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A Bandwidth Efficient Channel Estimation Algorithm for MIMO-SCFDE

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Abstract— **This paper proposes a novel method for channel estimation in a single-carrier MIMO system with frequency-domain equalization/detection. To this end, we construct novel short MIMO training sequences that have constant envelope in the time domain to preclude the peak-to-mean power problem encountered in many systems that utilize the frequency domain for data recovery. Simultaneously, the spectrum in the frequency domain is flat except for a grid of nulls for predefined frequency tones. Such construction allows us to interpolate over nulled tones to recover the full set of MIMO channel state information (CSI). The interpolation is performed by a specially crafted algorithm which iteratively reconstructs the full CSI by a series of FFT and IFFT operations. This algorithm is extremely bandwidth efficient in that the total training overhead required to obtain full CSI is just one block period.**

I. Introduction

The detection of information symbols transmitted over a broadband wireless channel is an example of a more general problem of inference in latent variable models. A common way of reducing the complexity of this detection problem is to first estimate the channel, then use this estimate to recover the information at the receiver.

A very popular low-complexity broadband technique that has seen extensive research in the past decade is orthogonal frequency division multiplexing (OFDM). Lately, however, much attention has been focused on another broadband technique, namely single-carrier (SC) transmission with frequency-domain equalization (FDE) [1], [2], [3]. SCFDE systems promise equal complexity and performance to OFDM systems [4] without the high peak-to-average power (PAPR) problem that plagues OFDM systems. Although channel estimation in OFDM systems has been studied extensively in both the single and multi-antenna cases, this topic has been left relatively unaddressed for SC-FDE systems.

In [5] and [6], recursive reconstructive (RR) channel estimation methods were proposed for single-input single-output (SISO) OFDM and multiple-input multiple-output (MIMO) OFDM systems, respectively. This method uses repeated IFFT and FFT operations to reconstruct the channel response. An important advantage in using this method in MIMO systems is that the entire channel response can be obtained after one transmission period, thus allowing more information to be transferred over a given time period. In this paper, the RR algorithm is extended to MIMO-SCFDE systems by employing novel training sequences based on modified Chu sequences [7].

The paper is organized as follows. Section II mathematically describes the system and channel. In section III, the RR algorithm is described and the novel design of the training sequences is detailed in section IV. Convergence, mean square error, and complexity properties of the RR algorithm are discussed in section V. Simulation results and discussions are presented in section VI and conclusions are given in section VII.

Notation: In this paper, vectors are represented by lowercase bold-faced letters and matrices are denoted by uppercase bold-faced letters. Underlined vectors/matrices represent concatenated vectors/matrices. The transpose of a matrix is denoted by $\{\cdot\}^T$ while the conjugate transpose of a matrix is denoted by $\{\cdot\}^H$. $\mathcal{E}\{\cdot\}$ denotes the expectation operation and $\mathrm{tr}\{\cdot\}$ denotes the trace of a matrix. $\mathbf{I}_N$ represents the $N \times N$ identity matrix and $\mathbf{F}$ is the $N \times N$ normalized FFT matrix.

II. System and Channel Description

Consider a wideband MIMO-SCFDE system with n_T transmit antennas and n_R receive antennas in which a length-N baseband sequence is modulated onto a single carrier waveform at each transmit antenna for transmission across a wireless channel. The received baseband sequences are equalized in the frequency domain. A cyclic prefix is added to each sequence prior to transmission and removed from each of the received sequences. We assume that the cyclic prefix is greater than or equal to the channel memory order, thereby preventing interblock interference and aiding the equalization process.

Let $\bar{\mathbf{t}}_q$ be the length-N time domain training sequence transmitted from antenna q, and let $\mathbf{h}_{p,q}$ be a length-N column vector that denotes the complex frequency response of the channel between transmit antenna q and receive antenna p. We denote the channel coefficient on the nth tone by $h_{p,q;n}$. The entire channel can be represented by the vector $\underline{\mathbf{h}} = \left[\mathbf{h}_{1,1}^T, \mathbf{h}_{1,2}^T, \ldots, \mathbf{h}_{n_R,n_T}^T\right]^T$.

An N-point FFT is employed prior to equalization at the receiver of a MIMO-SCFDE system. In the frequency domain, the received signal at the pth antenna on the nth tone can be mathematically described by

$$y_{p;n} = \sum_{q=1}^{n_T} h_{p,q;n} \overline{x}_{q;n} + \eta_{p;n} \tag{1}$$

where $\overline{x}_{q;n}$ denotes the nth tone of the training sequence transmitted from the qth antenna and the noise term $\eta_{p;n} \sim \mathcal{CN}\left(0, \sigma^2\right)^1$. As shown in (1), the signal received at the pth antenna is a superposition of all of the transmitted training signals, which complicates symbol detection and training. For data detection, the received signal is equalized and transformed back into the time domain with an IFFT. For channel estimation, however, the RR algorithm can be employed following the FFT as described in the next section.

III. Recursive Reconstruction for MIMO-SCFDE Systems

The advantage of using the RR algorithm is that it allows the transmitted sequence to be nulled on certain frequency tones, causing the transmitted training sequences to be orthogonal in the frequency domain. Thus, (1) reduces to

$$y_{p;n} = h_{p,q;n}\overline{x}_{q;n} + \eta_{p;n}, \qquad n \in \mathbf{\Omega}_q \tag{2}$$

where $\mathbf{\Omega}_q$ is the set of tones over which training data is transmitted from the qth antenna. The nulled tones are then reconstructed at the receiver to provide a full channel estimate. A visualization of the RR process is illustrated in Figure 1.

The RR algorithm executes the following steps for each channel path to reconstruct the entire frequency response [5].

1) Obtain initial channel estimate.
2) Convert channel estimate into the time domain and window significant taps.
3) Convert this time domain signal back into the frequency domain.
4) Replace the values of the frequency tones in Ω_q with the initial estimate in step 1 (ignore this step for the last iteration).
5) Repeat steps 2-4.

A. Obtaining the Initial Estimate, $\widehat{\underline{\mathbf{h}}}^{(0)}$

The initial estimate can be obtained by simply dividing the received signals on the tones in $\mathbf{\Omega}_q$ by the unique transmitted training signals.

$$\widehat{h}^{(0)}_{p,q;n} = \begin{cases} \frac{y_{p;n}}{\overline{x}_{q;n}} & n \in \mathbf{\Omega}_q \\ 0 & n \notin \mathbf{\Omega}_q \end{cases} \tag{3}$$

B. Mathematical Description of the Algorithm

The complete channel estimate after i iterations for all channel paths, $\widehat{\underline{\mathbf{h}}}^{(i)}$ is given by

$$\widehat{\underline{\mathbf{h}}}^{(i)} = \underline{\mathcal{K}}\widehat{\underline{\mathbf{h}}}^{(0)} \tag{4}$$

where $\underline{\mathcal{K}} = diag\left\{\sum_{m=0}^{i}\mathbf{K}_{1,1}^{m}, \ldots, \sum_{m=0}^{i}\mathbf{K}_{n_R,n_T}^{m}\right\}$ and $\widehat{\underline{\mathbf{h}}}^{(0)}$ is the initial channel estimate. The matrix $\mathbf{K}_{p,q}$ is a $N \times N$ matrix defined by

$$\mathbf{K}_{p,q} = \mathbf{W}_{F,q}\mathbf{F}\mathbf{W}_{T,p,q}\mathbf{F}^{-1} \tag{5}$$

[1] $\mathcal{CN}$ denotes the complex normal distribution.

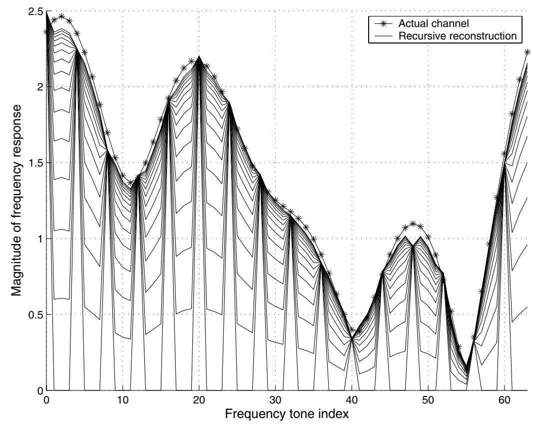

Fig. 1. Illustration of the RR process for a single channel in a MIMO system where $n_T = 4$, $N = 64$, and SNR = 20 dB.

$\mathbf{W}_{F,q}$ and $\mathbf{W}_{T,p,q}$ are rectangular windowing matrices where $\mathbf{W}_{F,q}$ is an $N \times N$ diagonal matrix with 1's at the indices given by the complement of $\mathbf{\Omega}_q$ and $\mathbf{W}_{T,p,q}$ is an $N \times N$ matrix with 1's on the main diagonal where time delay taps exist and zeros elsewhere for the channel path between transmit antenna q and receive antenna p. Equation (4) can be rewritten in terms of the actual channel frequency response $\underline{\mathbf{h}}$ by noting that the initial estimate

$$\widehat{\mathbf{h}}^{(0)}_{p,q} = \left(\mathbf{I} - \mathbf{W}_{F,q}\right)\left(\mathbf{h}_{p,q} + \eta\right) \tag{6}$$

where η is the $N \times 1$ noise vector. Letting

$$\begin{aligned} \underline{\mathbf{\Gamma}} &= diag \quad \Bigg\{\sum_{m=0}^{i}\mathbf{K}_{1,1}^{m}\left(\mathbf{I} - \mathbf{W}_{F,1}\right), \ldots, \\ &\qquad \sum_{m=0}^{i}\mathbf{K}_{n_R,n_T}^{m}\left(\mathbf{I} - \mathbf{W}_{F,n_T}\right)\Bigg\}, \end{aligned}$$

we can rewrite (4) to give

$$\widehat{\underline{\mathbf{h}}}^{(i)} = \underline{\mathbf{\Gamma}}\left(\underline{\mathbf{h}} + \underline{\eta}\right). \tag{7}$$

The matrix $\underline{\mathbf{\Gamma}}$ mathematically represents the RR algorithm performed for each antenna after i iterations where frequency tones are strategically nulled such that the training sequences are unique and orthogonal to each other in the frequency domain. The design of training sequences with these properties is addressed in section IV.

IV. Training Sequence Design

As previously mentioned, the intentional nulling of frequency tones in the transmitted training sequences is a key requirement for bandwidth efficiency in using the RR algorithm. One such "frequency-selective" training sequence can be constructed for transmission from the first antenna by simply repeating any arbitrary length-(N/n_T) sequence, denoted by $\mathbf{t}_1 \triangleq (t_{1;0}, \ldots, t_{1;N/n_T-1})$, n_T times to give

$\bar{\mathbf{t}}_1 \triangleq (\bar{t}_{1;0}, \ldots, \bar{t}_{1;N-1})$. For this method, N must be evenly divisible by n_T. The nth tone of the FFT of $\bar{\mathbf{t}}_1$ is given by

$$\bar{x}_{1;n} = \begin{cases} \chi_{1;n}, & n \bmod n_T = 0 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

where $\chi_{1;n} \in \mathbb{C}$. Equation (8) suggests that the set of nonzero frequency tones for the sequence $\bar{\mathbf{t}}_1$ is $\mathbf{\Omega}_1 = \{0, n_T, 2n_T, \ldots, N - n_T\}$. The proof of (8) is given in the Appendix.

To refrain from transmitting from multiple antennas on a given frequency tone, the set of nonzero frequency tones $\mathbf{\Omega}_q$ for a given sequence may be shifted for use by other antennas. This shifting is accomplished by progressively rotating the phase of the length-N time-domain training sequence such that the kth element of the sequence transmitted from the qth antenna for $q = 2, 3, \ldots, n_T$ is given by

$$\bar{t}_{q;k} = \bar{t}_{1;k} e^{j2\pi f_q k} \quad (9)$$

where f_q is the desired frequency shift for the sequence related to the qth transmit antenna. This result follows from a property of Fourier transforms.

Random training sequences are not desirable for this technique because in general a sequence will not have a constant envelope in the frequency domain, resulting in sensitivity to noise. This problem is remedied through the use of Chu sequences as base training sequences, which have a constant envelope in both the time domain and the frequency domain [7]. Although Chu sequences are constructed from an infinite alphabet of constant-modulus symbols, they can be realized with direct digital synthesis (DDS) devices.

V. Performance Analysis

A. Convergence

From the equations above, the term $\sum_{m=0}^{i} \mathbf{K}_{p,q}^{m}$ is a geometric series. Thus, under noiseless conditions, convergence occurs if $\|\mathbf{K}\| < 1$ or the spectral radius $\rho(\mathbf{K}) < 1$ where $\rho(\mathbf{K}) \triangleq \max_i |\lambda_i(\mathbf{K})|$ and $\lambda_i(\mathbf{K})$ are the eigenvalues of $\mathbf{K}$ [8]. In the limit, as $i \to \infty$, the algorithm converges to:

$$\lim_{i\to\infty} \left(\sum_{m=0}^{i} \mathbf{K}^m \right) = (\mathbf{I} - \mathbf{K})^{-1} \quad (10)$$

$(\mathbf{I} - \mathbf{K})$ will be a full-rank square matrix and invertible if the elements of the main diagonal in $\mathbf{K} \neq 1$ thus ensuring the existence of $(\mathbf{I} - \mathbf{K})^{-1}$. Divergence only occurs if

$$\begin{gathered} \operatorname{span}\left(\mathbf{F}^{-1}\mathbf{h}_{p,q}\right) \subseteq \operatorname{span}\left(\mathbf{W}_T\right) \\ \text{and} \\ \operatorname{span}\left(\mathbf{h}_{p,q}\right) \subseteq \operatorname{span}\left(\mathbf{W}_F\right) \end{gathered} \quad (11)$$

are simultaneously satisfied [6].

B. Mean-Squared Error

The error covariance matrix, $C_{\epsilon\epsilon}$, for this algorithm is given by

$$C_{\epsilon\epsilon} = \left(\underline{\mathbf{I}} - \underline{\mathbf{\Gamma}}\right)\underline{\mathbf{R}}_h \left(\underline{\mathbf{I}} - \underline{\mathbf{\Gamma}}^H\right) + \underline{\mathbf{\Gamma}}\underline{\sigma}^2\underline{\mathbf{\Gamma}}^H \quad (12)$$

and

$$MSE \propto \operatorname{tr}\{C_{\epsilon\epsilon}\} \quad (13)$$

where $\underline{\mathbf{R}}_h = \mathcal{E}\left\{\underline{\mathbf{h}}\underline{\mathbf{h}}^H\right\}$ and $\underline{\sigma}^2 = \mathcal{E}\left\{\underline{\eta}\underline{\eta}^H\right\}$ is the noise variance. Note that some error is introduced by the algorithm itself while some error is introduced by noise as expected [6]. In Figure 2, the MSE of the RR channel estimate is plotted against SNR for various numbers of iterations. An important conclusion that can be drawn from Figure 2 is that the performance of the RR algorithm is very sensitive to the number of iterations that are implemented.

C. Complexity

The complexity γ of the algorithm, which is based on the number of multiplications required, is given by [6]

$$\gamma = n_T n_R \left(2iN \log_2 N + \mu\right) \quad (14)$$

where μ is the number of divisions needed to obtain the initial estimate. Note that the complexity of this algorithm increases linearly with the number of iterations performed and does not directly depend on the length of the channel.

VI. Simulation Results

The performance of the proposed algorithm was examined by simulating several different MIMO-SCFDE systems and observing the bit error rate for each system. All simulated systems utilized $n_T = 4$ transmit antennas and $n_R = 4$ receive antennas. For each system, packets of 1024 information bits were mapped to 16-QAM symbols, which were then partitioned into $N = 64$ symbol blocks. The blocks were then encoded according to one of two space-time processing techniques: spatial multiplexing (SM) [9] and space-time block codes (STBC) [2], [10]. A cyclic prefix of 10 symbols was appended to each block at the output of the space-time encoder to combat interblock interference. The packet was then transmitted through a MIMO wireless channel where each channel path was modeled as having 11 discrete taps, which were normalized such that the average power of each tap was 1/11. Furthermore, each tap was independently faded following a Rayleigh profile. In total, 10000 channel realizations were generated for a given SNR in each performance simulation.

At the receiver, two methods of estimating the channel were employed for each system: the RR method and a least squares (LS) method, first proposed in [11] for OFDM systems with transmit diversity and modified here for use with SCFDE. As a benchmark, one system was assumed to have perfect knowledge of the channel. Using knowledge of the estimated channel, MMSE equalization was then performed on a block-by-block basis in the frequency domain to recover the transmitted message. The results of this performance analysis are illustrated in Figure 3.

It is interesting to note from Figure 3 that the systems employing the RR method perform better than the system using LS channel estimation and one training block period. The LS method outperforms the RR method only when four block periods are used for LS channel estimation. Furthermore,

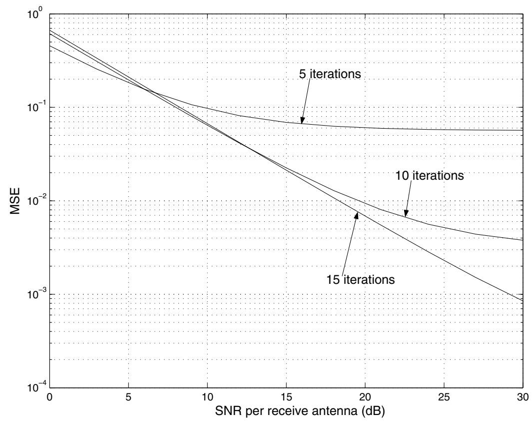

Fig. 2. MSE vs. SNR for various number of iterations. ($n_T = n_R = 4$, $N = 64$, 11-tap channel with Rayleigh fading)

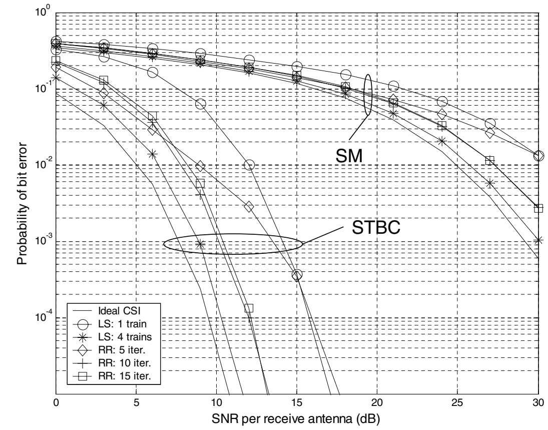

Fig. 3. Performance of STBC/SM-SCFDE systems. ($n_T = n_R = 4$, 16-QAM, 11-tap channel with Rayleigh fading, $N = 64$)

only 1.5 dB is gained over the RR method when LS estimation is implemented with the increased training overhead. The merits of the RR method are, therefore, clearly observed in Figure 3.

VII. Conclusions

In this contribution we have developed a novel technique for MIMO wireless systems that utilize frequency domain equalization/detection. The originality of the method is based on specially designed training sequences and the detection algorithm. Specific construction of the training sequences combined with the time limited property of the wireless channel (FIR filter) allows for efficient utilization of the bandwidth with minimal noise amplification during the estimation procedure. The estimation procedure itself, as a series of FFT and IFFT operations, is computationally efficient. Furthermore, the estimation complexity can be reduced via hardware acceleration of FFT/IFFT blocks. The proposed method offers a very competitive complexity/performance trade-off compared with the LS estimation technique. In fact, in a fair comparison (the same bandwidth utilization) it outperforms the LS estimation technique with the complexity advantage dependent upon the channel length.

Appendix

Proof of (8):

Given the sequence $\mathbf{t}_1 \triangleq (t_{1;0}, \ldots, t_{1;K-1})$, where $K = N/n_T$ and $\gcd(N, n_T) = n_T$, we construct a sequence

$$\begin{aligned}\bar{\mathbf{t}}_1 &\triangleq (\bar{t}_{1;0}, \ldots, \bar{t}_{1;N-1}) \\ &= (\underbrace{t_{1;0}, \ldots, t_{1;K-1}}_{\text{repeated } n_T \text{ times}}, \ldots).\end{aligned}$$

Omitting the subscript 1 for brevity, the nth component of the discrete Fourier transform of $\bar{\mathbf{t}}$ is given by

$$\begin{aligned}\bar{x}_n &= \sum_{k=0}^{N-1} \bar{t}_k e^{-j\frac{2\pi nk}{N}} \\ &= \sum_{\ell=0}^{n_T-1} \left(\sum_{k=\ell K}^{(\ell+1)K-1} \bar{t}_k e^{-j\frac{2\pi nk}{N}} \right) \\ &= \sum_{\ell=0}^{n_T-1} \sum_{k=0}^{K-1} t_k e^{-j\frac{2\pi nk}{N}} e^{-j\frac{2\pi n}{N}\ell K} \\ &= x_n^{(n_T)} \underbrace{\sum_{\ell=0}^{n_T-1} e^{-j\frac{2\pi n\ell}{n_T}}}_{E}\end{aligned}$$

where $x_n^{(n_T)} \triangleq \sum_{k=0}^{K-1} t_k e^{-j\frac{2\pi nk}{n_T K}}$. It is sufficient to show that

$$E = \begin{cases} n_T, & n \bmod n_T = 0 \\ 0, & \text{otherwise.} \end{cases}$$

We can write $n = mn_T + b$ where $m \in \mathbb{N}$ and $b = 0, 1, \ldots, n_T - 1$. Thus, when $b = 0$, we have the condition $n \bmod n_T = 0$, and when $b \neq 0$, we have the condition $n \bmod n_T \neq 0$. Therefore, we can write

$$\begin{aligned}E &= \sum_{\ell=0}^{n_T-1} e^{-j2\pi m\ell} e^{-j\frac{2\pi b\ell}{n_T}} \\ &= \sum_{\ell=0}^{n_T-1} e^{-j\frac{2\pi b\ell}{n_T}} \\ &= \begin{cases} n_T, & b = 0 \\ 0, & \text{otherwise.} \end{cases}\end{aligned}$$

□

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