathbf{I}$, but some default solution is assumed to be known about the problem. In the initial iteration no such solution is known so the equalizer/detector reduces to the MMSE MIMO-SCFDE one. As soon as some initial estimate is available, we rely on the central limit theorem and assume that each estimated symbol closely follows a Gaussian distribution. The Gaussianity assumption becomes more robust as the dimensionality of the problem (both in terms of the number of transmit antennas but also in terms of the memory order of the frequency selective channels) increases. Asymptotic normality has been proved for linear MUD receivers in [11]. We assume that the two problems are fundamentally the same, so asymptotic normality should also hold for the TR estimate in the absence of prior solution. In the presence of the prior solution the Gaussianity of the estimate is discussed in [10]. This allows for making hard decisions (in the time domain) only for symbols which satisfy some posterior probability of error criterion. Those, which lie outside the required decision boundaries, are left unchanged. These semi-hard decisions are crucial for significant performance gains[1]. A proposal solution vector is constructed, which consists of both soft and hard estimates, the latter of which we are confident that they are correct. The semi-hard decisions are transformed back in the frequency domain and organised (in groups of N_T) in order to be incorporated as prior solutions in each of the K TR estimators, as indicated in figure 1. This Bootstrapping process is repeated for a number of iterations giving increased importance to the proposal solution in each iteration, by increasing λ^2.

The estimator used in each iteration is given by:

$$\begin{array}{r} \hat{\mathbf{x}}_k^i = (\lambda_i^2\mathbf{I}_{N_t} + \mathbf{G}^H\mathbf{G})^{-1}(\lambda_i^2\tilde{\mathbf{x}}_k^{i-1} + \mathbf{G}^H\mathbf{y}_k) \\ 0 \le k \le K-1, 1 \le i \le N \end{array} \tag{13}$$

where $\tilde{\mathbf{x}}_k$ is the proposal solution to the problem. Clearly, $\tilde{\mathbf{x}}_0 = 0$ which reduces (13) to the MMSE estimator by choosing $\lambda_0^2 = \sigma^2$. We can manipulate (13) further in order to gain a better appreciation of the effect of the prior solution. So by substituting (5) in (13) and expressing $\tilde{\mathbf{x}}$ as the true solution pertubated by some error term $\mathbf{x}_e$ we have:

$$\begin{array}{l} \hat{\mathbf{x}} = (\lambda^2\mathbf{I}_{N_t} + \mathbf{G}^H\mathbf{G})^{-1}(\lambda^2(\mathbf{x}+\mathbf{x}_e) + \mathbf{G}^H(\mathbf{G}\mathbf{x}+\mathbf{n})) \Rightarrow \\ \hat{\mathbf{x}} = \mathbf{x} + (\lambda^2\mathbf{I}_{N_t} + \mathbf{G}^H\mathbf{G})^{-1}(\lambda^2\mathbf{x}_e + \mathbf{G}^H\mathbf{n}) \end{array} \tag{14}$$

where for simplicity of representation the indices have been dropped. Classical TR (i.e. without having a prior solution) introduces a bias in the estimation in an attempt to stabilize the ill-conditioned design matrix and thus limit the variance in the solution. The balance between bias and variance in the estimation is controlled by λ^2 which should ideally be chosen to minimize the MSE. We see from (14) that $\tilde{\mathbf{x}}$ effectively offers an estimate for the bias cancellation term. In the case where $\tilde{\mathbf{x}} = \mathbf{x}$ the bias is completely removed and the noise term can be made arbitrarily small by letting $\lambda^2 \to \infty$. This observation helps to gain some understanding to how the iterative procedure can offer significant performance gains; the partial cancellation of the bias term by the prior solution makes it safe to be more daring and choose bigger values of λ^2 in subsequent iterations. This helps to limit the noise term in (14) without affecting significantly the bias term. The new TR estimate is of reduced variance and an even better estimate for the bias cancellation term is available in the next iteration. In the case where no prior solution is offered to the problem $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}$ so there is a constant error term present (bias): $-(\lambda^2\mathbf{I}_{N_t} + \mathbf{G}^H\mathbf{G})^{-1}\lambda^2\mathbf{x}$.

The developed algorithm for MIMO-ZTBFE is summarised in a form of a pseudo code in table I.

[1]Iterative TR appeared in the literature previously [12][13], however to the best of our knowledge never in conjunction with semi-hard decisions, and never in any form in digital Communications problems

TABLE I

ALGORITHM SUMMARY

Initialise:

For $0 < k < K-1$

$\hat{\mathbf{x}}_k = (\lambda_0^2 N_T \mathbf{I}_{N_t} + \mathbf{G}_k^H \mathbf{G}_k)^{-1} \mathbf{G}_k^H \mathbf{y}_k$

End

$\hat{\mathbf{X}}_F = \mathcal{R}\{\hat{\mathbf{x}}_{1:K}\}$

Bootstrap recursions:

While $i < \{i_{\max}\}$

$\hat{\mathbf{X}}_T = \mathcal{IFFT}\left\{\hat{\mathbf{X}}_F\right\}$

$\tilde{\mathbf{X}}_T = \mathcal{SHD}\left\{\hat{\mathbf{X}}_T, \Delta_i\right\}$

$\tilde{\mathbf{X}}_F = \mathcal{FFT}\left\{\tilde{\mathbf{X}}_T\right\}$

$\tilde{\mathbf{x}}_{1:K} = \mathcal{R}^{-1}\left\{\tilde{\mathbf{X}}_F\right\}$

For $0 < k < K-1$

$\hat{\mathbf{x}}_k = (\lambda_i^2 N_T \mathbf{I}_{N_t} + \mathbf{G}_k^H \mathbf{G}_k)^{-1} (\lambda_i^2 \tilde{\mathbf{x}}_k + \mathbf{G}_k^H \mathbf{y}_k)$

End

$\hat{\mathbf{X}}_F = \mathcal{R}\{\hat{\mathbf{x}}_{1:K}\}$

End

Terminate:

$\hat{\mathbf{X}}_T = \mathcal{IFFT}\left\{\hat{\mathbf{X}}_F\right\}$

IV. Reduced Complexity Implemenatation and Parameter Choice

In a direct implementation of the algorithm, the complexity of the Bootstrap TR detector is linear to the MMSE detector's complexity, as a function of iterations. The need for recalculating $(\lambda_i^2 \mathbf{I}_{N_t} + \mathbf{G}^H \mathbf{G})^{-1}$ in each iteration, arises because it is important for the performance of the algorithm to update (increase) the value of the regularization parameter. A reduced complexity implementation of the bootstrap algorithm for MUD was developed in [5]. This iterative algorithm, based on the matrix inversion lemma, which given the inverse of matrix $\mathbf{M}^{-1}$ can efficiently recompute $(\mathbf{M}+\mathbf{D})^{-1}$ where $\mathbf{D} = diag(d_1, d_2, .., d_N)$ i.e. some diagonal matrix. The number of computations required to implement the $\mathcal{MIL}$ formula can be reduced by appreciating the fact that $\mathbf{s}$ has a single non-zero element.

The number of complex multiplications is a very useful metric of an algorithm complexly. Table II lists complexities for the investigated MIMO systems. It is assumed that a number multiplications needed to compute a matrix inverse is $\mathbf{A}^{-1} \sim \frac{1}{2}n^2(n+1)$ and to compute an FFT/IFFT is $\sim K\log(K)$. The proposed reduced complexity algorithm brings down the number of required complex multiplications from $\sim \frac{1}{2}n^2(n+1)$ to $\sim n^2$ needed to update an inverse of a matrix. It has to be stressed however that the presented calculations are not perfect estimates, since the exact numbers will be dictated by a particular implementation. Also, one may take into account a fact that consecutive $\mathbf{G}_k$ are in general similar and apply iterative methods, bringing down the computational burden.

The parameter choice is not an easy task even in standard TR problem, where only a single parameter λ^2 is to be optimised. Here, the iterative formulation of TK with an additional parameter Δ complicates the matters even further (also λ_i^2 is no longer fixed). The parameter Δ sets a decision boundary in the semi-hard decisions concept. Reference [10] sheds some light on the parameter choice issue. It is shown there, that parameter choice is fortunately not a critical issue, as the semi-hard processing on the soft estimate results in an estimate of reduced variance for a wide range of values of Δ.

V. Numerical Results

In this section we investigate the performance of the proposed MIMO-ZTBFE system and compare it with some popular orthogonal multi-carrier systems. For simulation purposes we use a simple MIMO channel with $L = 11$ taps, all i.i.d. complex circular Gaussian ($\mathcal{N}\left(0, (2L)^{-1}\right)$ per dimension). The values for the ZTBFE parameters are $\Delta_i = i^{-.4}\big/\sqrt{2N_T}$ and $\lambda_i = i^{1.5}\sigma^2$, where i is the iteration number. All systems use FFTs of size 64 and QPSK. Additionally we assume that the channels are perfectly known at the receiver. We investigate two MIMO configurations of equal numbers of transmit and receive antennas. This is a challenging case since, for $N_R >> N_T$, the problem is typically well-conditioned and both MMSE and ZF can provide results that virtually coincide with ML. It can be observed form figures 2 and 3 that ZT-BFE operates with an advantage of more than 8 dB (at BER = 10^{-3}) over equivalent MMSE based MIMO-SCFDE system. Direct comparison with MLSE is impossible since the number of states is $4^{8\times 10} \approx 1.5 \times 10^{48}$. Rather than that we benchmark our system against ML (exhaustive search with number of states $4^8 = 65536$) CP-OFDM. This cannot be treated as an absolute lower bound, since uncoded OFDM does not utilise frequency diversity, nevertheless it is a useful point of reference. As can be seen the difference is ≈ 1 dB. More importantly both 8×8 and 16×16 cases attain the same performance after 21 iterations, which suggests that the proposed detection scheme achieves full diversity.

Figure 4 depicts the performance of a coded ZTBFE system. In this system, the ZTBFE detector is followed by a rate .5 turbo decoder of parallel concatenated convolutional codes (the constituent encoders are [7,5]). The turbo loop does not comprise the ZTBFE - the iterations in the figure refer to the turbo code iterations. We benchmark our coded system against Turbo-MIMO-OFDM with APP decoder in the first SISO block. The second SISO block uses the same decoder as the constituent decoders in our system. Here the iterations in the figure refer to outer iterations. Both systems use 4Tx by 4Rx antennas, and the cases are equivalent in terms of FFT sizes (256), modulation (QPSK) and coding redundancy (rate = .5). Both systems achieve reliable transmission of $BER = 10^{-6}$ at $SNR = 8dB$. The complexity of our system is drastically reduced (see table II). In fact this is the binary turbo decoder that dominates the complexity curve of our system.

VI. Conclusions

We have developed a new class of equalisation/detection scheme for MIMO communications. The scheme is based on iterative bootstrap detector, which was originally proposed for multi-user detection in CDMA [5]. We have demonstrated excellent performance over wideband channels. The complexity is only moderately greater than that of MMSE. Since FFT/IFFT operations can be hardware accelerated, the complexity can be reduced even further when implemented in hardware.

TABLE II

COMPLEXITY ORDER OF THE INVESTIGATED SYSTEMS

System	Number of complex multiplications
MMSE(ZF)-SCFDE	$K\left[(N_T+N_R)\log(K)+N_T\left(2N_RN_T+N_R+\frac{1}{2}N_T^2+\frac{1}{2}N_T\right)\right]$
ML-CPOFDM	$K\left[\log(K)(N_T+N_R)+2N_RM^{N_T}\right]$
ZTBFE	$i_{max}K\left[(N_T+N_R)\log(K)+N_T\left(2N_RN_T+N_R+\frac{1}{2}N_T^2+\frac{1}{2}N_T\right)\right]$
ZTBFE (RC)	$K\left[(N_T+N_R)\log(K)+N_T\left(2N_RN_T+N_R+\frac{1}{2}N_T^2+\frac{1}{2}N_T\right)\right]+(i_{\max}-1)K\left[(N_T+N_R)\log(K)+N_T(2N_RN_T+N_R+N_T)\right]$

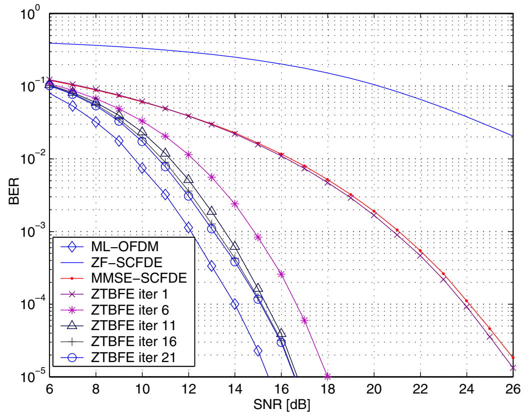

Fig. 2. Performance comparison of various multi-carrier schemes over MIMO wideband channels. $N_T = N_R$ = 8, L = 11, all taps i.i.d. Gaussian.

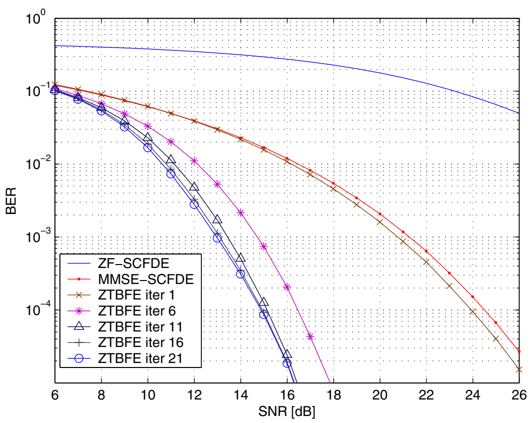

Fig. 3. Performance comparison of various multi-carrier schemes over MIMO wideband channels. $N_T = N_R$ = 16, L = 11, all taps i.i.d. Gaussian.

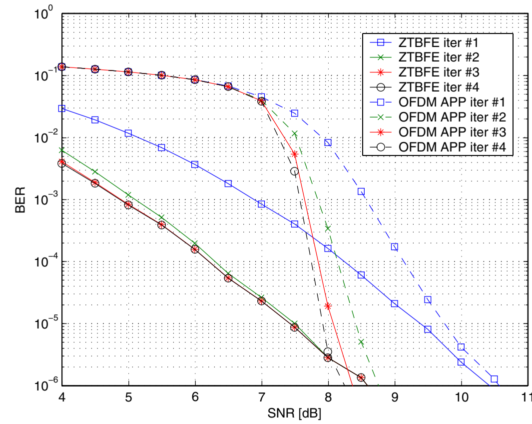

Fig. 4. Performance comparison of Turbo-MIMO-OFDM and coded ZTBFE over wideband channels. $N_T = N_R$ = 4, L = 16, all taps i.i.d. Gaussian.

ACKNOWLEDGMENTS

R. Piechocki would like to thank Toshiba TREL Ltd for sponsoring his research activities. C. Kasparis would like to acknowledge QinetiQ Ltd for the financial support.

REFERENCES

[1] G. Foschini, "Layered space-time architecture for wireless communication in fading environment when using multi-element antennas," *BLTJ, Autumn*, 1996.

[2] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Edison, "Frequency domain eqialization for single carrier broadband wireless systems," *IEEE Communications Magazine*, pp. 58 – 66, 2002.

[3] Shengli Zhou and Georgios B. Giannakis, "Space-time coding with maximum diversity gains over frequency selective fading channels," *IEEE Signal Processing Letters*, vol. 8, no. 10, pp. 269 – 271, 2001.

[4] J. Coon and M. Beach, "An investigation of mimo single carrier frequency domain mmse equalization," *in proc. of London Communications Symposium, UCL, 9-10 Sep 2002*, vol. 1, pp. 237 – 240, 2002.

[5] Christos Kasparis, Robert J. Piechocki, Paul N. Fletcher, and Andrew R. Nix, "A bootstap multi-user detection for cdma based on tikhonov regularization," *in proc. of IEEE International Conference on Acoustics, Speech, and Signal Processing 2003 (ICASSP03)*, 2003.

[6] B. Muquet, M. de Courville, G. B. Giannakis, Z. Wang, and P. Duhamel, "Reduced complexity equalizers for zero-padded ofdm transmissions," *in Proc. of Intl. Conf. on ASSP, Istanbul, Turkey, June 5-9*, 2000.

[7] A. Scaglione, G. Giannakis, and Brarbarossa, *Linear Precoding for Estimation and Equalisation of Frequency Selective Channels in Signal Processing Advances in Wireless and Mobile Comunications; Edited by G. Giannakis, Y. Hua, P. Stoica, and L. Tong*, Prentice Hall, 2001.

[8] A.N. Tikhonov and V.Y. Arsenin, *Solutions of Ill-Posed Problems*, Winston, Washington, D.C., 1977.

[9] P.C. Hansen, *Rank deficient and discrete ill-posed problems*, SIAM, Philadelphia, 1998.

[10] C. Kasparis, R.J. Piechocki, P.N. Fletcher, and A.R. Nix, "Bootstrap multi-user detection for ds-cdma based on iterated tikhonov regularization," *submitted to EURASIP Journal on Applied Signal Processing, July 2003*.

[11] D.Guo, S.Verdu, and L.K. Rasmussen, "Asymptotic normality of linear multiuser receiver outputs," *IEEE Trans. On Information Theory*, vol. 48, no. 12, pp. 3080 – 3095, 2002.

[12] J.T. King and D. Chillingworth, "Approximation of generalized inverses by iterated regularization," *Numerical Functional Analysis and Optimazation*, vol. 1, pp. 499 – 513, 1979.

[13] H.W. Engl, "On the choice of regularization parameter for iterated tikhonov regularization of ill-posed problems," *Journal of Approximation Therory*, vol. 49, pp. 55 – 63, 1987.