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REDUCED COMPLEXITY EQUALIZATION OF MIMO SYSTEMS WITH A FIXED-LAG SMOOTHED M-BCJR ALGORITHM

C. M. Vithanage [♣] [◇], C. Andrieu [♣], R. J. Piechocki [◇] and M. S. Yee [♣]

[♣] Department of Mathematics, University of Bristol, Bristol BS8 1TW, UK.

[◇] Center for Communications Research, University of Bristol, Bristol BS8 1UB, UK.

[♣] Toshiba Research Europe Ltd., Bristol BS1 4ND, UK.

Email: {C.Vithanage, C.Andrieu, R.J.Piechocki}@bristol.ac.uk and Mong.Yee@toshiba-trel.com

ABSTRACT

M-BCJR algorithm is a reduced state version of the BCJR algorithm and selects a set of active states in the forward recursion based on an estimation of the filtered probability distribution of states at each time. We propose to use instead an estimation of the fixed-lag smoothed probability distribution of states with a non zero lag. Our implementation uses a Gaussian approximation to estimate these distributions with a low complexity, using the principle of Probabilistic Data Association (PDA). The performance of the M-BCJR can be seen to be greatly improved as a result while remaining robust against changes in the channel multipath profile.

Key words - BCJR algorithm, M algorithm, Probabilistic data association, MIMO equalization.

1. INTRODUCTION

Optimal equalization of practical MIMO systems is a challenging problem since trellis based efficient algorithms such as the BCJR algorithm [1] are intractable due to the huge trellis sizes. Hence to reap the benefits of trellis based equalization, reduced complexity variants of the optimal algorithm need to be employed. Use of trellises based on part of the channel memory is suggested in [2] while grouping of the transmitted signal constellation is used in [3] to reduce the size of the trellis. Another alternative is the M-BCJR [4], which selects a set of active states for each time instant following the M algorithm which was used originally for source coding [5] and also for Viterbi type sequence decoding as in [6]. M-BCJR algorithm is more suitable for use in minimum phase channels, and hence gives variable performance with changes in the channel multipath profile. In this work we present a modified M-BCJR algorithm which is based on a fixed-lag smoothed state selection scheme and showing much improved performance.

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2. SYSTEM MODEL AND THE M-BCJR ALGORITHM

Let us consider an m transmit antenna n receive antenna spatial multiplexing system as shown in Fig. 1, where the channel memory of each transmit antenna to receive antenna path is L . Considering a discrete time complex baseband analysis, let the fading coefficient of the d^{th} channel tap from j^{th} transmit antenna to the i^{th} receive antenna be denoted by $h^{i,j}(d)$ for $d \in [0, L]$, $j \in [1, m]$ and $i \in [1, n]$. We will consider a quasistatic Rayleigh fading channel where each $h^{i,j}(d)$ remains constant during a frame of transmission and changes independently from one frame to another. We will also assume perfect channel state information at the receiver.

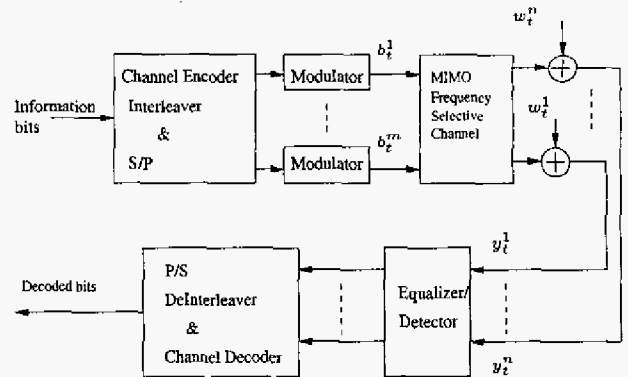


Fig. 1. The System Model

Let b_t^j and y_t^i denote the symbol transmitted by the antenna j and signal received by the antenna i respectively at the t^{th} time instant. Let us assume that the symbol b_t^j is chosen from the set $B = \{a_1, a_2, \dots, a_N\}$ with cardinality N and let the frame length be T . The received signal y_t^i consists of the convolution of the channel impulse response and a sequence of symbols transmitted up to $L+1$ time instants and an additive white Gaussian noise w_t^i ;

$$y_t^i = \sum_{d=0}^L \left[\sum_{j=1}^m h^{i,j}(d) b_{t-d}^j \right] + w_t^i. \quad (1)$$

Now, denoting vector transpose operation by $(\bullet)^\dagger$, let us denote the received vector at time t as $\mathbf{y}_t = (y_t^1, \dots, y_t^n)^\dagger$ and the symbols transmitted by all antennas at time t by $\mathbf{b}_t = (b_t^1, b_t^2, \dots, b_t^m)^\dagger$ which is termed a space-time symbol. Let us define the state of the equalizer at time instant t as $\mathbf{s}_t = (\mathbf{b}_t^\dagger, \dots, \mathbf{b}_{t-L+1}^\dagger)^\dagger$. For a set $\{\mathbf{v}_i\}$ and $j \leq k$, let $\mathbf{v}_{j:k}$ denote the sequence $\mathbf{v}_j, \mathbf{v}_{j+1}, \dots, \mathbf{v}_k$.

Optimum equalization to minimize the symbol error rates, detects the transmitted space-time symbols based on their marginal posterior distributions $p(\mathbf{b}_t | \mathbf{y}_{1:T})$. These distributions can be evaluated by first computing the marginal distributions of the state transitions: $p(\mathbf{s}_{t-1}, \mathbf{s}_t | \mathbf{y}_{1:T})$, which can be done by marginalizing from the joint distribution of $p(\mathbf{s}_{0:T}, \mathbf{y}_{1:T})$ (\mathbf{s}_0 denotes the initial state of the equalizer). The BCJR algorithm can be used to compute these marginal distributions efficiently with a complexity which is linear in T , by making use of the fact that $\mathbf{s}_{0:T}$ forms a Markov chain. In its implementation, the BCJR algorithm involves a forward recursion through the trellis formed by the possibilities of the state sequence $\mathbf{s}_{0:T}$, in which the computation $\gamma(\mathbf{s}_{t-1}, \mathbf{s}_t) = p(\mathbf{y}_t, \mathbf{s}_t | \mathbf{s}_{t-1})$ and the recursive calculations of the metric $\alpha(\mathbf{s}_t) = p(\mathbf{s}_t, \mathbf{y}_{1:t})$ are made, and a backward recursion in which the recursive calculations of the metric $\beta(\mathbf{s}_t) = p(\mathbf{y}_{t+1:T} | \mathbf{s}_t)$ are made for all $t \in \{1, T\}$ and $\mathbf{s}_{t-1}, \mathbf{s}_t \in B^{mL}$.

From these, the marginals of the state transitions can be evaluated by

$$p(\mathbf{s}_{t-1}, \mathbf{s}_t | \mathbf{y}_{1:T}) \propto \alpha(\mathbf{s}_{t-1}) \gamma(\mathbf{s}_{t-1}, \mathbf{s}_t) \beta(\mathbf{s}_t), \quad (2)$$

from which the marginal posterior distributions of the transmitted symbols can be derived. This algorithm although efficient and optimal becomes intractable for MIMO systems since the implementation complexity is exponential in m and L .

The M -BCJR algorithm is a reduced state suboptimal version of this where instead of considering the whole state space of \mathbf{s}_t , M active states are selected at each time instant in the forward recursion of the algorithm based on the metric $\alpha(\mathbf{s}_t)$. Only the transitions emanating from the active states are considered active when performing the computations of the BCJR algorithm. The backward recursion as well follows through the selected active states, and thus the complexity of the algorithm becomes dependent on M instead of N^{mL} . Now we can see that $\alpha(\mathbf{s}_t) \propto p(\mathbf{s}_t | \mathbf{y}_{1:t})$. Hence the active state selection criterion of the M -BCJR algorithm is the filtered probability distribution of states at each time. Instead, we propose to make the active state selection of each time instant based on the fixed-lag smoothed

probability distribution of states, $p(\mathbf{s}_t | \mathbf{y}_{1:t+L'})$ with a lag $L' > 0$. Two such implementations are described in sections 3.1 and 3.2 where these fixed-lag smoothed densities of states are estimated with a low complexity using the principle of Probabilistic Data Association (PDA). While the MBCJR-PDA-1 algorithm presents the implementation of the main ideas, the MBCJR-PDA-2 algorithm is a further reduced complexity version which is more suitable for practical implementation.

3. PROPOSED VARIANTS OF THE M-BCJR ALGORITHM

3.1. MBCJR-PDA-1 algorithm

The proposed algorithm differs from M-BCJR in respect of the active state selection stage in the forward recursion of the algorithm. Let us denote the set of active states at time t by Ω_t and the set of states contending to be as such by Ξ_t . It can be seen that for a given Ω_{t-1} , there are up to MN^m states in Ξ_t . Now say we have decided Ω_{t-1} and calculated $\alpha(\mathbf{s}_{t-1})$ for the active states and are interested in selecting Ω_t . We will make this selection based on $p(\mathbf{s}_t | \mathbf{y}_{1:t+L'})$. Noting that

$$p(\mathbf{s}_{t-1}, \mathbf{s}_t | \mathbf{y}_{1:t+L'}) \propto \alpha(\mathbf{s}_{t-1}) \gamma(\mathbf{s}_{t-1}, \mathbf{s}_t) p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t),$$

we can approximate $p(\mathbf{s}_t | \mathbf{y}_{1:t+L'})$ by

$$Z(\mathbf{s}_t) \sum_{\mathbf{s}_{t-1} \in \Omega_{t-1}} \alpha(\mathbf{s}_{t-1}) \gamma(\mathbf{s}_{t-1}, \mathbf{s}_t) p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t), \quad (3)$$

where the normalization constant $Z(\mathbf{s}_t)$ ensures that $p(\mathbf{s}_t | \mathbf{y}_{1:t+L'})$ is a proper probability mass function. Hence provided we know $p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t)$ for each $\mathbf{s}_t \in \Xi_t$, the required fixed-lag smoothed distribution can be computed. The received values $\mathbf{y}_{t+1:t+L'}$ and the transmitted modulation symbols that have energy components returned within time $[t+1, t+L']$ can be related as

$$\mathbf{y} = \mathbf{A}\mathbf{b} + \mathbf{w}, \quad (4)$$

where $\mathbf{b} = (\mathbf{b}_{t-L+1}^\dagger, \dots, \mathbf{b}_{t+L'}^\dagger)^\dagger$, \mathbf{y} and \mathbf{w} are given by $\mathbf{y} = (\mathbf{y}_{t+1}^\dagger, \dots, \mathbf{y}_{t+L'}^\dagger)^\dagger$ and $\mathbf{w} = (w_{t+1}^1, \dots, w_{t+L'}^n)^\dagger$. \mathbf{A} is a $nL' \times m(L+L')$ matrix with banded Toeplitz structure given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}(L) & \mathbf{H}(L-1) & \dots & \mathbf{H}(0) & \dots & 0 \\ 0 & \mathbf{H}(L) & \dots & \mathbf{H}(1) & \mathbf{H}(0) & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & \mathbf{H}(L) & \dots & \mathbf{H}(0) \end{bmatrix}, \quad (5)$$

where $\mathbf{H}(d)$ denotes an $n \times m$ matrix with the $(i, j)^{th}$ element being $h^{i,j}(d)$ for $d \in [0, L]$. In the matrix multiplication $\mathbf{A}\mathbf{b}$ of (4), there is a column of \mathbf{A} involved with the element b_k^l . Let us denote that column by \mathbf{h}_k^l . Now (4) can be written as

$$\mathbf{y} = \sum_{\substack{k \in \{t-L+1, \dots, t\} \\ l \in \{1, 2, \dots, m\}}} \mathbf{h}_k^l b_k^l + \sum_{\substack{k \in \{t+1, \dots, t+L'\} \\ l \in \{1, 2, \dots, m\}}} \mathbf{h}_k^l b_k^l + \mathbf{w}. \quad (6)$$

For a given state $\mathbf{s}_t \in \Xi_t$, b_k^l is known only for $k = t - L + 1, \dots, t$ and $l = 1, 2, \dots, m$. Hence the probability density function $p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t)$ is actually a Gaussian mixture making accurate evaluation of $p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t)$ for the particular received values computationally too intensive, especially since the computations need to be done for all $t \in [1, T]$ and also for every $\mathbf{s}_t \in \Xi_t$. Instead we resort to the use of the principle of probabilistic data association. The main idea behind PDA is the approximation of a complex probability distribution such as a mixture of Gaussians by a single moment matched Gaussian distribution. This concept was introduced to communications research in [7] for the synchronous multi user detection of CDMA systems, and was applied to the soft decision equalization of MIMO wireless systems in [8].

In doing so, we can moment match a single Gaussian distribution to $p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t)$, with matched mean and covariance given by,

$$\begin{aligned} \bar{\boldsymbol{\mu}} &= \sum_{\substack{k \in \{t-L+1, \dots, t\} \\ l \in \{1, 2, \dots, m\}}} \mathbf{h}_k^l b_k^l + \sum_{\substack{k \in \{t+1, \dots, t+L'\} \\ l \in \{1, 2, \dots, m\}}} \mathbf{h}_k^l E(b_k^l) \\ \bar{\boldsymbol{\Sigma}} &= \boldsymbol{\Sigma} + \sum_{\substack{k \in \{t+1, \dots, t+L'\} \\ l \in \{1, 2, \dots, m\}}} \mathbf{h}_k^l (\mathbf{h}_k^l)^\dagger \text{Var}(b_k^l). \quad (7) \end{aligned}$$

Here $(\bullet)^\dagger$ denotes the conjugate transpose operation, $\boldsymbol{\Sigma}$ is the covariance matrix of \mathbf{w} given by $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_{nL'}$, with σ^2 being the noise variance on each receive antenna and \mathbf{I}_p denoting the $p \times p$ identity matrix. $E(b_k^l)$ and $\text{Var}(b_k^l)$ are the mean and variance of the modulated symbol transmitted by antenna l at time index k , which can be computed from any available prior information such as when used in a turbo equalization scheme or when this algorithm is iterated itself to obtain better defined maxima in the posterior marginal distributions.

Now, we can obtain an approximation to the likelihood $p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t)$ given by

$$\hat{p}(\mathbf{y} | \mathbf{s}_t) \propto \exp \left(-(\mathbf{y} - \bar{\boldsymbol{\mu}})^\dagger \left(\bar{\boldsymbol{\Sigma}} \right)^{-1} (\mathbf{y} - \bar{\boldsymbol{\mu}}) \right), \quad (8)$$

where $(\bullet)^{-1}$ denotes matrix inversion operation. Thus (8) is used to estimate the fixed-lag smoothed probability of the state \mathbf{s}_t in (3). From the fixed-lag smoothed probability distribution thus derived for the states in Ξ_t , we can choose M states which have the largest probability and consider them to be the set of states in Ω_t . Finally the metric $\alpha(\mathbf{s}_t)$ is calculated as in the BCJR algorithm for the selected active states. The same backward recursion of the M-BCJR algorithm is performed to estimate the posterior probabilities of the states, state transitions and the space-time symbols. We can also note at this point that the inverse of the covariance matrix $\bar{\boldsymbol{\Sigma}}$ needs to be computed only once per frame transmission when there is no prior information on the transmitted symbols. This is the case when the algorithm is used in a serially concatenated equalizer decoder system without any iterations within the equalizer. Otherwise there is scope for the use of the matrix inversion lemma to reduce the complexity of computing the matrix inverses at each time instant.

Now, the proposed scheme can be seen to consist of two main functions in its forward recursion. For each time instant t , initially the active state selection will be performed based on an estimate of the fixed-lag smoothed probability distribution of states. Thereafter, the calculation of the metric $\alpha(\mathbf{s}_t)$ of BCJR will be performed for the selected active states. Therefore we have in effect decoupled the processes of active state selection and the calculation of metrics which are used for the estimation of the posterior probabilities.

3.2. MBCJR-PDA-2 algorithm

In the evaluation of the fixed-lag smoothed probabilities of the states via (3), estimation of $p(\mathbf{y}_{t+1:t+L'} | \mathbf{s}_t)$ needs to be performed for every $\mathbf{s}_t \in \Xi_t$ and also $\gamma(\mathbf{s}_{t-1}, \mathbf{s}_t)$ needs to be calculated for each of the MN^m state transitions emanating from $\mathbf{s}_{t-1} \in \Omega_{t-1}$. The number of computations in both these schemes is exponential with m . This can be avoided by the following formulations.

Noting that

$$p(\mathbf{s}_{t-1}, \mathbf{s}_t | \mathbf{y}_{1:t+L'}) \propto \alpha(\mathbf{s}_{t-1}) p(\mathbf{b}_t | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'}) p(\mathbf{y}_{t:t+L'} | \mathbf{s}_{t-1}),$$

we can approximate $p(\mathbf{s}_t | \mathbf{y}_{1:t+L'})$ by

$$\tilde{Z}(\mathbf{s}_t) \sum_{\mathbf{s}_{t-1} \in \Omega_{t-1}} \alpha(\mathbf{s}_{t-1}) p(\mathbf{b}_t | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'}) p(\mathbf{y}_{t:t+L'} | \mathbf{s}_{t-1}),$$

where $\tilde{Z}(\mathbf{s}_t)$ is the normalization constant ensuring that the probability mass function $p(\mathbf{s}_t | \mathbf{y}_{1:t+L'})$ is proper. The received values at time instants t to $t + L'$ can be related as,

$$\tilde{\mathbf{y}} = \tilde{\mathbf{A}}\mathbf{b} + \tilde{\mathbf{w}}, \quad (9)$$

where $\tilde{\mathbf{b}} = (\mathbf{b}_{t-L}^\dagger, \dots, \mathbf{b}_{t+L'}^\dagger)^\dagger$, $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{w}}$ are given by $\tilde{\mathbf{y}} = (\mathbf{y}_t^\dagger, \dots, \mathbf{y}_{t+L'}^\dagger)^\dagger$, $\tilde{\mathbf{w}} = (\mathbf{w}_t^\dagger, \dots, \mathbf{w}_{t+L'}^\dagger)^\dagger$ and $\tilde{\mathbf{A}}$ is a $n(L'+1) \times m(L'+1)$ matrix of the usual banded Toeplitz structure. We can proceed as before to estimate $p(\mathbf{y}_{t:t+L'} | \mathbf{s}_{t-1})$. Let us see how $p(\mathbf{b}_t | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'})$ can be estimated. First we will make the approximation of assuming the transmitted antenna symbols of each time instant are independent of each other even after the observations through the channel. This results in

$$p(\mathbf{b}_t | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'}) \approx \prod_{j=1}^m p(b_t^j | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'}).$$

Again, denoting the column of $\tilde{\mathbf{A}}$ involved with the element b_t^j of $\tilde{\mathbf{b}}$ in the matrix multiplication $\tilde{\mathbf{A}} \tilde{\mathbf{b}}$ of (9) by \mathbf{h}_t^j , we can equivalently estimate $p(b_t^j | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'})$ since \mathbf{h}_t^j is known. Now for this estimation, we will make the approximation of assuming $\mathbf{h}_t^j b_t^j$ to be a continuous random variable. Thereby, it is possible to approximate the probability distribution of $\mathbf{h}_t^j b_t^j$ given \mathbf{s}_{t-1} and $\mathbf{y}_{t:t+L'}$ with a moment matched Gaussian distribution and evaluate the probability $p(b_t^j | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'})$ for each $\mathbf{s}_{t-1} \in \Omega_{t-1}$ and $b_t^j \in B$. These approximations are based on the work of [8], and enables us to estimate the fixed-lag smoothed distribution of states at each time instant in a computationally simple manner. Furthermore, since an estimation of $p(\mathbf{b}_t | \mathbf{s}_{t-1}, \mathbf{y}_{t:t+L'})$ is made, we will also select the S most probable state transitions emanating from each active state of time $t-1$ in deciding the states of Ξ_t . Hence now, there are only up to MS states contending to be included in Ω_t instead of the possibly much larger MN^m . After estimating the fixed-lag smoothed probability distribution for these states in Ξ_t , the remaining steps of the algorithm are similar to those of the MBCJR-PDA-1 algorithm.

The order of computations of the MBCJR-PDA-1 and MBCJR-PDA-2 algorithms are seen to be dominated by the larger of $(nL')^2 MN^m$ and $MN^m \log_2(MN^m)$ and the larger of $n^2(L'+1)^2 MN^m$ and $MS \log_2(MS)$ respectively for non iterative implementations with uniform priors. Since the order of computations of the M-BCJR algorithm is dominated by the larger of $(L+1)nmMN^m$ and $MN^m \log_2(MN^m)$, the second proposed algorithm is computationally simpler compared to M-BCJR for the same number of active states, for large N and m (for example, in the setup of Fig. 3, $m = n = 2$, $N = 8$, $M = 8-32$, $L = 4$, $L' = 12$ and say $S = 4$).

The ideas of these two algorithms can also be applied to the T-BCJR algorithm presented in [4] which will give further savings in computations at high signal to noise power ratios, with the disadvantage of having variable decoding times.

4. SIMULATION RESULTS

We have simulated the proposed algorithms with the channel code being a rate half turbo code using two $(5, 7)_8$ convolutional codes and the decoder performing 4 turbo decoding iterations. The non iterative algorithms were compared against the BCJR and M-BCJR algorithms for small state spaces. These algorithms were implemented in their logarithms for better numerical stability. When the trellis sizes are too large for the evaluation of optimal performance, we have compared with the soft output linear MMSE scheme proposed for MIMO systems in [9]. Fig. 2 shows the performance of MBCJR-PDA-1 in a system with 2x2 BPSK transmission employed in a 5-tap multipath channel that has the channel tap energy distribution as [0.227 0.46 0.688 0.46 0.227]. Each frame transmission contained 144 data bits. The optimal equalizer has a total of 256 states. The MBCJR-PDA-1 achieves near optimal uncoded performance using only 4 active states and performs better than the M-BCJR scheme that uses 16 active states.

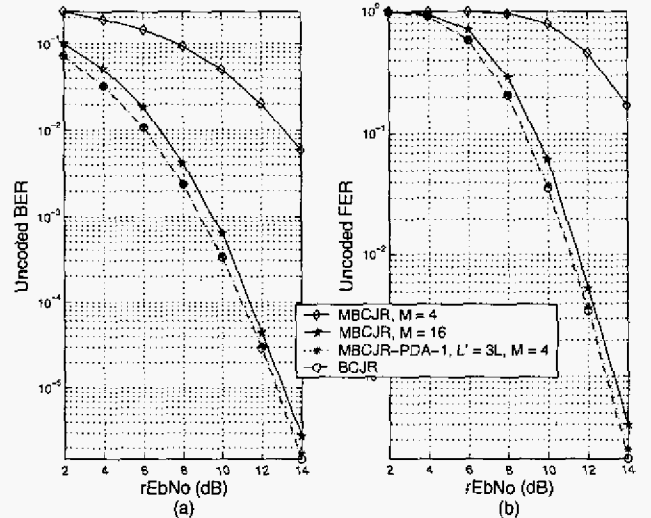


Fig. 2. Uncoded performance with 2x2 BPSK transmission into a 5-tap triangular delay profile channel

Fig. 3 shows the performance of MBCJR-PDA-1 and MBCJR-PDA-2 in a system with 2x2 8PSK transmission employed in the same channel. The optimal equalizer have more than 16 million total states. Each frame contained 1152 data bits in this case. The MBCJR-PDA-1 was performing 3dB better than the MMSE scheme at a bit error rate of 10^{-4} by using only 16 active states. The MBCJR-PDA-2 scheme achieved this performance with $M = 32$. Even though there are 64 state transitions emanating from each active state in this system, the second algorithm with S set to 4 or 64 performed nearly the same, thus indicating a large saving in computations. The M-BCJR algorithm fails

to give the same performance for practical values of M .

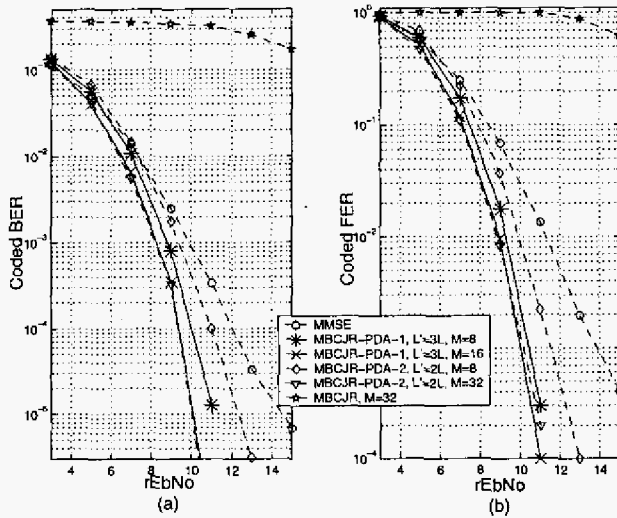


Fig. 3. Coded performance with 2x2 8PSK transmission into a 5-tap triangular delay profile channel

Fig. 4 shows the performance of MBCJR-PDA-2 in a system with 1x1 8PSK transmission employed in uniform delay profile channels with 3 and 10 taps. Frames contained 144 data bits. The optimal equalizers have 64 and over 134 million states respectively. The MBCJR-PDA-2 when used with $L' = 3L$ was performing 1.5dB and 1dB better than the MMSE scheme at a bit error rate of 10^{-3} by using only 8 and 64 active states respectively.

In simulating the algorithms, a lag of $3L$ was nearly able to achieve the attainable performance of the algorithms for a given number of active states and was having only a slightly superior performance compared to a lag of $2L$.

5. CONCLUSIONS

We have presented improved versions of the M-BCJR algorithm by changing the active state selection criterion. As a result the new algorithms are able to compete in error rate performance with state of the art algorithms like the soft output linear MMSE scheme [9], even when the total number of possible states is huge.

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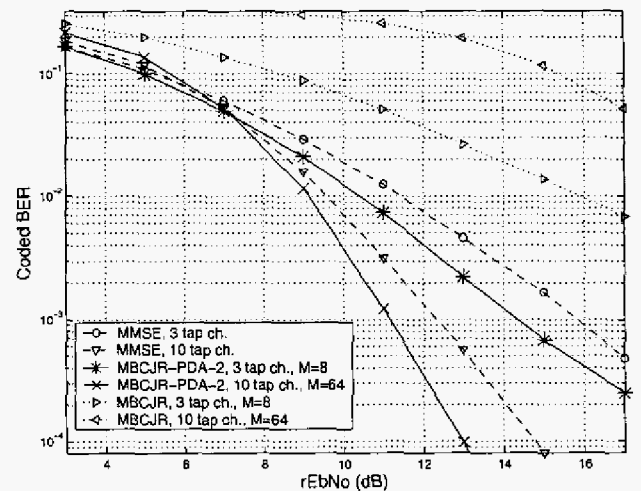


Fig. 4. Coded performance with 1x1 8PSK transmission into uniform delay profile channels. $L' = 3L$