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# Algorithms for Flexible Equalisation in Wireless Communications

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**Abstract :** In this paper the requirements for a flexible equaliser architecture for wireless communications are discussed and illustrated with results from simulation. Decision Feedback Equalisers are compared in terms of their performance and computational requirements. It has been found that the recursive modified Gram Schmidt algorithm provides superior BER performance to the least squares lattice decision feedback lattice and the conventional RLS algorithm. In addition the problem of synchronising the equaliser to achieve best performance and exploit the multipath propagation of the wideband channel is addressed. For a two path Rayleigh fading channel model it is shown that best performance is obtained by synchronising the frame to the arrival time of the first multipath rather than the dominant path.

## I. Introduction

There are several major TDMA systems currently in operation or being deployed throughout the world [1]. In most systems equalisation is required to maintain acceptable performance. With the development of future generations of universal mobile communication systems (UMTS/MBS) there is likely to be significant interest in a flexible equaliser architecture which can be configured to compensate for highly variant channel conditions. In this paper the issues concerning the choice of algorithm for such a flexible equaliser are discussed. The training data in current TDMA systems is located at either the start (a preamble) or centre (a midamble) of a transmitted packet. It is proposed therefore that the equaliser is trained off-line by buffering each received frame [7]. This allows symbol timing and frame synchronisation to be determined prior to training. The choice of equaliser length and exponential weighting factor required for starting up the DFE are described in section II. Due to the very large time delay spread of the new ETSI Hiperlan standard [7], a low complexity algorithm is proposed for this system in section II. The issue of frame synchronisation is discussed in section III.

## II. Equaliser Algorithms

Decision Feedback Equalisers (fig1) trained by the recursive least squares (RLS) algorithm are chosen because of their fast convergence and relatively low complexity [2]. In addition, the basic structure of the equaliser is

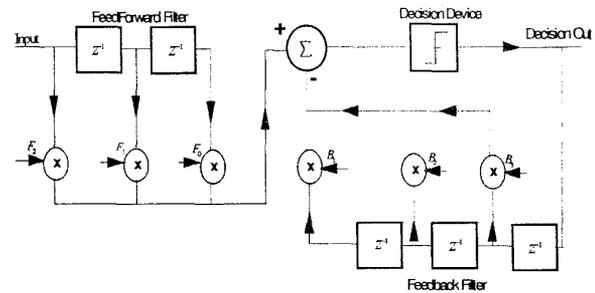


Figure 1 Transversal DFE

independent of system type and it is therefore relatively simple to reconfigure the equaliser for each system. Important factors that affect the performance and determine how the equaliser is set up are the equaliser length and the exponential weighting factor applied to the input data. The length can be selected from a channel impulse response (CIR) estimate derived from the synchronisation sequence. The clocking rate must be adjusted to deal with the different data rates. The appropriate training sequence must also be supplied, together with the modulation scheme for decision directed feedback. The four least-square algorithms considered here (a-priori forms) are the Direct form RLS, Complex Fast Kalman Algorithms based on transversal filter structures [2,4], the Lattice Least Squares Decision Feedback equaliser [5] and the Recursive Modified Gram Schmidt algorithm [6].

### II.1 Simulation Study

For the study, simulated GSM and NADC(IS-54) system models have been developed based on the recommendations in [3] and [4]. In the NADC simulation, root raised cosine filtering at the receiver and transmitter ( $\alpha = 0.35$ ) is used. The use of both  $\pi/4$ -DQPSK and QPSK has been considered. For the GSM system, Gaussian pulse shaping in the transmitter is used. At the receiver a Butterworth five pole filter was used with 3-dB bandwidth  $0.5/T$  [2]. The results obtained using QPSK modulation are shown in fig 2 for a two path channel with independently faded paths of equal mean power. For each frame a new random channel realisation was used. The performance of the RMGS algorithm was found to be marginally better than the LSL and DRLS algorithms. The use of decision

directed update was found to cause a significant loss in performance.

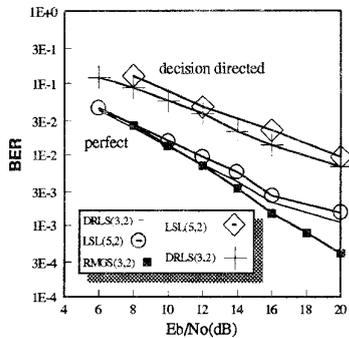


Figure 2: Comparative performance of DFE equalisers at 20mph over a two path channel.

### II.2 Equaliser Time Span and Complexity

The computational complexity of any algorithm is dependent on the required equaliser length. For the NADC and JDC systems the time dispersion covers at most two symbol periods so a short equaliser can be used. In [4] it has been shown that for ISI spanning less than a third of a symbol period an equaliser will degrade performance and should therefore be switched out. The complexity of the algorithms are compared below based on the number of feedforward ( $N_1$ ) and feedback ( $N_2$ ) stages, where  $N=N_1+N_2$  is the total number of stages. For the CFKA  $p=2$  or 3 depending on whether fractionally or symbol spaced feedforward taps are used.

Alg	Comp $\times$	Comp $\div$	Comp $+$
1	$2N+1$	0	$2N-1$
2	$2.5N^2 + 3.5N$	2	$2N^2 + 2N$
3	$(N+6)p^2 + (7N+2)p + 2N$	$2p^2$	$(N+5)p^2 + (6N-2)p + 2N$
4	$17N_1 + 49N_2 - 43$	$2N_1$	$11N_1 + 23N_2 - 20$
5	$2.5N^2 + 9N$	$N^2/2 + 2N$	$N^2 + 3N$
6	$5N^3/6 + N^2/2 - N/2$	$N$	$5N^3/6 + N^2/2 - N/2$

(1) LMSDFE, (2)DRLS, (3)CFKA (4)LSLDFE (5) RMGS, (6)Cholesky Decomposition (also requires N square roots)

Table 1:Relative Complexity of DFE Algorithms

The Cholesky decomposition method in Table 1 is used to compute the DFE tap weights of a transversal filter directly from the Wiener Hopf equations. The relatively high computational count for this approach is offset by the fact that this operation is carried out only once for the packet. It is therefore most appropriate for quasi static channels when channel tracking is not required. The computational flow is also less regular and might make implementation difficult.

The fast RLS algorithms only provide significant computational savings when the equaliser length is greater than five symbols [2]. For the short CIRs occurring on the

NADC and DECT systems these fast algorithms may not be the most appropriate choice. However, the modularity of the LSL and RMGS algorithms can be exploited to increase processing speed which is particularly important for high speed systems. The length of the equaliser is also restricted by the amount of training data provided and therefore relatively short equalisers must be used. The complexity of the DRLS and RMGS algorithms are therefore comparable to the fast algorithms. However the need for fractionally spaced feedforward taps will increase the number of equaliser taps.

### II.3 Channel Tracking

Channel time variation requires the use of tracking. The GSM system has specified the use of a midamble training sequence which minimises the channel variation across the packet. For the NADC system the longer frame duration results in significant time variation. For a DFE the choice of the tracking parameter is critical and must be chosen to balance the algorithm noise introduced and the need to track rapid channel variations. Figure 3 shows the performance of the RMGS algorithm for the NADC simulation (50mph) as a function of the weighting factor and  $E_b/N_0$ . At low values of  $E_b/N_0$  a low value of weighting factor introduces an unacceptable level of additional noise and so larger values are required. This is true of the other algorithms, but it was found that the optimum values for the weighting factor in these algorithms were generally larger. The results indicate that performance degrades rapidly when channel tracking is not used and therefore the tracking parameter should generally be kept small if the rate of change of the channel is unknown.

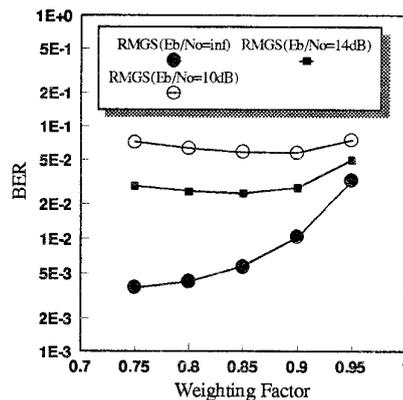


Figure 3: Effect of varying weighting factor as a function of  $E_b/N_0$

### II.4 Combined LMS/CIR Estimation Equaliser

The proposed Hiperlan standard [7] is being developed to support data rates of approximately 20Mb/s. At this high rate the ISI is predicted to cover up to seven symbol periods. An implementation of the Square Root Kalman algorithm for use on this system is currently being developed but requires the use of four C40 processors in

parallel [7]. An alternative approach is to compute the equaliser tap weights directly from the Wiener-Hopf equations formed from an estimate of the CIR. For illustration, the performance of this type of approach is compared with the DRLS algorithm over a simplified form of the model adopted in [9].

The approach adopted here is to use the solution derived from the Wiener-Hopf equations (without noise estimation) to initialise the tap weights of the equaliser. The equaliser is then fine adjusted using the LMS algorithm, by reusing the training data. This approach does not require any assumptions regarding the correlation of the interfering noise. The Wiener-Hopf equations were solved using a Cholesky factorisation of the data correlation matrix. A 48 symbol synchronisation sequence was used as in [9] to derive the channel estimate and carry out frame synchronisation. The performance of the proposed method is compared with the conventional RLS algorithm in fig 4 over a channel with a RMS delay spread of approximately 20ns. The effect of varying the excess bandwidth of the receive filtering has also been varied.

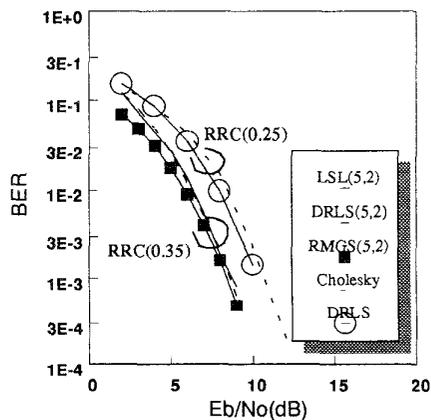


Figure 4: Comparison of DFEs over a stationary channel

The difference in performance between the DRLS algorithm and LMS based algorithm is very slight. This could be due to the slightly larger excess mean square inherent in the LMS algorithm. However convergence of the LMS algorithm was possible within the given training length. The comparatively low complexity of this type of approach over the DRLS algorithm makes it attractive for the Hiperlan system. This technique can also be applied to the other systems at reasonable computational expense, provided channel variations are not too rapid.

### III. Synchronisation

The issue of frame synchronisation is described in this section. A 2-Path channel model that has been used

extensively in previous simulation studies [10] is considered. In general, the CIR is represented by the complex transfer function (for multipaths separated by a symbol period)

$$H(z) = h_0 + h_1 z^{-1} \quad (1)$$

where  $h_i$  is a Rayleigh fading process. A real CIR will be considered here for ease. It is assumed that the transmitted data and channel multipaths are uncorrelated. For a minimum phase impulse response the optimum equaliser coefficients are given by

$$F_0 = 1/h_0, B_1 = -h_1/h_0, F_k = B_k = 0 \quad K \geq 1 \quad (2)$$

where  $F_k$ ,  $k = 0..N_f$  and  $B_k$ ,  $k = 1..N_b$  are the feedforward and feedback equaliser taps respectively (Figure 1). For a non minimum phase channel, the equaliser tap weights are dependent on the burst timing adopted. In [10] it was assumed that the received sample at time  $t$  from which an estimate of  $d(t)$  is made is given by

$$u(t) = h_0 d(t+1) + h_1 d(t) + \eta(t) \quad (3)$$

where  $\eta(t)$  is additive Gaussian noise. In this case the ISI is entirely precursor and the feedback filter is not used. By fixing the starting point at the arrival time of the first multipath, the ISI will now be entirely post cursor and the decision feedback effect is obtained. The sample  $u(t+1)$  contains the desired symbol as well and therefore the feedforward filter contains signal energy from the two multipaths. In [10] because of the timing adopted, a variable reference tap was used, which was set at the penultimate feedforward filter tap. This has the same effect as sampling the first arriving ray. For the non minimum phase CIR, the equaliser tap values are given by [8]

$$F_0 = B_1/h_1, F_k = \left(\frac{-h_0}{h_1}\right)^{k-1} \left(\frac{1 - (h_0/h_1)B_1}{h_1}\right) \quad (4)$$

This solution in terms of the feedback tap has also been obtained from the Wiener-Hopf equations. The exact solution in [8] was obtained by choosing  $B_1$  to minimise the noise power at the output of the equaliser. The power spectral density of the noise is  $N_0$  and therefore the noise power at the equaliser output is

$$N_{tot} = N_0 \sum_{k=0}^{N_f} |F_k|^2 = \frac{N_0}{h_1^2} \left[ B_1^2 + (1 - HB_1)^2 G \right] \quad (5)$$

where

$$G = \frac{1 - H^{2N_f}}{1 - H^2}, H = h_0/h_1$$

The total noise is minimised when

$$B_1 = H \left( \frac{1 - H^{2N_f}}{1 - H^{2(N_f+1)}} \right) \quad (6)$$

This solution assumes that the residual ISI is negligible and that noise is present. It has been found that as the noise

power becomes vanishingly small, in order to minimise the residual ISI, only the feedforward filter tap  $F_0$  will be non zero. To account for this, the residual ISI power contributed by the last feedforward tap is included in (5) which leads to a modified solution for  $B_1$ .

$$B_1 = \frac{HGN_0 + |h_0|^2 H^{2N_f}}{N_0 + H^2GN_0 + H^{2N_f+1}|h_0|^2} \quad (7)$$

To compare performance for minimum and non minimum phase channels, the desired signal energy (obtained from the weighted sum of the components of the desired symbol in the input samples contained in the feedforward section) is compared to the total interference (noise and residual ISI). Assuming  $|h_0|^2 + |h_1|^2 = 1$  and an Eb/No value of 20dB the ratio of the power in the desired symbol to the interfering power as a function of the relative power in the two paths is shown in Fig 5.

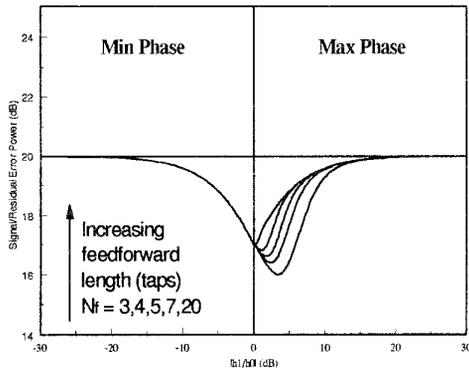


Figure 5: Effect of Channel Impulse Response on DFE performance

As the feedforward filter length is increased the DFEs performance for a non minimum phase CIR approaches that for the minimum phase channel. This is verified by allowing  $N_f$  to tend to infinity in (5). In addition, as  $H \rightarrow 1$ ,  $B_1 \rightarrow 1$  and all the feedforward filter taps apart from the first tend to zero, which is consistent with the minimum phase solution (2). A time reversal equaliser uses the energy from whichever path is currently dominant and can therefore achieve the same performance, for both minimum and non minimum phase channels, but with fewer taps.

If the dominant path in a non minimum phase CIR is used for synchronisation, only the feedforward filter is used and therefore, the noise power at the equaliser output is approximated by (ZF criterion)

$$N_{tot} = \frac{N_0}{|h_1|^2} \left( \frac{1 - H^{2(N_f+1)}}{1 - H^2} \right) \quad (8)$$

As  $H \rightarrow 1$  the total noise power tends to  $N_0(N_f + 1)/|h_1|^2$  which is clearly larger than in (5). Relaxing the ZF criterion

by allowing some residual ISI at the equaliser output will reduce the performance penalty.

#### IV. Conclusions

In this paper various algorithms have been considered for the training of a flexible equaliser. DFE type structures have been selected because of their lower computational complexity relative to MLSE algorithms. The results shown here indicate that the RMGS algorithm provides marginally superior BER performance to both the DRLS algorithm and LSL DFE. In addition it is capable of tracking very fast time varying channels. Its complexity is similar to the DRLS algorithm and for short time spans its computational requirements compared to the LSL are favourable. It also shares with the LSL a high degree of modularity which is attractive for high speed applications. The RMGS algorithm is therefore a potentially attractive solution for a flexible architecture. The importance of frame synchronisation and its effect on the performance of the DRLS DFE was also highlighted. For a two path channel, synchronising the frame to the first arriving ray reduces the total noise power at the equaliser output compared to synchronising to the dominant ray.

#### V. Acknowledgements

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