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## **MIMO Channel Capacity in Co-Channel Interference**

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# MIMO Channel Capacity in Co-Channel Interference

Matthew W. Webb, Mark A. Beach, and Andrew R. Nix

**Abstract**—In this paper we investigate the performance of MIMO systems in co-channel interference. We present a way of explicitly examining the effect of interference on the MIMO sub-channel gains. Using this, we derive upper and lower asymptotic bounds on capacity with many interferers and in high interference-to-noise ratio. Simulation results are presented for these bounds and for the performance of MIMO systems in a variety of situations as a function of the number of co-channel interferers and as a function of interference-to-noise ratio. Unusually, it is seen that, with a slight modification of the MIMO channel matrix to incorporate interference, higher correlation in the channel yields higher capacity. We also simulate the effect of *a priori* knowledge of the channel and interference covariance at the transmitter and evaluate the capacity gain that such knowledge offers.

**Index Terms**—MIMO, co-channel interference, capacity, number of interferers, interference-to-noise ratio.

## I. INTRODUCTION

IN recent years, the use of antenna arrays at both ends of the link in wireless communication has attracted much research interest. These so-called multiple-input multiple-output (MIMO) systems have been shown to offer unprecedented spectral efficiencies under mild constraints [1]–[4]. Since the discovery of means to exploit these dramatic spectral efficiencies via layered architectures [5]–[8] and space–time coding [9]–[15], there has been work undertaken to understand the behaviour of these systems in the presence of spatially (and temporally) white interference and noise [16]–[23].

With various assumptions regarding channel and interference knowledge at the transmitter, the information theoretic capacity of such systems has been investigated [18] in the presence of both spatially-white and spatially-coloured interference using relevant power-allocation schemes. Future wireless systems will offer the user multi-rate data services, with the rates offered a function of the various features of the service such as real-time demands, QoS, video quality, etc. In general, a user’s transmit power will be proportional to their data-rate and so there has been some work carried out recently [24] investigating how the distribution of interference power affects capacity.

Additionally, the interference in a wireless system will in general have some spatial structure — it will be spatially coloured. Initial investigations into the behaviour of MIMO in such interference have been presented in [18], [24], [25]. The work in [24] extends that in [18] by giving a more explicit form to the interference and investigating how the capacity behaves with fixed total interference-plus-noise power and varying number of interferers. Such a constraint is reasonable since there is likely to be some means of controlling the other

users in our cell, particularly if they are all subscribed to the same network operator. In [24] it is concluded that MIMO performs with greater spectral efficiency with few, high-power users than with many, low-power ones but no explanation is given.

In [25] a closed form solution for the capacity in interference is derived in the limit of a large, equal number of antennas on all users. This provides an analytical description of the result in [24], using the same model.

In this work, we employ the model of [24], which is also used in [25], to provide a more rigorous understanding of the results therein. We provide an explicit explanation of the conclusions in [24] and derive asymptotic lower bounds on the capacity in the presence of many interferers and high interference-to-noise ratio (INR). We investigate more generally the performance of MIMO with varying numbers of interferers and differing transmit and receive array sizes. Simulation results are presented for a wide range of situations, including the asymptotic cases already mentioned.

Though we arrive at some of the same predictions as [25], we have a rather different setting of numbers of interferers and INR instead of array size. Our new approach offers a more explicit understanding of the manner in which interference impinges on the performance of MIMO systems, regardless of their size. In particular, we make clear the effect of the degrees-of-freedom of the receiving array in MIMO and are able to use this insight to explain some additional features of the capacity curves we draw.

The paper is organized as follows. In Section II we introduce the system model and in Section III use this to determine the information-theoretic capacity of such systems. Section IV introduces a new tool for analyzing MIMO systems in the presence of interference. We present upper and lower bounds on the capacity in such situations. In Section V we simulate the capacity for varying numbers of interferers with varying numbers of antennas along with the asymptotic bound we derive in Section IV. We also consider the effect of changing the number of antennas the desired user has at each end of the link, and the effect of different knowledge of the channel and interference covariance under such scenarios. In Section VI, we turn our attention to INR and investigate how the capacity is affected by the number of antennas at each end of the link as well as on the the interferers. The bounds from Section IV are also simulated and compared to our predictions. We draw our conclusions in Section VII.

## II. SYSTEM MODEL

Following the analysis in [24], we consider a single-user, narrowband link with co-channel interference from other users

whom we have some control over. This control is nominal – we mean that we can assume equal power transmissions from all of our interfering users, subject to some fixed, total interference power. The issue of power control in interference limited MIMO has been investigated in [26], [27]. Nevertheless, power-control could be used to ensure the validity of our equal power assumption. We equip the desired user with  $n_T$  transmit antennas and  $n_R$  receive antennas. Each interfering user has  $n_I$  transmit antennas, but uses the same set of  $n_R$  receive antennas as the desired user. We model the received signal vector as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \underbrace{\sqrt{\frac{P_I}{Ln_I}} \sum_{i=1}^L \mathbf{G}_i \mathbf{s}_i^I}_{\mathbf{n}} + \mathbf{w} \quad (1)$$

where  $\mathbf{y} \in \mathbb{C}^{n_R \times 1}$ ,  $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$  is the MIMO channel matrix of the desired user and  $\mathbf{s} \in \mathbb{C}^{n_T \times 1}$  their transmit signal vector with unit power. The number of interferers is  $L$ , the fixed total interference power is  $P_I$ ,  $\mathbf{G}_i \in \mathbb{C}^{n_R \times n_I}$  is the channel matrix of the  $i^{\text{th}}$  interferer,  $\mathbf{s}_i^I \in \mathbb{C}^{n_I \times 1}$  their transmit signal vector with unit power and  $\mathbf{w} \in \mathbb{C}^{n_R \times 1}$  is additive white Gaussian noise (AWGN) at the receiver, with  $\mathbf{E}\{\mathbf{w}\mathbf{w}^\dagger\} = \sigma^2 \mathbf{I}_{n_R}$  where  $\dagger$  denotes complex-conjugate transpose (hermitian). We will denote the received interference-plus-noise vector as  $\mathbf{n} \in \mathbb{C}^{n_R \times 1}$ . The channel matrices  $\mathbf{H}$  and  $\mathbf{G}_i$  are all mutually independent and assumed quasi-static being uncorrelated from frame to frame. As usual, the elements of  $\mathbf{H}$  and  $\mathbf{G}_i$  are assumed independent and identically distributed complex Gaussian with zero mean and unit variance split equally across both real dimensions. All of the transmitted signals  $\mathbf{s}$  and  $\mathbf{s}_i^I$  and the receiver noise  $\mathbf{w}$  are mutually independent also. All of the interferers have the same power in (1).

We assume that the interference is co-channel. That is, all the users in the system, whether desired or interfering, transmit in the same bandwidth. There is an implicit assumption that all system users are transmitting the whole time and that the receiver has sufficient dynamic range to linearly process all incoming signals. The channel model described above is narrowband; it assumes that there is exactly zero ISI and hence represents frequency-flat fading.

It is possible to show that the covariance matrix of the interference-plus-noise vector is

$$\mathbf{R}_0 = \mathbf{E}\{\mathbf{n}\mathbf{n}^\dagger\} = \frac{P_I}{Ln_I} \sum_{i=1}^L \mathbf{G}_i \mathbf{G}_i^\dagger + \sigma^2 \mathbf{I}_{n_R}$$

with  $\mathbf{R}_0 \in \mathbb{C}^{n_R \times n_R}$  and that the covariance matrix of the received signal is

$$\mathbf{E}\{\mathbf{y}\mathbf{y}^\dagger\} = \mathbf{H}\Sigma_s\mathbf{H}^\dagger + \mathbf{R}_0$$

where  $\Sigma_s = \mathbf{E}\{\mathbf{s}\mathbf{s}^\dagger\}$ .

### III. CHANNEL CAPACITY

The channel capacity is derived in detail in [24] and we do not propose to repeat this derivation here. Instead, we provide the important results of that analysis which we will use

and then derive some bounds on the performance of MIMO systems in interference. As defined in [24]:

$$\mathbf{R} = \frac{1}{\sigma^2} \mathbf{R}_0 = \frac{P_I}{\sigma^2 Ln_I} \sum_{i=1}^L \mathbf{G}_i \mathbf{G}_i^\dagger + \mathbf{I}_{n_R} \quad (2)$$

where  $\frac{P_I}{\sigma^2}$  is the INR and  $\mathbf{R}$  has the same dimensions as  $\mathbf{R}_0$ . This leads to the capacity being formulated as

$$C = \max_{\substack{\text{tr}(\frac{\Sigma_s}{\sigma^2}) \leq \frac{P_T}{\sigma^2}}} \left\{ \log_2 \det \left[ \mathbf{I}_{n_R} + \left( \mathbf{R}^{-1/2} \mathbf{H} \right) \frac{\Sigma_s}{\sigma^2} \left( \mathbf{R}^{-1/2} \mathbf{H} \right)^\dagger \right] \right\}. \quad (3)$$

Equation (3) suggests considering  $\mathbf{R}^{-1/2} \mathbf{H}$  as a ‘combined’ channel. As a result, the capacity in (3) is equivalent to the capacity of the combined channel  $\mathbf{R}^{-1/2} \mathbf{H}$  in spatially white noise. With this interpretation, the authors of [24] derive the capacity in spatially coloured interference and noise.

#### A. Both channel and interference covariance information known at the transmitter

By applying water-filling at the transmitter with the combined channel  $\mathbf{R}^{-1/2} \mathbf{H}$ , the channel capacity is found to be

$$C = \sum_{i=1}^{n_T} \log_2 (1 + p_i \lambda_i) \quad (4)$$

and the optimal transmit signal covariance matrix is

$$\Sigma_s = \sigma^2 \mathbf{U} \text{diag}(p_1, \dots, p_{n_T}) \mathbf{U}^\dagger$$

where

$$\mathbf{H}^\dagger \mathbf{R}^{-1} \mathbf{H} = \mathbf{U} \Lambda \mathbf{U}^\dagger$$

$\lambda_1, \dots, \lambda_{n_T}$  are the eigenvalues of  $\mathbf{H}^\dagger \mathbf{R}^{-1} \mathbf{H}$ ,  $\mathbf{U}$  is a unitary matrix whose columns are the eigenvectors of  $\mathbf{U} \Lambda \mathbf{U}^\dagger$ ,

$$p_i = \left( \mu - \frac{1}{\lambda_i} \right)^+, \quad (5)$$

where  $\mu$  is chosen such that

$$\sum_{i=1}^{n_T} p_i = \frac{P_T}{\sigma^2}$$

and  $(x)^+$  denotes the larger of 0 and  $x$ .

The problem of the channel changing too rapidly to allow us this complete knowledge is an area of current research; see [28], [29] for example. Nevertheless, in Sections V-B and VI-B we will assume, for the purposes of comparison, that such knowledge is available.

#### B. Neither channel nor interference covariance information known at the transmitter

In this case, the transmitter should apply uniform power allocation across the transmit antennas i.e.  $\Sigma_s = (P_T/n_T) \mathbf{I}_{n_T}$  and the capacity is given by

$$C = \log_2 \det \left( \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \mathbf{R}^{-1} \mathbf{H} \mathbf{H}^\dagger \right) \quad (6)$$

#### IV. BOUNDS ON CAPACITY IN INTERFERENCE

We consider the Singular Value Decomposition (SVD) of  $\mathbf{R}$  and of  $\mathbf{H}$ . Note that if  $\mathbf{G}_i \in \mathbb{C}^{n_R \times 1}$   $\mathbf{R}$  is Hermitian positive definite. If  $n_I \neq 1$  then  $\mathbf{R}$  is in general only Hermitian positive semi-definite [30]. In either case,

$$\mathbf{R} = [\mathbf{u}_1 \cdots \mathbf{u}_{n_R}] \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & & \\ 0 & \cdots & & \lambda_{n_R} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1^\dagger \\ \vdots \\ \mathbf{u}_{n_R}^\dagger \end{bmatrix} \quad (7)$$

and

$$\mathbf{H} = [\mathbf{v}_1 \cdots \mathbf{v}_{n_R}] \begin{bmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \gamma_N & \\ 0 & \cdots & & 0 \\ 0 & \cdots & & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{w}_1^\dagger \\ \vdots \\ \mathbf{w}_{n_T}^\dagger \end{bmatrix} \quad (8)$$

where

$$\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^\dagger \quad \text{and} \quad \mathbf{H} = \mathbf{V}\mathbf{\Gamma}\mathbf{W}^\dagger, \quad (9)$$

$N = \min(n_R, n_T)$  and all of  $\mathbf{U} \in \mathbb{C}^{n_R \times n_R}$ ,  $\mathbf{\Lambda} \in \mathbb{C}^{n_R \times n_R}$ ,  $\mathbf{V} \in \mathbb{C}^{n_R \times n_R}$ ,  $\mathbf{\Gamma} \in \mathbb{C}^{n_R \times n_T}$ ,  $\mathbf{W} \in \mathbb{C}^{n_T \times n_T}$  are unitary matrices. We will choose to arrange the singular values such that  $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_{n_R}$  and  $\gamma_1 \geq \gamma_2 \geq \cdots \gamma_N$ . Note that (when not transposing) we are using column vectors. Hence, the combined channel from (3),

$$\mathbf{R}^{-1/2}\mathbf{H} = \mathbf{U}\mathbf{\Lambda}^{-1/2}\mathbf{U}^\dagger\mathbf{V}\mathbf{\Gamma}\mathbf{W}^\dagger. \quad (10)$$

In [31] the familiar result of ‘orthogonal spatial sub-channels’ is derived via the SVD and linear operations at transmit and receive. Without interference, whence  $\mathbf{R} = \mathbf{I}$ , the transmitter left multiplies  $\mathbf{s}$  by  $\mathbf{W}$  and the receiver left-multiplies the received signal by  $\mathbf{V}^\dagger$ . Substituting these into (1) yields (without interference):

$$\mathbf{y} = \mathbf{\Gamma}\mathbf{s} + \mathbf{w}. \quad (11)$$

In our case, the equivalent of the linear operations would be for the transmitter to left-multiply  $\mathbf{s}$  by  $\mathbf{W}$  and the receiver to left-multiply the received signal by  $\mathbf{U}^\dagger$ . Substituting these into (1) yields the equivalent of (11) in interference:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{s} + \mathbf{w} \quad (12)$$

where

$$\mathbf{\Phi} = \mathbf{\Lambda}^{-1/2}\mathbf{U}^\dagger\mathbf{V}\mathbf{\Gamma} \quad (13)$$

serves the purpose of  $\mathbf{\Gamma}$  but with, in general, off-diagonal elements. It is shown in the Appendix that

$$\mathbf{\Phi} = \begin{bmatrix} \gamma_1 \tilde{\lambda}_1 (\mathbf{u}_1 \cdot \mathbf{v}_1) & \cdots & \gamma_N \tilde{\lambda}_1 (\mathbf{u}_1 \cdot \mathbf{v}_N) \\ \vdots & \ddots & \vdots \\ \gamma_1 \tilde{\lambda}_{n_R} (\mathbf{u}_{n_R} \cdot \mathbf{v}_1) & \cdots & \gamma_N \tilde{\lambda}_{n_R} (\mathbf{u}_{n_R} \cdot \mathbf{v}_N) \end{bmatrix} \mathbf{0}_{n_R \times \chi} \quad (14)$$

where  $\tilde{\lambda}_i \triangleq \lambda_i^{-\frac{1}{2}}$ . The partition recognizes the fact that, if  $n_R < n_T$ ,  $\mathbf{\Phi}$  will be ‘padded’ with  $\chi = (n_T - n_R)^+$  columns of zeros. Compared to  $\mathbf{\Gamma}$ , the diagonal elements have been

scaled by factors due to interference and due to the relative orientations of the column-spaces of  $\mathbf{R}$  and  $\mathbf{H}$ . Additionally, the off-diagonal elements are no longer zero, depending on similar interference scale factors, but also on the ‘cross-correlation’ between the column spaces. We acknowledge that  $\mathbf{\Phi}$  does not recover  $\mathbf{\Gamma}$  in the absence of interference, but we will see that we do recover the system performance nevertheless. A brief examination of the scale factors is in order.

First though, a simplification. Noting that all transmitted signals are independently and identically distributed complex Gaussian by assumption, we can model interferers with multiple antennas by grouping together multiple single-antenna interferers. In effect, there is no difference between having MIMO interferers and SIMO interferers under this assumption. Thus, without loss of generality and for the sake of simplicity of expression, the remainder will develop on the basis of single-antenna interferers with the understanding that references to ‘an interferer’ should be taken to mean ‘an interfering antenna’ except where indicated otherwise. We will specifically simulate MIMO interferers in Section V-A.

It is easily verified that the structure of  $\mathbf{R}$  is such that its singular values satisfy

$$\lambda_i \begin{cases} \geq 1 : i \leq L \\ = 1 : i > L \end{cases} \quad (15)$$

since such an Hermitian matrix has  $n_R$  degrees-of-freedom and they are ‘used up’ in turn as interferers are added; the matrix  $(\mathbf{R} - \mathbf{I})$  remains singular and rank-deficient until  $L \geq n_R$  though  $\mathbf{R}$  itself is never singular and is always full rank. Additionally, the smaller the off-diagonal elements of  $\mathbf{R}$ , the closer  $\mathbf{U}$  and  $\mathbf{\Lambda}$  will be to a scaled identity. Thus:

$$\begin{aligned} \tilde{\lambda}_i &\leq 1 & i = 1 \dots n_R \\ \mathbf{u}_j \cdot \mathbf{v}_k &\leq 1 & j, k = 1 \dots n_R \end{aligned} \quad (16)$$

and interference will, in general, decrease the diagonal elements and distort the off-diagonal elements of  $\mathbf{\Phi}$  compared to  $\mathbf{\Gamma}$ . The effect of this remains unclear at this stage however. In the remainder, we will sometimes be implicitly indexing the  $\gamma_i$  beyond  $N$ . To avoid complicated subscripts and conditions, we will understand that

$$\gamma_i = \begin{cases} (\mathbf{\Gamma})_{i,i} : 1 \leq i \leq N \\ 0 : \text{otherwise.} \end{cases} \quad (17)$$

We seek to determine the capacity in terms of  $\mathbf{\Phi}$ . We will consider here that we opt for equal-power assignment at the transmitter, and see from [31] that the capacity without interference is

$$\begin{aligned} C &= \log_2 \det \left( \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \mathbf{H}\mathbf{H}^\dagger \right) \\ C &= \log_2 \det \left( \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \mathbf{H}^\dagger \mathbf{H} \right) \end{aligned} \quad (18)$$

$$C = \sum_{i=1}^N \log_2 \left( 1 + \frac{P_T}{\sigma^2 n_T} \gamma_i^2 \right). \quad (19)$$

Here, our channel is  $\mathbf{R}^{-1/2}\mathbf{H}$ :

$$\begin{aligned}
C &= \log_2 \det \left[ \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \mathbf{R}^{-1/2} \mathbf{H} \left( \mathbf{R}^{-1/2} \mathbf{H} \right)^\dagger \right] \\
C &= \log_2 \det \left( \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \mathbf{U} \mathbf{\Lambda}^{-1/2} \mathbf{U}^\dagger \mathbf{V} \mathbf{\Gamma} \mathbf{W}^\dagger \mathbf{W} \mathbf{\Gamma} \mathbf{V}^\dagger \mathbf{U} \mathbf{\Lambda}^{-1/2} \mathbf{U}^\dagger \right) \\
C &= \log_2 \det \left[ \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \left( \mathbf{\Lambda}^{-1/2} \mathbf{U}^\dagger \mathbf{V} \mathbf{\Gamma} \right) \left( \mathbf{\Gamma} \mathbf{V}^\dagger \mathbf{U} \mathbf{\Lambda}^{-1/2} \right) \right] \\
C &= \log_2 \det \left( \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \mathbf{\Phi} \mathbf{\Phi}^\dagger \right) \\
C &= \log_2 \det \left( \mathbf{I}_{n_R} + \frac{P_T}{\sigma^2 n_T} \mathbf{\Phi}^\dagger \mathbf{\Phi} \right) \quad (20)
\end{aligned}$$

and (20) serves the purpose of (18) in interference. Note that in the absence of interference all three matrices in the SVD of  $\mathbf{R}$  are the identity so in this case

$$\begin{aligned}
\mathbf{\Phi} &= \left[ \gamma_1 \mathbf{v}_1 \quad \cdots \quad \gamma_N \mathbf{v}_N \mid \mathbf{0}_{n_R \times \chi} \right] = \mathbf{\Phi}_0 \\
\mathbf{\Phi}_0^\dagger \mathbf{\Phi}_0 &= \begin{bmatrix} \gamma_1^2 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \gamma_N^2 & \\ 0 & \cdots & & \ddots \end{bmatrix} = \mathbf{\Gamma}^2 \quad (21)
\end{aligned}$$

where  $\mathbf{\Phi}_0^\dagger \mathbf{\Phi}_0 \in \mathbb{C}^{n_T \times n_T}$ , since  $\|\mathbf{v}_i\|_2^2 = 1 \forall i$  and  $\mathbf{v}_i \cdot \mathbf{v}_j = 0$  for  $i \neq j$  by construction and we have recovered the interference free capacity. In general, the elements of  $\mathbf{\Phi}^\dagger \mathbf{\Phi} \in \mathbb{C}^{n_T \times n_T}$  are

$$(\mathbf{\Phi}^\dagger \mathbf{\Phi})_{i,j} = \begin{aligned} & \gamma_i \gamma_j \left[ \tilde{\lambda}_1^2 (\mathbf{u}_1 \cdot \mathbf{v}_j) (\mathbf{u}_1 \cdot \mathbf{v}_i)^* + \cdots \right. \\ & \left. + \tilde{\lambda}_{n_R}^2 (\mathbf{u}_{n_R} \cdot \mathbf{v}_j) (\mathbf{u}_{n_R} \cdot \mathbf{v}_i)^* \right] \quad (22) \end{aligned}$$

if  $1 \leq \{i, j\} \leq N$  and zero otherwise. Note the diagonal elements are purely real and sums dependent on the absolute values of the dot-products of the singular vectors. Note that  $\mathbf{\Phi}^\dagger \mathbf{\Phi}$  is Hermitian but not, in general, definite.

#### A. Bound on the capacity with many interferers

Recall from Section I that the interferers' channel matrices are, like the desired user's, circularly symmetric complex Gaussian distributed with zero mean and unit variance split equally across both real dimensions. The first term in (2), then, is the covariance matrix of the sum of  $Ln_I$  Gaussians. Its off-diagonal elements are non-zero reflecting the colour of the noise. The central-limit theorem can be invoked to show that, with many interferers, this covariance matrix becomes that of a spatially white Gaussian, with all non-diagonal elements zero

i.e. as  $Ln_I \rightarrow \infty$ ,

$$\begin{aligned}
\frac{P_I}{\sigma^2 Ln_I} \sum_{i=1}^L \mathbf{G}_i \mathbf{G}_i^\dagger &\rightarrow \frac{P_I}{\sigma^2} \mathbf{I}_N \\
\mathbf{R} &\rightarrow \left( \frac{P_I}{\sigma^2} + 1 \right) \mathbf{I}_N
\end{aligned}$$

whence

$$\begin{aligned}
\mathbf{\Phi} &\rightarrow \left( \frac{P_I}{\sigma^2} + 1 \right)^{-1/2} \mathbf{\Phi}_0 \\
\mathbf{\Phi}^\dagger \mathbf{\Phi} &\rightarrow \left( \frac{P_I}{\sigma^2} + 1 \right)^{-1} \begin{bmatrix} \gamma_1^2 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \gamma_N^2 & \\ 0 & \cdots & & \ddots \end{bmatrix}.
\end{aligned}$$

Then, in the limit of large  $L$ , from (20),

$$C = \sum_{i=1}^L \log_2 \left[ 1 + \frac{P_T}{(\sigma^2 + P_I) n_T} \gamma_i^2 \right] \quad (23)$$

which provides a convenient comparison with (19). The SNR has been replaced by the signal to interference-plus-noise ratio (SINR). This should not be too surprising since the interference now appears spatially white and is subsumed within the thermal noise already present. This fact is emphasized by the fact that (23) does not depend on  $n_I$ , the number of antennas on each interferer.

#### B. Bounds on the capacity in high INR

In this part, we will examine three possible bounds for the capacity in high INR. By 'high' we will mean asymptotically high, that is  $\frac{P_I}{\sigma^2} \rightarrow \infty$  with some fixed number of interferers. In Section V we will simulate these bounds and compare them to the performance observed in practise.

We observe from the definition of  $\mathbf{U}$  as a unitary matrix that any increase in the INR will result in an increase in the singular values of  $\mathbf{R}$ . Recalling (15), it is easy to see that, for a given fixed number of interferers, increasing INR will decrease the  $\tilde{\lambda}_i$  for  $i \leq L$ . However, for  $i > L$ , increasing INR will have no effect; these  $\tilde{\lambda}_i$  will remain fixed at unity. It is on this basis that we will derive our limits.

Consider first the straightforward effect of the reduction of some  $\tilde{\lambda}_i$  to zero in the case of  $L < n_R$ :

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{0}_{1 \times n_R} \\ \vdots \\ \mathbf{0}_{1 \times n_R} \\ \phi_{L+1} \\ \vdots \\ \phi_{n_R} \end{bmatrix} \quad (24)$$

where we denote the  $i^{\text{th}}$  row of (14) as  $\phi_i$ . Thus, the direct manifestation of interference is to reduce the rows of  $\mathbf{\Phi}$  controlled by those singular values of  $\mathbf{R}$  it affects. In the limit of high INR, these rows' elements vanish. The simplest

limit we can formulate is to compute the capacity given by substituting (24) in (20). It is worth pointing out at this point that both bounds mentioned so far, and those following, are all statistical in nature. They depend on the channel and interference parameters and only bound the performance in a particular instantiation of channel and interference. Thus, they must be simulated along with channel capacity to determine their ergodic and outage values.

Equation (24) allows a further insight. Provided  $L < n_R$  the system will be able to support some capacity, regardless of INR. If, though,  $L \geq n_R$ , the capacity will decline to zero as the INR becomes large. This highlights the effect of degrees-of-freedom in a MIMO system. Intuitively, the MIMO system is able to ‘support’ as many interferers as it has degrees-of-freedom, and these degrees-of-freedom are set by the number of receiving antennas.

This equation allows one final, straightforward prediction. Obviously, the rows will only be the zero-vector in the limit of high INR, and will reduce toward it as INR increases. This predicts that MIMO systems will perform more efficiently (more bps/Hz) in lower INR regimes. This has been observed by simulation in [24]. More generally, we may say that MIMO systems prefer a noise-dominated regime to an interference dominated one.

We temporarily change direction and consider trying to achieve as ‘nearly as possible’ the interference-free capacity. That is, we keep the channel gains  $\gamma_i$  and the interference factors  $\tilde{\lambda}_i$  as they are but seek to restore the system as closely as possible to the interference-free state. This we do by allowing the vectors in  $\mathbf{U}$  and  $\mathbf{V}$  to change. This is not intended as a practical solution to interference but does offer a means of examining the best-case effect a particular set of interference factors could yield.

It can be seen from (22) that each non-zero element of  $\Phi^\dagger \Phi$  contains, in general, a factor of each  $\tilde{\lambda}_i$  but that how much effect each of these has is controlled by the dot-products. Then, considering we seek to recover (21), we would want  $\Phi^\dagger \Phi$  to be of the same diagonal form but with the effect of the  $\tilde{\lambda}_i$  as small as possible. Since  $\tilde{\lambda}_i$  are either zero or one, the least effect we can arrange for is for them to zero out the smallest  $L$  of the diagonal elements of (22) and leave the larger elements untouched. In general, when the interference factors are not at their limiting values, we would arrange for the smallest  $\tilde{\lambda}_i$  to multiply the smallest  $\gamma_i$ . Since  $\tilde{\lambda}_i$  are ordered non-decreasing whilst  $\gamma_i$  are ordered non-increasing, they are paired ‘in reverse’ i.e.

$$\Phi^\dagger \Phi = \begin{bmatrix} (\gamma_1 \tilde{\lambda}_{n_R})^2 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & (\gamma_i \tilde{\lambda}_{n_R+1-i})^2 & \\ & & & \ddots \\ & & & & (\gamma_N \tilde{\lambda}_{n_R+1-N})^2 & 0 \\ 0 & \cdots & & & & \ddots \end{bmatrix}$$

$$\Phi^\dagger \Phi \rightarrow \begin{bmatrix} \gamma_1^2 & 0 & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ & & \gamma_{n_R-L}^2 & 0 \\ 0 & \cdots & & \ddots \end{bmatrix} \text{ in high INR}$$

which requires (from (22)) choosing  $\mathbf{U}$  and  $\mathbf{V}$  such that

$$\mathbf{u}_i \cdot \mathbf{v}_j = \begin{cases} 1 : i + j = n_R + 1 \\ 0 : \text{otherwise} \end{cases} \quad (25)$$

where the condition on the first line imposes the reversed pairing we mentioned. In this case

$$\Phi = \begin{bmatrix} 0 & \cdots & & 0 \\ \vdots & \ddots & & \vdots \\ & & 0 & \ddots \\ & & \gamma_{n_R-L} & \\ \gamma_1 & 0 & \cdots & 0 \end{bmatrix} \mathbf{0}_{n_R \times \chi} \quad (26)$$

Substituted in (20), this will yield the capacity closest to (18) in interference. This bound behaves in a manner similar to (24). It will predict a non-zero upper-bound provided  $L < n_R$  and a zero bound otherwise.

Returning to lower bounds, we see that, instead of choosing  $\mathbf{U}$  and  $\mathbf{V}$  to ensure the interference factors do minimal damage to the channel gains, we could choose  $\mathbf{U}$  and  $\mathbf{V}$  to have them do maximal damage. This would yield the capacity furthest from that available without interference. Instead of pairing smallest interference factor with smallest channel gain, in this case we would pair the smallest interference factor with the largest channel gain and, in the limit of high INR, this would zero that gain completely. Then,

$$\Phi^\dagger \Phi = \begin{bmatrix} (\gamma_1 \tilde{\lambda}_1)^2 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & (\gamma_N \tilde{\lambda}_N)^2 & \\ & & & 0 \\ 0 & \cdots & & \ddots \end{bmatrix}$$

$$\Phi^\dagger \Phi \rightarrow \begin{bmatrix} 0 & \cdots & & 0 \\ 0 & \ddots & & \vdots \\ & & \gamma_{L+1}^2 & \\ & & & \ddots \\ & & & & \gamma_N^2 & 0 \\ 0 & \cdots & & & & \ddots \end{bmatrix} \text{ in high INR}$$

which requires (from (22)) choosing  $\mathbf{U}$  and  $\mathbf{V}$  such that

$$\mathbf{u}_i \cdot \mathbf{v}_j = \begin{cases} 1 : i = j \\ 0 : i \neq j \end{cases} \quad (27)$$





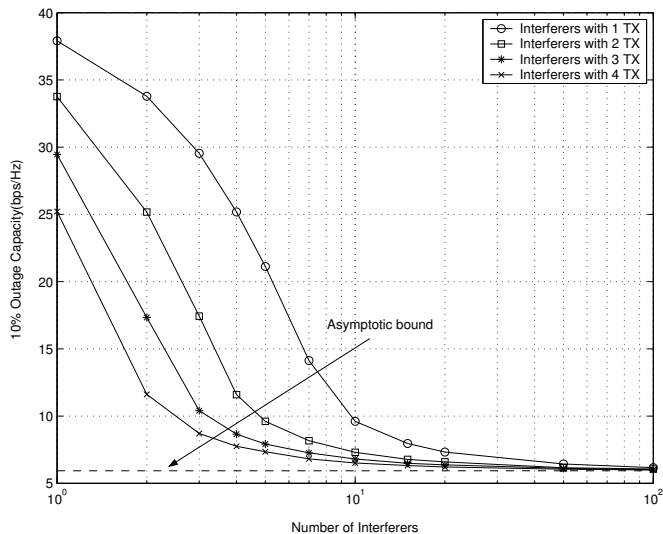


Fig. 3. 10% outage capacity versus number of interferers for MIMO interferers with different numbers of TX antennas. User of interest has 8 TX and 8 RX. Curves shown only for unknown channel and interference.

In Figs. 2 and 3, we focus on the system performance with MIMO interferers of different sizes (by ‘size’ we refer to the number of antennas,  $n_I$ , on each interferer). Fig. 2 shows how  $C_{0.1}$  varies in a  $4 \times 4$  system and Fig. 3 is for an  $8 \times 8$  system. Also shown is the asymptotic lower bound predicted by (23). As expected, the  $8 \times 8$  system saturates later than the  $4 \times 4$  system, since it has more degrees-of-freedom with which to mitigate interference. However, the absolute loss-per-antenna is the same in both systems, about 4 to 5bps/Hz per antenna before saturation. Thus, the  $8 \times 8$  system does not afford better protection against interference than the  $4 \times 4$  system; it just starts out with more capacity to lose in the first place.

In comparing the behaviour of the systems in higher-order interference we would expect there to be a correspondence between, say, 2 interferers with 1 antenna each and 1 interferer with 2 antennas since the transmitted signals are Gaussian by assumption. This is clearly seen to be the case.

From the plot of the spatially white interference lower bound, we can see that even with a few tens of interferers it serves as a very accurate prediction of actual capacity. Note that this too has been plotted at its 10% outage across the 10 000 channel instantiations used. We could have taken as our measure of the bound the lowest capacity computed, but this would not seem to provide a direct comparison with the other (outage) curves.

*Remark 1:* Recall that, as  $L$  increases,  $\mathbf{R}$  has smaller and smaller off-diagonal elements. Thus, as  $\mathbf{R}$  becomes whiter the capacity falls. We see that MIMO systems perform more efficiently the more spatially coloured the interference. This conclusion was also reached in [25], though by different means.

*Remark 2:* In the combined channel  $\mathbf{R}^{-1/2}\mathbf{H}$ ,  $\mathbf{R}$  affects the correlations in the channel. The gradual whitening of  $\mathbf{R}$  represents a decrease in the additional correlation of the columns/rows of the channel. Once  $\mathbf{R} = \mathbf{I}$ , there is no additional correlation at all. However, as  $\mathbf{R}$  whitens (with more

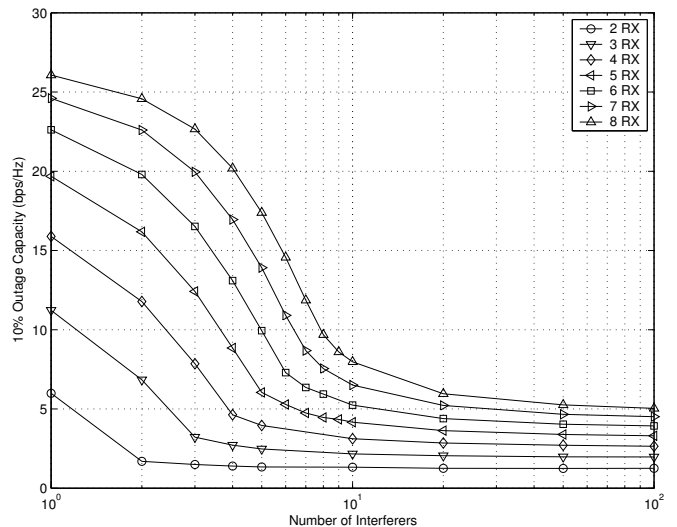


Fig. 4. 10% outage capacity versus number of interferers for different numbers of RX antennas. All interferers have 1TX antenna, desired user has 4TX antennas. Curves shown only for unknown channel and interference.

interferers) the capacity falls. This is unusual — we are accustomed to wanting as little correlation as possible to maximize the capacity but here find that higher correlation achieves that goal. We think this may be explained intuitively by remembering that  $\mathbf{R}$  is a covariance matrix, and the off-diagonal, cross-correlation elements show the ‘predictability’ of the interference. The receiver is able to exploit this predictability to mitigate it. As these off-diagonal elements decrease, the interference is less and less structured.

We turn now to evaluating the effect of having a rectangular MIMO system (i.e.  $n_R \neq n_T$ ), wishing to see if there is any impact on the performance in interference. Fig. 4 focuses on the effect of changing the number of receiving antennas and Fig. 5 on the effect of changing the number of transmitting antennas. In both these figures, all the interferers have 1 transmit antenna. In Fig. 4 we see the usual result that adding a receive antenna always increases MIMO capacity — it collects more of the transmitted power. We see that adding receive antennas provides better resilience to interference. It raises the curve as expected and, since the system now has more degrees-of-freedom, it saturates later. This we would expect from (24) since  $\Phi$  has more rows. Note also that adding receive antennas reduces the bps/Hz cost per interfering antenna. In going from four receive antennas to eight, the loss decreases from  $\sim 4$ bps/Hz to  $\sim 2$ bps/Hz.

In Fig. 5, we observe the expected fall in capacity as we remove transmit antennas and the small and diminishing increase as we add them. Without more receive antennas to collect more of the transmitted power (which effectively strengthens the channel’s eigenmodes), we would not expect any substantial gain. On the other hand, removing transmit antennas does not decrease the saturation population — we can support as many interferers as there are receiving antennas since this limits the degrees-of-freedom as explained in Section IV. As a result, it is worth noting that there is little point providing multiple transmit antennas with large numbers of

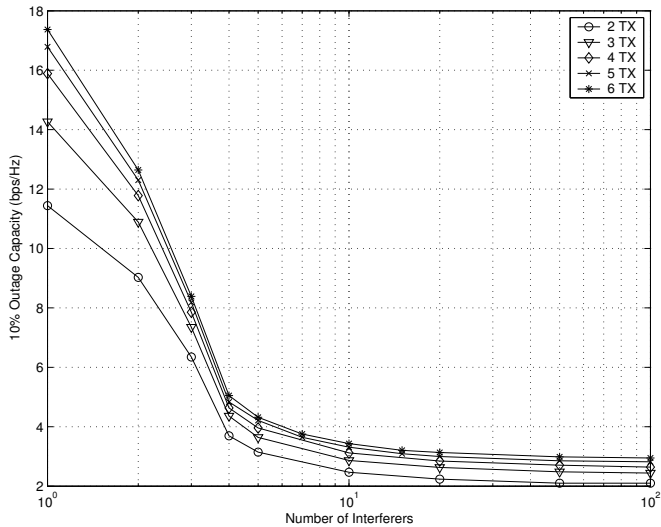


Fig. 5. 10% outage capacity versus number of interferers for different numbers of TX antennas. All interferers have 1TX antenna, desired user has 4RX antennas. Curves shown only for unknown channel and interference.

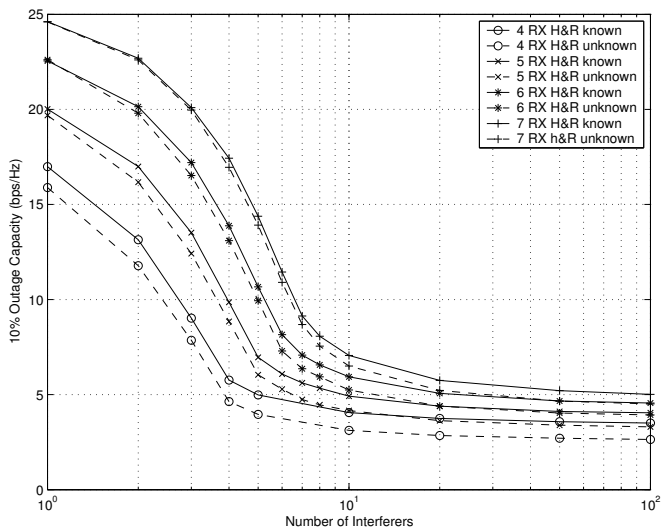


Fig. 6. 10% outage capacity versus number of interferers for  $n_R > n_T$ . All interferers have 1TX antenna, desired user has 4TX antennas. Solid lines are with both channel and interference information, dashed lines with neither.

interferers, since all the curves shown exhibit closely similar capacities beyond the saturation point. There is very little to be gained in this region; by tripling the number of antennas from 2 to 6, the capacity grows only from 2bps/Hz to 3bps/Hz.

### B. Effect of knowledge of channel and interference covariance

We consider two scenarios here, similar to those in the previous section:  $n_R > n_T$  in Fig. 6 and  $n_R < n_T$  in Fig. 9 with  $n_T = 4$  and  $n_I = 1$ . Whilst the likelihood of knowing both  $\mathbf{H}$  and  $\mathbf{R}$  at the transmitter and implicitly in real-time is debatable, the comparison is interesting nevertheless.

From Fig. 6, we see that the benefit of knowing channel and interference diminishes as we add receive antennas. In particular, the gain with few interferers almost vanishes by the

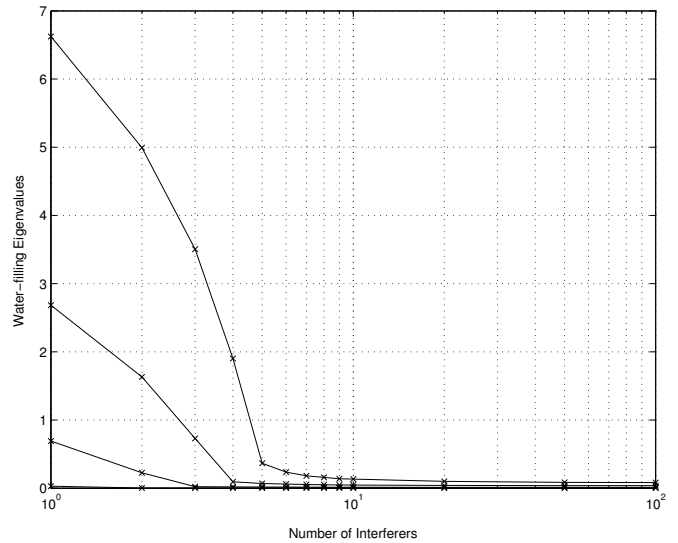


Fig. 7. 10% 'outage' values of ordered water-filling eigenvalues of MIMO system where desired user has 4 TX and 5 RX antennas. Each interferer has 1 TX antenna.

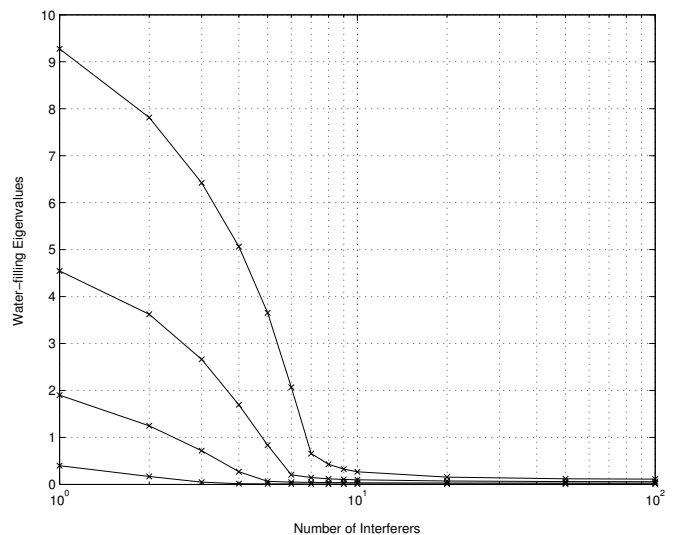


Fig. 8. 10% 'outage' values of ordered water-filling eigenvalues of MIMO system where desired user has 4 TX and 7 RX antennas. Each interferer has 1 TX antenna.

time we have 7 receive antennas, but is approximately 1bps/Hz for the case of 4 receive antennas. It is also apparent that the gain increases as there are more interferers though this effect is more pronounced with higher numbers of receive antennas.

These trends may be explained by noting that water-filling (at a particular SNR) depends critically on the eigenvalues of  $\mathbf{H}^\dagger \mathbf{R}^{-1} \mathbf{H}$ . The 10% 'outage' values of the eigenvalues for a  $5 \times 4$  system and a  $7 \times 4$  system are shown in Figs. 7 and 8 respectively. It can be seen that, apart from the fact that the range of the ordered eigenvalues is larger in the 7 receive antenna case, their actual values are also greater. Recall from (5) that the power allocation to the eigenmodes depends on the *reciprocal* eigenvalues. Thus, a higher-valued set of eigenvalues will demand power allocations that are more nearly equal. That is, the water-filling scheme looks more

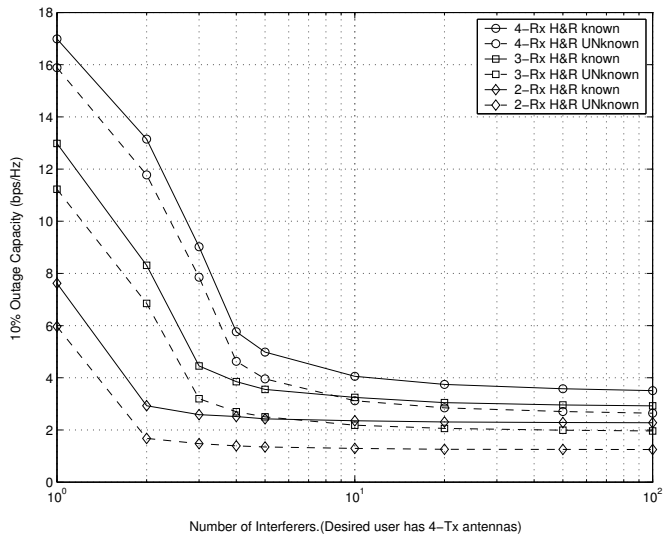


Fig. 9. 10% outage capacity versus number of interferers for  $n_R < n_T$ . All interferers have 1 TX antenna, desired user has 4 TX antennas. Solid lines are with both channel and interference information, dashed lines with neither

like equal power allocation when there are larger numbers of receive antennas, and hence the two capacities are closer to one another. An inverse argument explains the larger gain afforded by water-filling as the number of interferers increase, since it is clear that as this happens, the eigenvalues fall.

Additionally, Fig. 6 shows that with many interferers, a higher capacity is achieved by reducing the number of receive antennas by one and ensuring full channel and interference information is known at the transmitter. Thus, in a system with many more interferers than receive antennas, a slightly lower complexity MIMO architecture should be used and this should also make it easier to achieve full channel feedback to the transmitter. This observation is accounted for by again remembering that water-filling powers are assigned in accordance with the reciprocal eigenvalues. With many interferers, it is likely that one or more eigenmodes will be so weak that equal-power allocation would waste a substantial fraction of the power on this eigenmode whereas water-filling would assign it little, if any as its reciprocal would be so much larger than the others. For example, in Fig. 8, with 100 interferers, the reciprocal eigenvalues are the set  $\{8.9, 17.2, 38.6, 140.8\}$ . At an SNR of 20dB water-filling would assign no power to the largest reciprocal whilst equal power allocation would ‘waste’ 25% of the available power on this eigenmode. A similar effect is observed in Fig. 9 with fewer receive than transmit antennas.

## VI. SIMULATION RESULTS FOR INR

We now fix the desired user to have  $n_T = n_R = 4$  antennas and each interferer to have  $n_I = 1$  transmit antenna, and vary the INR. We fix the SNR at 15dB. We employ a range of INR from 0 to 30dB since we wish to explore the performance when the system is dominated by interference rather than by noise as would be the case if  $\text{INR} < 0\text{dB}$ .

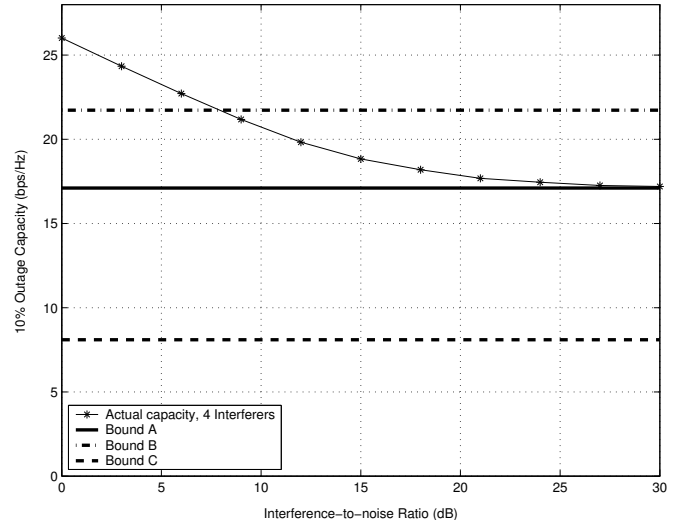


Fig. 10. 10% outage capacity and asymptotic bounds versus INR. Desired user has 8 TX and 8RX antennas. 4 interferers, each with 1 TX antenna, SNR=15db. Curves shown only for neither channel nor interference information.

### A. Bounds

We begin by simulating the bounds derived in Section IV. Fig. 10 shows capacity as a function of INR for an  $8 \times 8$  system with 4 interferers and Fig. 11 the same with 7 interferers. The three horizontal lines are identified as follows:

**Bound A:** Equation (24)

**Bound B:** Equation (26)

**Bound C:** Equation (28)

All three bounds are plotted at the 10% ‘outage’ level. We see that Bound A provides a tight asymptotic lower bound as expected, whilst Bound C is very loose. In the case of 4 interferers, Bound C underbounds by about 9bps/Hz and with 7 interferers by about 4.5bps/Hz. This looseness is as predicted in Section IV and would require the interferers to conspire in the worst possible manner. Similarly, the upper bound, Bound B, is also rather optimistic as expected. It over bounds the asymptotic capacity by about 4bps/Hz with 4 interferers and by about 1.5bps/Hz with 7 interferers and would require the interferers to conspire in the best possible manner. Of these three bounds, Bound A is clearly the most useful (and the easiest to calculate). It seems to us that whilst upper bounds to capacity are usually of more interest than lower bounds, in interference we would want to know the worst that the situation could possibly be rather than the best.

It can also be seen that Bound A is reached more quickly with fewer interferers, and in these cases serves as a good capacity prediction even at moderate INR. This fact is direct from (24) by observing that the fewer rows of  $\Phi$  affected by interference, the smaller their maximum overall degrading effect on the capacity can be and hence their effect will expire sooner.

Fig. 12 shows the variation of all three bounds with number of interferers in an  $8 \times 8$  system and Fig. 13 in an  $8 \times 5$  system. In Fig. 12, Bound C is seen to remain loose throughout (until convergence), whilst the upper bound, Bound B,

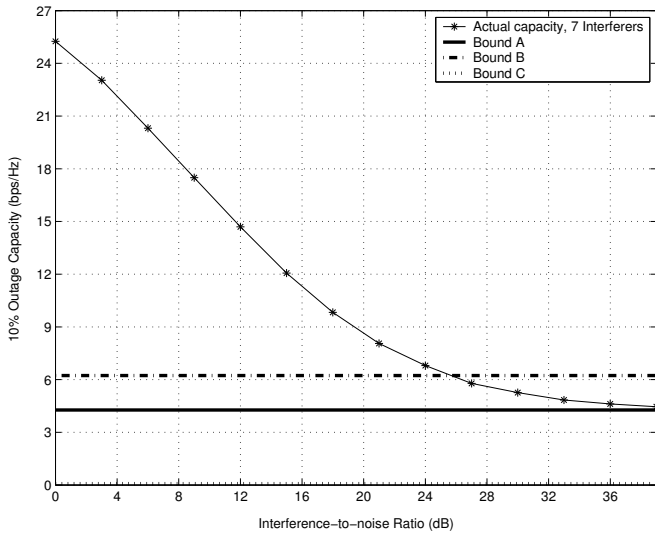


Fig. 11. 10% outage capacity and asymptotic bounds versus INR. Desired user has 8 TX and 8RX antennas. 7 interferers, each with 1 TX antenna, SNR=15dB. Curves shown only for neither channel nor interference information.

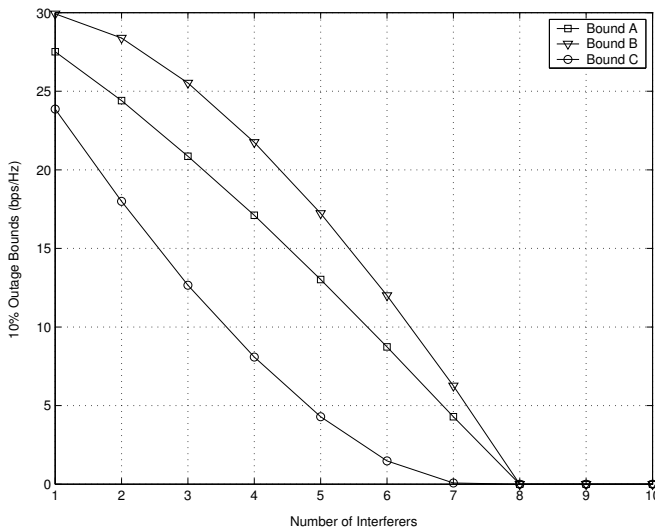


Fig. 12. 10% outage asymptotic bounds versus number of interferers. Desired user has 8 TX and 8 RX antennas, each interferer has 1 TX antenna, SNR = 15dB. Curves shown only for neither channel nor interference information.

gradually improves as an estimate of the true asymptotic capacity but is still loose. The tight bound is almost linear in the number of interferers, consistently losing about 4 to 5bps/Hz per interferer in both cases which matches our observations on Figs. 2 and 3. Fig. 12 shows all three bounds converging to zero at  $L = n_R$  as expected though Bound C is practically zero at  $L = 7$ . Fig. 13 shows that Bound C is noticeably looser in a system with  $n_T < n_R$ , and falls to zero at  $L = \min(n_R, n_T) = n_T$  as expected from Section IV. Unlike in the  $n_T = n_R$  case, the upper bound at first becomes looser before tightening. This is because there is some immunity afforded to this bound by the reverse pairing of the channel gains  $\gamma_i$  and interference factors  $\tilde{\lambda}_i$ . We must have at least  $(n_R - n_T)^+$  interferers before any of the zero

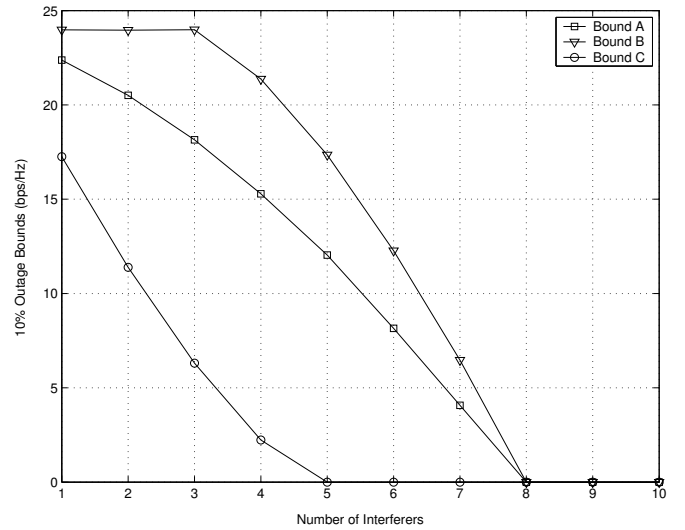


Fig. 13. 10% outage asymptotic bounds versus number of interferers. Desired user has 5 TX and 8 RX antennas, each interferer has 1 TX antenna, SNR = 15dB. Curves shown only for neither channel nor interference information.

$\tilde{\lambda}_i$  affect the channel gains. In this region only Bound A is informative since the other two are suffering from their loose nature more seriously at low numbers of interferers.

### B. Effect of channel and interference covariance knowledge

Figs. 14 and 15 compare the performance versus INR of an  $8 \times 8$  and a  $16 \times 16$  system with various numbers of interferers, and different conditions of channel and interference knowledge.

It is seen that, while  $L < n_R$ , having more interferers means that water-filling on the basis of knowing both  $\mathbf{H}$  and  $\mathbf{R}$  gives an increasing gain over equal power allocation. For example, in the  $8 \times 8$  case the gain with 2 interferers is about 2.5bps/Hz whilst with 4 interferers it is about 3.5bps/Hz at high INR. In the  $16 \times 16$  case, these gains are about 3bps/Hz and 5bps/Hz respectively.

For the curves with  $L \geq n_R$  the benefit of knowing  $\mathbf{H}$  and  $\mathbf{R}$  remains nearly constant, though is slightly larger at moderate INRs. In the higher INR region though, the curves begin to converge toward zero. This supports our prediction from Section IV and the convergence of the curves from Section VI-A since  $\Phi$  has only  $n_R$  rows with which to support interferers.

## VII. SUMMARY

We have introduced a new tool for examining the effect of co-channel interference in MIMO systems. The new matrix investigated here makes more explicit than before the effect of such interference in these systems. We have used  $\Phi$  to show that adding an interfering antenna has the effect of eliminating one of the well-known MIMO sub-channels (in the limit of high INR) but leaving the others unchanged. This observation allowed us to derive bounds on the performance in high INR. These bounds have been simulated and seen to provide limits of varying tightness. It was also seen that MIMO

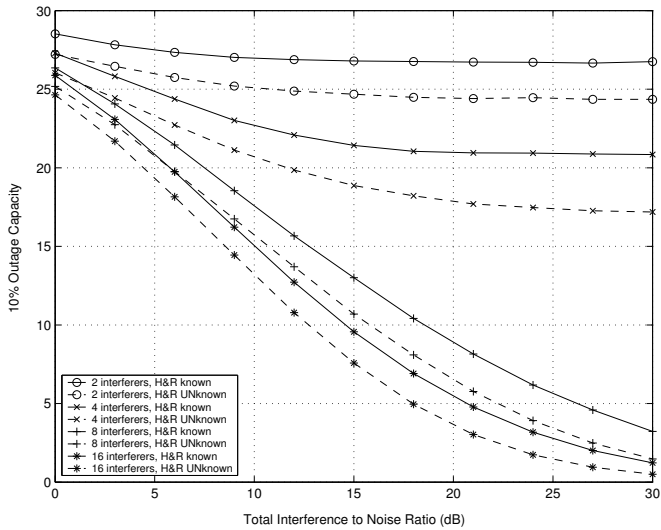


Fig. 14. 10% outage capacity versus INR for an  $8 \times 8$  system with various numbers of interferers. Each interferer has 1 TX, SNR = 15dB. Solid lines are with both channel and interference, dashed lines with neither.

systems perform more efficiently the more spatially coloured the interference they experience, and, with many interferers, their overall effect is a reduction of the prevailing SNR.

By simulation, we examined the effect of having more or less receive antennas than transmit antennas. As expected, there was an advantage in having  $n_R > n_T$ , but little benefit for  $n_T > n_R$ . The impact of having full transmit-side knowledge of channel and interference compared to having no such knowledge was assessed. It was found that such knowledge provided a small gain which diminished as the system was equipped with more receive antennas. These simulations also showed that transmit-side channel knowledge with water-filling is able to compensate for having one less receive antenna and no such knowledge.

## VIII. ACKNOWLEDGMENTS

The authors wish to thank Debbie Raynes, Tim Mousley and Chris Williams for their insightful discussions and helpful suggestions.

## APPENDIX

Here, we show that  $\Phi$  has the general form shown in (14). Recall from (13):

$$\Phi = \Lambda^{-1/2} \mathbf{U}^\dagger \mathbf{V} \mathbf{T}$$

$$= \begin{bmatrix} \tilde{\lambda}_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \tilde{\lambda}_{n_R} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1^\dagger \\ \vdots \\ \mathbf{u}_{n_R}^\dagger \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \cdots \mathbf{v}_{n_R} \end{bmatrix} \begin{bmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \gamma_N \\ 0 & \cdots & & \ddots \end{bmatrix} \quad (\text{A.29})$$

where all the matrices are of the dimensions given in Section IV and  $N = \min(n_R, n_T)$ . The middle two matrices combine

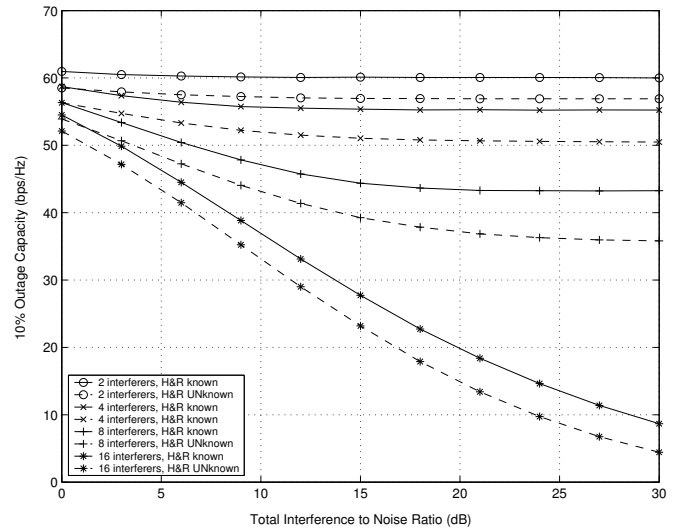


Fig. 15. 10% outage capacity versus INR for a  $16 \times 16$  system with various numbers of interferers. Each interferer has 1 TX, SNR = 15dB. Solid lines are with both channel and interference, dashed lines with neither.

to give a matrix of dot-products:

$$\begin{bmatrix} \mathbf{u}_1^\dagger \\ \vdots \\ \mathbf{u}_{n_R}^\dagger \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 & \cdots & \mathbf{v}_{n_R} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 \cdot \mathbf{v}_1 & \cdots & \mathbf{u}_1 \cdot \mathbf{v}_{n_R} \\ \vdots & \ddots & \vdots \\ \mathbf{u}_{n_R} \cdot \mathbf{v}_1 & \cdots & \mathbf{u}_{n_R} \cdot \mathbf{v}_{n_R} \end{bmatrix}. \quad (\text{A.30})$$

With pre(post)-multiplication by diagonal matrices, the diagonal elements multiply the rows (columns) of (A.30) element-wise. Remembering that the  $\gamma_i = 0$  for  $i > N$ , the last  $\chi = (n_T - n_R)^+$  columns of  $\Phi$  will be multiplied by zero. Thus,

$$\Phi = \begin{bmatrix} \gamma_1 \tilde{\lambda}_1 (\mathbf{u}_1 \cdot \mathbf{v}_1) & \cdots & \gamma_N \tilde{\lambda}_1 (\mathbf{u}_1 \cdot \mathbf{v}_N) \\ \vdots & \ddots & \vdots \\ \gamma_1 \tilde{\lambda}_{n_R} (\mathbf{u}_{n_R} \cdot \mathbf{v}_1) & \cdots & \gamma_N \tilde{\lambda}_{n_R} (\mathbf{u}_{n_R} \cdot \mathbf{v}_N) \end{bmatrix} \mathbf{0}_{n_R \times \chi}, \quad (\text{A.31})$$

which is (14).

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