NEW DIRECTIONS FOR SITUATED COGNITION IN MATHEMATICS EDUCATION
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Edited by
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Chapter 7

‘We Do It A Different Way At My School’

*Mathematics homework as a site for tension and conflict*

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Abstract: This chapter draws on Wenger’s (1998) account of communities of practice to provide insights into the relationship between home and school mathematics practices and identities. The chapter presents and analyses an interaction between a 9-year-old boy and his mother as she attempts to help him with a mathematics homework task, consisting of a sheet of two-digit subtraction problems. The analysis reveals considerable tension and conflict at the boundary between home and school practices, as the different identities of mother and child negotiate with and challenge each other. These conflicts are exemplified by arguments about the appropriate methods for carrying out the subtractions, in which both participants justify their positions in terms of power and legitimacy instead of the underlying mathematical principles. One implication is that schools need to reconceptualise their approach to homework and parents’ role in supporting homework if such interactions are to be more supportive of children’s mathematics learning.

Key words: communities of practice, boundaries, identities, mathematics homework

1. INTRODUCTION

In the late 1980s and early 1990s many mathematics educators were drawn to the novel ideas about situated cognition and situated learning emanating from writers such as Brown, Collins, and Duguid (1989), Lave (1988) and Lave and Wenger (1991). These ideas were attractive to mathematics
educators as they challenged the traditional view embodied in much educational thinking that knowledge can be separated from the situations in which it is acquired and used. Instead, Lave and her colleagues argued that knowing and learning are essentially situated in social practices, and that in order to understand the nature of knowing and learning we need therefore to understand the nature of these practices. This meant that attention was drawn to the use of mathematics in everyday settings such as supermarkets, workplaces and homes, as well as to the acquisition of mathematics in the classroom (e.g. Watson, 1998).

Our own particular and longstanding interest is with the different worlds which young (pre-school and primary school) children inhabit as they move between home and school – and other places beside (e.g. Greenhough and Hughes, 1998; Hughes, 1986 and 2001; Tizard and Hughes, 1984). We are interested in the ways in which these different worlds are present and interpenetrate – or create obstacles between each other in events and practices. We are also interested in what happens to individual children as they move between these different worlds – how they present themselves in each world, whether they experience them as similar or dissimilar, and how they make sense of any dissimilarities or discontinuities which they may experience. While our focus here is on mathematics, we are interested in these issues across the school curriculum and beyond.

In some of the early writing of situated theorists these kinds of issues were only sketchily addressed. For example, the practices studied by Lave and Wenger are considered primarily in isolation from other practices, and there is little sense of participants moving between a number of different practices. As others have pointed out (e.g. Walkerdine, 2007) a somewhat static and singular view of practice can come across from these writings. More recently, though, Wenger (1998) has given greater recognition to the plurality and dynamic nature of practice, and the ways in which individuals move between multiple communities of practice. For example, he suggests that organisations such as factories and schools might be more productively viewed as constellations of communities of practice, which can be linked together in various ways. He also pays particular attention to the boundaries between different communities of practice, and looks at ways in which continuities across these boundaries can be maintained. One way is through boundary objects, a term originally used by Star and Griesemer (1989) to describe “objects that serve to coordinate the perspective of different constituencies for some purpose” (Wenger, p. 106). A second way of maintaining continuity is through the practice of brokering, which occurs when individuals use their membership of multiple communities of practice “to transfer some element of one practice into another” (ibid., p. 109). Wenger points out that “the job of brokering is complex. It involves
The multiple membership of different communities of practice is also central to Wenger’s conceptualisation of identity. He argues that an identity should not be regarded as a static or singular entity, but instead should be viewed as ‘a nexus of multimembership’. This notion of identity as a nexus means that work frequently has to be done to reconcile the different forms of membership forming the nexus. Indeed, Wenger proposes that:

The work of reconciliation may be the most significant challenge faced by learners who move from one community of practice to another. For instance, when a child moves from a family to a classroom, when an immigrant moves from one culture to another, or when an employee moves from the ranks to a management position, learning involves more than appropriating new pieces of information. Learners must often deal with conflicting forms of individuality and competence as defined in different communities (p. 160, emphasis added)

Wenger suggests that this process of reconciliation may not be easy, and that membership of multiple communities of practice may involve tensions and conflicts that are never fully resolved. At the same time, he makes clear that in his view “multimembership and the work of reconciliation are intrinsic to the very concept of identity” (p. 161)

While Wenger’s work provides an important conceptual backdrop to this chapter, we will also draw on more recent work by Street, Baker and Tomlin (2005). This work represents one of the most far-reaching attempts to date to analyse the nature of home and school mathematics. Here, we will briefly describe some of the key constructs used by these authors.

Like Wenger, Street et al. see themselves as developing a ‘social approach’ to learning, although in their case the focus is specifically on numeracy. They argue for a perspective “which sees the social in terms of context, values and beliefs, social and institutional relations” (p. 17). They also refer to this as an ‘ideological’ model of numeracy:

From this perspective social relations refer to positions, roles and identities of individuals in relation to others in terms of numeracy. Social institutions and procedures we see as constitutive of control, legitimacy, status and the privileging of some practices over others in mathematics… (ibid., p. 17).

Street et al. also make an important distinction between numeracy events and numeracy practices. Drawing on an earlier definition of a literacy event by Heath (1983), they define numeracy events as “occasions in which a numeracy activity is integral to the nature of the participants’ interactions
and their interpretative processes” (ibid., p. 20). Numeracy practices, in contrast, are said to focus on “the conceptualisations, the discourse, the values and beliefs, and the social relations that surround numeracy events as well as the contexts in which they are located” (ibid., p. 20). Numeracy practices are also said to be “broad notions about the ways numeracy is dealt with in different contexts and settings” (ibid., p. 21).

In addition, Street et al. make an important distinction between domain and site. Drawing again on previous work in literacy, this time by Barton and Hamilton (1998), they distinguish between ‘sites’ – as the actual places where the activities take place – and ‘domains’ – as areas of activity not located in specific places. Applying this to the distinction between home and school provides the 2 x 2 grid shown in Table 1 below:

<table>
<thead>
<tr>
<th>Domain: schooled numeracy practices</th>
<th>Domain: out-of-school numeracy practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>School site</td>
<td>Dates, birthdays, aspects of data and measuring, Pokemon cards, money, playground games</td>
</tr>
<tr>
<td>Working on number bonds, counting, calculating. Numbers of children away and in class.</td>
<td>Pocket money, time, laying the table, shopping, setting the video, home discipline, ‘symbolic’ uses of number systems, ‘finger counting’, door numbers, jigsaws and calendars</td>
</tr>
<tr>
<td>Home site</td>
<td>Homework, commercially marketed texts, counting up and down stairs, patterns on car number plates, door numbers</td>
</tr>
</tbody>
</table>

Like Street et al., we are interested in the relationship between home and school mathematics practices, and what happens when children move between them. In an earlier study (Hughes and Greenhough, 1998) we approached these issues by looking at children aged 5-7 years playing a similar mathematical game in two settings, with a parent at home and with a teacher at school. As well as being interested in what this told us about the boundaries between home and school, we were also interested in the ways in which children might or might not make connections across these boundaries. We observed that the children spontaneously made connections between the two settings, for example assuming that the rules of the game were the same in each setting. We also noticed examples of where the adult’s lack of awareness of what had happened in the other setting had a significant effect on how the game was played. For example, one child used a measuring ruler as a number line in the school setting, but when she suggested this at home her mother refused on the grounds that it was irrelevant to the activity.

In this chapter we explore these issues further by looking at a 9-year-old boy carrying out a piece of mathematics homework at home. The data takes
the form of a transcript of the conversation which ensues when the boy’s mother attempts to help him. We will focus in particular on the different worlds which are present in the conversation, and the different ways in which these worlds relate to each other, in an attempt to increase our understanding of the different practices of home and school mathematics, and of the boundaries between them. In so doing, we are explicitly following a suggestion made by Lave\textsuperscript{11} that homework can provide an interesting perspective on these issues, “because it moves back and forth between home and school, and actually to the bowling alley, burger bar and so on”. In other words, by studying an object such as homework which crosses the boundaries between different communities of practice, we can learn something about those communities in particular and something about boundary crossing more generally.

2. **RYAN, HIS MOTHER AND HIS HOMEWORK**

In this part of the chapter we present a description of a numeracy event, as defined by Street et al., 2005, involving a 9-year-old boy called Ryan (a pseudonym) and his mother. The event occurs in the living room of the family home while Ryan is doing his mathematics homework. We will first provide a verbatim account of the event as it occurred, and then present an analysis of the event in terms of the different practices and identities involved.

The event was captured on video by Ryan’s mother as part of her involvement in the numeracy strand of the Home School Knowledge Exchange project. The overall aim of the project was to develop and implement programmes of home school knowledge exchange activities and look at their impact on children, teachers and parents. The numeracy strand of the project involved children in Years 4 and 5 from four contrasting primary schools in Bristol and Cardiff. In each school six children were chosen for more intensive study, on the basis of gender and attainment, and in-depth interviews were carried out with these children, their teachers and their parents. Ryan was one of these ‘target’ children, selected at random from a group of low-attaining boys (see Winter, Salway, Yee, and Hughes, 2004, for more details of the numeracy strand of the project).

\textsuperscript{11} Situated cognition in mathematics, *Seminar held at Oxford University, Department of Educational Studies May 3rd, 1996*
As part of the family’s involvement in the project, Ryan’s mother was loaned a video camera and asked to record mathematics events which took place in the home. This request was made after a long interview in which the kinds of mathematics taking place at home had been explored. When Ryan’s mother returned the camera the tape was mostly filled with the homework event, although it also contained some footage of Ryan and his brother playing games outside.

At the start of the event Ryan is doing his homework on a box file balanced on top of a pouffe. He does not look happy. His mother is sitting on the floor next to him peering over his work. The work is in the form of a sheet headed ‘takeaway revision work’

As can be seen from Figure 1, the worksheet consists of a number of subtraction calculations involving two-digit numbers. On the worksheet these calculations are printed in horizontal form, with an empty box in which to place the answer (e.g. \(33 - 16 = \square\)). However, Ryan’s teacher has also written each calculation in a vertical form e.g.

\[
\begin{array}{c}
33 \\
-16 \\
\hline
\end{array}
\]

next to the horizontal form. In addition, next to each calculation is an empty number line with the number which has to be subtracted from (the *minuend*) printed at the right-hand end. For the first calculation, Ryan’s teacher has added 16 dots and numbers to the number line, counting back from the minuend. These dots and numbers represent the number which has to be subtracted (the *subtrahend*). The answer to the calculation (17) can therefore be read off from the left-hand end of the number line.

The homework sheet thus affords a number of ways of carrying out the calculation. This is consistent with current teaching methods in primary mathematics in England, as laid out in the National Numeracy Strategy (DfEE, 1999). In particular, children are encouraged to develop a range of mental and informal written methods for addition and subtraction calculations before they are introduced to standard written procedures.
For subtraction calculations such as these, where the number in the units column for the subtrahend is greater than that for the minuend, the currently favoured standard procedure is one of decomposition. This means that 1 is taken from the tens column of the minuend and 10 is added to the units column, as shown below:

\[
\begin{array}{c}
33 - 16 = \\
33 \\
- 16 \\
\hline
17 \\
18 \\
19 \\
20 \\
21 \\
22 \\
23 \\
24 \\
25 \\
26 \\
27 \\
28 \\
29 \\
30 \\
31 \\
32 \\
33 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
43 - 15 = \\
43 \\
- 15 \\
\hline
28 \\
29 \\
30 \\
31 \\
32 \\
33 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
84 - 17 = \\
84 \\
- 17 \\
\hline
67 \\
68 \\
69 \\
70 \\
71 \\
72 \\
73 \\
74 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
61 - 13 = \\
61 \\
- 13 \\
\hline
48 \\
49 \\
50 \\
51 \\
52 \\
53 \\
54 \\
55 \\
56 \\
57 \\
58 \\
59 \\
60 \\
61 \\
62 \\
63 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
52 - 18 = \\
52 \\
- 18 \\
\hline
34 \\
35 \\
36 \\
37 \\
38 \\
39 \\
40 \\
41 \\
42 \\
43 \\
44 \\
45 \\
46 \\
47 \\
48 \\
49 \\
50 \\
51 \\
52 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
95 - 19 = \\
95 \\
- 19 \\
\hline
76 \\
77 \\
78 \\
79 \\
80 \\
81 \\
82 \\
83 \\
84 \\
85 \\
86 \\
87 \\
88 \\
89 \\
90 \\
91 \\
92 \\
93 \\
94 \\
95 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
71 - 12 = \\
71 \\
- 12 \\
\hline
59 \\
60 \\
61 \\
62 \\
63 \\
64 \\
65 \\
66 \\
67 \\
68 \\
69 \\
70 \\
71 \\
72 \\
73 \\
74 \\
75 \\
76 \\
77 \\
78 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\text{Figure 7.1. Ryan’s homework sheet}
\end{array}
\]

However there is an alternative method which was favoured in the past, called equal addition. Here 10 is added to the units column in the minuend,
while at the same time 1 is added to the tens column in the subtrahend (see below)

\[
\begin{array}{c}
33
-16 \\
\hline
\end{array}
\]

becomes

\[
\begin{array}{c}
3\ 13 \\
-2\ 6 \\
\hline
\end{array}
\]

It is not clear which procedure Ryan’s teacher wants him to use for these calculations, and there are no instructions on the sheet to provide guidance. Nevertheless, the fact that there are several different ways of carrying out these calculations is crucial for understanding the conversation which follows.

The conversation starts as Ryan is working on the calculation 84 - 17. He has already attempted the first two calculations.

1. M  What’s that you’re doing?
2. C  My work (sounds defensive)
3. M  What’s that? Let’s see
4. C  It’s my work (He uses his arm to cover the part of his sheet he is working on. His body language generally suggests “get out of my face”.) (Looks at the video camera.)
   It’s on record, mum (defensive and accusatory)
5. M  {What’s/it’s?} take away 15, take away 43
   {You’ve} just dropped off one right? 12 
   No because I just wanted to know if that was the way you were doing it, if it was the same as what I was doing
6. C  I do it a different way from you
   (He has now gone back to the first calculation 33 – 16.)
   3 take away 6, I can’t do that
7. M  (Takes camera off the tripod to get closer to the work.)
8. C  (Closes eyes and sighs.)
   {I keep doing them wrong}
   (Puts head on arm.)
9. M  Well go on to the next {one} then
10. C  Can you stop holding it too close

12 We use the following conventions in this transcript:
() contains a description of non-verbal behaviour or our comment
{word} shows some uncertainty about what was said
[ ] simultaneous speech
.. a slight hesitation or change of direction in what is said
… omission
(Mum has taken the camera off the tripod so that the sheet can be seen more clearly.)
That’s why I hate it (presumably referring to the camera/filming)
11. M Go on to the next one then
12. C I am (with emphasis and an element of accusation)
13. M Right
14. C (Appears to write a number at the end of the number line next to the calculation 61-13)
15. M Have you no\(^\text{13}\) to do this? (pointing to the filled in number line next to the first calculation 33 – 16) Put the same as what.. across {t}here at the top, no?
16. C It’s there already for me, Miss done it
17. M Oh that’s what it’s there for, right
18. C {Mum, you’re speaking}
19. M I know
20. C I’m just doing all that, why is that there
21. M I know, because I don’t... I don’t understand why you’ve no put it there, here, there and there (pointing to the empty lines below)
22. C I don’t have to put it all down there (argumentatively and upset)
23. M Oh right
24. C It’s going to waste all my time.. Miss said
25. M But you’re no in any hurry.
26. C (Sort of tuts and puts his arm down.) Mum, I just want to play out
27. M Well, Ryan, you’ve got to do your homework first
28. C Can you stop speaking, I can’t concentrate
29. M Right, sorry
30. C (By this point he has written 63 next to 61-13=)
   (Works on the remaining calculations in the vertical format, then transfers the answers to the horizontal format, whispering to self.)
   (Seems to finish with a slight bang of the hand holding the pencil.)
   (Returns to the second calculation where he earlier completed the vertical format but did not transfer the answer to the horizontal format.)
31. M Right, can I check them?
32. C (rubbing out) I haven’t done one (Writes 32 next to 43 – 15 = )
   Right
   (Bangs fist down on the work, as if to indicate he has finished.)

\(^{13}\) Ryan’s mother was partly educated in Scotland
Figure 7-2. The homework sheet after Ryan’s first attempt to complete it

Figure 2 shows the answers which Ryan has given to each calculation at this point. As can be seen, only the first one is correct. His most common mistake is simply to subtract the smaller number from the larger number in
the units column, instead of using one of the standard methods described above. For example, for 43 – 15 he has subtracted 3 from 5, followed by subtracting 1 from 4, getting an incorrect answer of 32.

33. M That’s it, finished?
34. C Yeh
35. M Right, all this.. see this here (Points to the vertical format of 61 – 13)
36. C Yeh
37. M It says 61 take away 13 (Points to the horizontal format.)
38. C Miss put it there for me (Points to the vertical format.)
39. M Oh she’s put it there, right
40. C Yeh
41. M To make it easier for you, right
42. C Yeh
43. M Right, well let’s have a look. That’s.. I don’t think that’s right is it?
    That one there (pointing to 43 – 15 = 32 in vertical format) That’s
44. C 4, 5, no 4 \[8 \] 2
45. M I think you can’t.. you can’t take 5, you can’t take..
46. C You have to take 3 away from 5 (rising intonation) 4, 3, 2. You
don’t get it, do you?
47. M No, because if I was doing a take away sum, I’d put
48. C (Raising voice, sounds indignant) It’s the way I do it
49. M Stroke that, you say stroke that (pointing to 43 – 15 = ) and take
    away one.. a 10
50. C It’s the way I do it, we do it a different way
    ...
    They’re tens (pointing to the calculation 43 - 15 in vertical format)
51. M That’s a 4 (points to the 4)
52. C Tens and units (pointing to the 3)
53. M A unit, so it’s.. what.. take one unit away from 4 (rising intonation)
54. C That’s a ten, the 4
55. M Yeh
56. C And there’s the units, the 3
57. M To take em.. to be able to take 5 away frae 3 you have to put one
    unit off the 4 and put it onto the 3, do you not?
58. C No
59. M Well why.. you have to
60. C You don’t, not in my school we don’t, we do it a different way
61. M But it’s no.. that’s no your answer 32, 15 take away..
62. C I’ll do it again then
63. M Let me see, I may be wrong, let me see right, em.. 43 right, take
    away 15, that’s 33.. no, that’s not
64. C (Rubs out.) \{Let me do\} \{do a thing then\} (truculently)
We Do It A Different Way At My School

65. M  Right, well that’s all I’m doing, asking you to do it
66. C  (Looks at the calculation with pencil poised above it.)
67. M  The first time you done it right, you crossed off a unit, that’s prop.. that’s right (The first calculation had a line across the tens part of the upper number.)
68. C  (Gets answer of 32 again.) I’ve got 33 again.. 32, that’s the way I do it (Tone has softened somewhat.)
69. M  But you stroked one unit off there, OK? (rising intonation, pointing to the 4 in 43)
70. C  Oh I get it now
71. M  And put one that you get there, yeh
72. C  (Puts a line across 4 and writes 3. Puts 1 before the 3 units.)  (Hesitates.)
73. M  You’re able to take 5 away from 13 now
74. C  (Sigh) (After a while writes 8 in the units column of the answer, then 2 in the tens column.)
75. M  [I’ll have to do this again} (somewhat crossly)
76.  (Wants to do this again)
77. M  [That’s right, 28, you had 48 the first time
78.  Right what about the next one?
79. C  (_puts a line across 4 and writes 3. Puts 1 before the 3 units.)
80. C  (Wants to do this again)

This numeracy event might appear at first sight to be somewhat mundane. Ryan is doing his maths homework, his answers to the calculations are incorrect, his mother tries to help him, and as a result he starts using an alternative procedure which provides the correct answers. Yet beneath this mundane appearance the event reveals a good deal about the nature of mathematical practices, boundaries and identities.

2.1 The practice of homework

First, we note that the site of the event is the family home. At the time of the recording, this was not a particularly happy place. Ryan’s mother and father were having difficulties in their relationship, and Ryan was undergoing counselling to help him cope with this. He was also having medical
problems which seemed to be related to this. However, when we returned a year later the situation had improved considerably.

While the event is taking place in the home site, it does not belong to the home domain (Street et al., 2005). It serves no function within the family, either as a piece of domestic business or as a leisure activity. Instead, the event is a homework task, part of a practice by which an element of school can legitimately enter the home and demand the child’s attention. This privileged status of homework is evident in the interchange which takes place on turns 26 and 27, when Ryan says “Mum, I just want to play out” and his mother replies “Well, Ryan, you’ve got to do your homework first”. Here we see a home norm relating to homework within which the mathematics interchanges are embedded. The mother has the power to insist that the homework is done even though she cannot necessarily create a scenario wherein the task is done well. However, her insistence that the homework is done may itself be embedded in interchanges with the school that demand that parents see to it that homework gets done. There is also the society view of what constitutes a good parent, which despite the difficulties in her life Ryan’s mother would like to be. For example, in her interview she said about his homework “I do make sure he’ll sit and finish it”.

While the school expects parents to make sure that homework gets done, it does not seem to encourage parental help or support. There are no instructions on the homework sheet, nor is there any information for potential helpers. Thus Ryan’s mother has to infer what the task is, as she tries to do on turn 15. This lack of support (or dialogue with parents) implies that although the task has been sent home, the way in which it is done is still being circumscribed by the school. The ownership and control of the task remain with the school – and specifically with Ryan’s teacher - who determines what is to be done and how it is to be done. It is the interactions which have already taken place at school between teacher and child which are intended to count, not those which might take place between parent and child. Thus we can see that the homework task comes into the home with strong boundaries around it which are intended to keep it firmly under the control of the school. However, as we shall see, these boundaries are challenged and renegotiated as the event unfolds.

The strong influence of the teacher on how the task is carried out can also be seen in the interchange which takes place at turns 24 and 25, concerning time. Ryan’s mother has suggested that he uses the number line method which the teacher has completed for the first calculation, but Ryan seemingly repeats his teacher’s view that this would take too much time. In practice, time is a key aspect when it comes to homework. School homework policies usually focus on time (in terms of how long homework should take for each year group) rather than the actual content of the homework. The
teacher therefore has to judge and get right the amount of time the task will take. Filling in the number lines may help provide a way to access the answers but they will be time consuming and have therefore probably been discouraged. The teacher has to operate within a school policy framework and does not want parents complaining to the headteacher that their children spend far too long on their homework.

In fact, the reply given by Ryan’s mother on turn 25 – “but you’re no in any hurry” - suggests that she is unlikely to subscribe to this view. Her view of time reflects a more out-of-school perspective on time, in which taking/wasting time is only important if you are short of time or are in a hurry or have other things to do. Ryan’s mother clearly thinks it is more important that Ryan spends time getting his homework correct than that he should do it quickly and badly.

2.2 Ryan’s school and home identities

Bringing the school into the home also means that Ryan’s identity in relation to school work becomes visible. At school, Ryan was a low-attamer. According to his class teacher, he had SEN\textsuperscript{14} support in class but still found it hard to listen and concentrate. His reading was particularly poor and this spilled over into other subjects. His teacher described him as being a “loveable rogue” who was “very active, likes sport, but doesn’t enjoy school work”. Another teacher who had taught Ryan for some maths lessons said that he was “not into all this work, he does it against the grain… I like Ryan but there’s not a lot there, maybe”.

This picture of Ryan struggling with school work was supported by observations of him in class. During a lesson on percentages Ryan was seen to be having difficulty understanding throughout the lesson, and there was little evidence by the end that he had grasped the basic ideas. However he tried to be helpful to the teacher, for example by sorting out a problem with the lead for the OHP projector.

As part of the project, Ryan had a few months before the video completed a self-report questionnaire on his attitude to mathematics. On a five-point scale, he gave the most negative response to over half the questions. For example, he said that he “hated maths”, found it “really hard” and thought he was “really bad” at it. However there were some areas where he was more positive, such as working out money problems and measuring.

When interviewed a year after the homework event took place Ryan was asked whether he thought he was different at home compared with school.

\textsuperscript{14} Special Educational Needs
He replied “loads”. He went on “(at home) I forget about everything, I just forget about school and play”. He thought that “in school I’m one person but when I come home I’m another person…naughtier at home than around the school”. He didn’t think his teacher knew what he was like at home and didn’t want her to know more about his home life.

In engaging with a school task at home, then, it is likely that Ryan was bringing with him an identity as someone who was struggling at school. Certainly he gave no indication of getting any enjoyment from the homework task; rather, it was an unpleasant chore to be completed before he could go off and play. His comments at turns 4 and 10 also suggest that he was not enjoying being filmed, unlike other children in the study who welcomed the opportunity to be the centre of attention. Ryan, in short, was a reluctant participant in this particular numeracy event.

2.3 Ryan’s mother and mathematics

We also need to consider Ryan’s mother’s identity in relationship to maths. When interviewed she made clear that her view of herself and maths is not singular – it depends on which aspect of maths is being considered. She says that at school she was good at her tables but she could not get long division into her head. She is not good at measuring or fractions. She is, however, good at budgeting and this includes the decision-making about which bills to pay as well as the mathematics.

Ryan’s mother reported that while she tried to help Ryan with his maths homework, she was often unable to do so and felt frustrated and ‘thick’ as a result: “I don’t know if it’s just the way they pronounce some things and he’s explaining it to me and I just hav’na a clue and I just can’t help him”. She felt that much of this was due to her being taught mathematical procedures differently from Ryan:

Mother: I can read it out to him but he always says I’m wrong because I’m not doing it properly.. so.. and we end up at loggerheads and I just.. I think well you need to just take it back to your teacher and say you can’t do it… “oh” she says, “I’ve showed him and I’ve showed him and I’ve showed him, but he just doesnae seem to take it in”.

Interviewer: So do you think that you are doing it a different way?

Mother: Oh, definitely. I had.. see that’s when I went to a meeting, the other week about the maths and everything, it’s like you’ll do your take away sum.. we used to do 10 to the top, 10 to the bottom, and she showed me, the teacher, you take 1 off the 8s it was and it came as 7 and you put that on there, the others. It was entirely different. But yet his dad does it the same.
These comments make clear that Ryan’s mother was taught the process of equal addition when she was at school, although she had recently learnt Ryan’s decomposition method from his teacher. They also suggest that Ryan is not slow to point out to her when he thinks she is using methods which are different from those of his school.

2.4 Tensions and conflicts during the homework event

We can now return to the homework event in the light of the above remarks on practices and identities. Throughout the event we can see tensions and conflicts emerging as Ryan’s mother tries to help and Ryan responds in various ways to her attempts. Thus right at the start of the event (turns 1 - 4) we can see Ryan’s initial defensive response to her interest, suggesting he does not find it welcome. On turn 5 Ryan’s mother justifies her interest in terms of wanting to see whether they were both using the same methods, which we now know was an ongoing issue between them. Ryan responds on turn 6 by emphasising this difference, suggesting that he is using the difference to try to keep his mother at bay. However he is ambivalent here, as he recognises that he is stuck (“I keep doing them wrong” on turn 8) and will have to allow his mother into the domain of his homework. This is not easy: as we have already noted, it is not at all clear how the homework task is meant to be tackled, or how a parent might help, and Ryan is clearly reluctant – or maybe unable – to provide an adequate explanation for his mother.

After Ryan has completed (incorrectly) the calculations for the first time (see Figure 2) his mother takes on a new role, that of checking his answers are correct (she says “can I check them” on turn 31 and “let’s have a look” on turn 43). This leads to further tension and conflict. Thus on turn 43 she somewhat hesitantly suggests that Ryan’s answer of 43 – 15 = 32 may not be correct, and says “I think you can’t.. you can’t take 5.. you can’t take”. Here we can possibly hear a voice from the time when she herself was a child in the maths classroom: part of the mantra for the take away calculation decision making is the recognition of ‘can’t’ if the number of units in the subtrahend is greater than in the other number, the minuend. Ryan’s response to this (“you have to take 3 away from 5”) has something everyday or matter of fact about it: if you can’t do something one way, find another way to do it. At the same time he accompanies this with a derogatory accusation of his mother’s ability to understand – maybe reflecting times when she has admitted not understanding the mathematics in his homework. He also calls on the authority of his school to emphasise the difference and justify his position (“It’s the way I do it, we do it a different way” on turn 50).
The sense of conflict here may also be heightened by the rather unusual language which Ryan’s mother is using to describe her method – she says “you say stroke that” on turn 49 (and again on turn 69) using a phrase with which Ryan is probably unfamiliar and which he may see as coming from another world. (It is interesting that she refers here to the physical action of putting a ‘stroke’ through a number, rather than seeing it as a mental process.) There is also an imprecision about her language which might well add to Ryan’s confusion. For example, on turns 53 and 57 she talks about taking a ‘unit’ from the 4 in 43, although in fact it is a ‘ten’. Indeed, Ryan corrects his mother at this point (turns 54 and 56) pointing out that the 4 is a ‘ten’ and the 3 is a ‘unit’. This may explain why he thinks she does not understand his decomposition method, although it is becoming clearer around turn 57 that she is in fact suggesting the same method as used in Ryan’s school. Nevertheless Ryan still resists this, and again calls on the authority of his school to justify his position. The nub of the conflict is revealed in stark terms in the following interchange:

59. M Well why.. you have to
60. C You don’t, not in my school we don’t, we do it a different way

Ryan’s mother persists with her belief that Ryan’s answer is incorrect and on turn 63 tries a different approach. She is somewhat hesitant here – “I may be wrong” – but perhaps surprisingly Ryan accepts her judgement that he has got the answer wrong and starts to rub out his answer. It is noteworthy that the method she uses to check accuracy is actually a mental calculation which starts by taking 10 of the 15 from 43. At this point she can see that the child’s answer is incorrect since she is already just about at the same number as his answer (33 compared with his 32) and she still has more to subtract. What is interesting here is that she does not use the method talked about earlier involving ‘stroking’ tens and so on. Rather she uses a more informal method involving a mental calculation of the kind which is encouraged within the National Numeracy Strategy, although she is presumably unaware of this.

Despite the confusion and conflict, something has been communicated to Ryan and on turn 70 he says “Oh I get it now”. This comment is justified by his subsequent behaviour, when he uses the decomposition method to complete correctly the calculation 43 – 15 = 28. However, his negative mood is not improved by this success. He states crossly on turn 74 that he has to repeat the rest of his work and on turn 80 joylessly admits that he now understands what he is doing. Perhaps he is more aware that not only did he fail to keep his mother out of his homework world but that he now has to repeat all his work – thus delaying even further the moment when he can go off and play.
3. **DISCUSSION**

Our analysis suggests that, beneath the surface of this particular homework event, the presence of a number of different worlds can be detected. Thus the event is an exemplar of the wider practice of homework, a practice which allows the school domain to legitimately enter and occupy the home site. With the practice comes a range of identities and presences. From the direction of school, we have Ryan’s school identity as a low-attaining pupil with strong negative feelings towards mathematics; there are also the presences of his class teacher, the architects of the school homework policy, the publishers of the homework sheet and even the writers of the mathematics curriculum being used at the time. From the direction of home there is Ryan’s home identity as someone who wants to forget about school and just play; there is also Ryan’s mother and the different identities she brings – as helper, checker and enforcer of homework - and as someone with her own strong and ambivalent feelings about maths. We can even detect the presence of her own experiences of learning mathematics despite their taking place at least 20 years previously. In addition, we should not forget the presence of the research team, represented through the video camera which records the event with an unforgiving detachment.

As we have seen, these identities and presences do not co-exist harmoniously. There is a great deal of conflict and tension, as the various identities negotiate with and challenge each other. Moreover, this challenge is not present in every aspect of the interaction. For example, Ryan does not challenge his mother’s insistence that he has to finish his homework before he can go out to play, possibly because he knows from experience that when his mother and the school are lining up on the same side he has ultimately little option. Instead, he vigorously challenges his mother’s understanding of mathematics, calling on the legitimacy of his school to justify his own incorrect methods and to overrule his mother’s attempts to persuade him otherwise. Thus we can see the clear presence of what Street et al. call issues of “control, legitimacy, status and the privileging of some practices over others in mathematics” (p. 17).

Unfortunately, it seems that the conflict and tension identified in this particular homework event are not atypical – either of Ryan or of homework more generally. As we saw earlier, Ryan’s mother reported that they were frequently ‘at loggerheads’ over homework, as he regularly challenged her understanding of his school mathematics. In a wider study of homework (Hughes and Greenhough, 2002) we also found that homework frequently engendered heightened emotions between parents and children, as parents tried to make sure homework was completed or struggled to find ways of helping their children: as one parent commented “we often end up at
screaming pitch”. Similar tensions around homework have also been reported by Solomon, Warin, and Lewis (2002).

To what extent does our analysis of what is going on in this event relate to Wenger’s (1998) framework for discussing communities of practice? We would suggest there are several fruitful areas of interplay.

First, the event can be seen as taking place at what Wenger terms a ‘boundary’ – in this case between home and school. At the same time, the event shows that this boundary is not a static or straightforward entity, but one which is dynamic and constantly being negotiated and renegotiated. A key factor in this negotiation is Ryan’s ambivalence between wanting to keep his mother out of the world of his school work, and wanting her in so that she can help him get the correct answers. He thus oscillates between having the boundary drawn tightly around him and his work – indeed at more than one point he creates a physical barrier with this arm between his mother and his homework sheet - and opening it up to allow his mother entry into the school domain.

If Ryan and his mother are operating at the boundary between home and school, then is it appropriate to describe the homework sheet as some kind of ‘boundary object’? In some ways it is. The homework sheet appears to play a similar role in this event to the claims processing form described by Wenger in his study. It is a physical object - in Wenger’s terms, the product of ‘reification’ - which has the potential to connect up different practices by moving in time and space between them. At the same time, the potential of this particular sheet to connect up home and school is very limited. As we have already observed, there are no instructions on the sheet or suggestions of ways in which parents might help. There is no attempt to translate the decontextualised mathematics of the subtraction calculations into an activity more familiar from the home domain (e.g. turning the subtractions into problems about shopping and money). Again, our previous research on homework suggests this is not atypical: homework has the potential to link home and school but for the most part this potential is not realised (Hughes and Greenhough, 2002).

In addition to boundary objects, Wenger describes the process of ‘brokering’ as another means by which connections can be made between communities of practice. As we saw earlier, a broker is essentially someone who is a member of two (or more) practices and uses this multimembership to make positive connections between the practices. In the homework event, Ryan is clearly a member of both the home and school practices, and potentially could use this – as other children might do – to create links between them. In reality, as we have seen, Ryan has little desire to do this. He would prefer the practices to be kept separate, and so his role is more often one of ‘blocker’ than ‘broker’.
In contrast, it is Ryan’s mother who is trying to play the role of broker in this event. She wants to bring whatever understandings she has about mathematics to help Ryan with his school work. Her problem, however, is that she is not a member of the school community and so lacks valuable information about how the school expects the calculations to be done. As she admitted in the interview, she had tried to overcome this lack of knowledge by attending a meeting at the school about the methods used to teach mathematics, but her knowledge was still patchy. This, together with her own lack of confidence and Ryan’s low opinion of her understanding, meant that her attempts at brokering frequently foundered.

It is also interesting to look at the homework event in the light of Wenger’s ideas about identity, and in particular his view that identity should be seen as a ‘nexus of multimembership’ which involves the important work of ‘reconciliation’. As we indicated earlier, both Ryan and his mother bring several facets of their identities to the homework event. For Ryan, though, there is little sign that the process of reconciliation has made much headway, if any. His interview comments make clear that he thinks he is very different at home and at school, and that when he is at home “I just forget about school and play”. In contrast, Ryan’s mother is more complex. Again there are several facets of her identity in evidence, such as her role as ‘good parent’, and her lack of confidence around maths, but these are not always working harmoniously together. Moreover, although she reports in interview that she has contemplated taking courses to improve her ability with mathematics, she has been inhibited from doing so by her perception that everyone in the class would be ‘more intelligent’ than her. Thus while Ryan’s mother has considered taking action that would help to reconcile aspects of her identity, her lack of self-confidence has prevented her from doing so.

Finally, we turn to the implications for mathematics education. No doubt there will be many mathematics educators who will find the content of this homework event somewhat depressing. The child is unhappy, and has a negative attitude towards many aspects of mathematics. The task is mundane, and makes no connection to real-life contexts or to his out-of-school life. The interaction between mother and child, although ultimately leading to the child adopting a correct procedure, is negative and bad-tempered. There is little appeal to mathematical principles to resolve disagreements, but instead regular references to power and legitimacy to decide which procedure should be adopted.

How might such a situation be improved? One suggestion would be for a fuller implementation of the principle, embodied in the National Numeracy Strategy, that children should be made aware that there are a range of different methods – all equally appropriate – for carrying out particular
calculations. We do not know enough about Ryan’s classroom to say whether or not he had been properly introduced to this principle, but if he had then he had clearly not internalised it. As we have seen, much of his difficulty with the homework stems from his reluctance to accept that there might be more than one way of doing it.

We would also suggest two further areas where practical steps could be taken to improve the interaction around mathematics which takes place between children and parents at home. First, there is much which can be done to improve the nature and quality of homework tasks. This would, however, require some fundamental rethinking about the purposes of homework and the role which parents – as well as family and peers – might be expected to play in the process. Thus if homework continues to be seen as a practice whose main purpose is to reinforce and extend the school curriculum, with the assumption that it will be carried out independently, then unstimulating and opaque worksheets such as Ryan’s will continue to be sent home. If on the other hand, homework is seen as a genuine way of making connections across home and school practices, involving other family members and peers in collaborative problem-solving, then it will lead to very different homework tasks and interactions around homework. For example, in our previous research on homework (Hughes and Greenhough, 2002) one class of students was set a mathematics assignment which required them to locate a number of items (like cosmetics) which were still in their original packaging. The students were asked to construct a chart showing the overall volume of the goods purchased as a percentage of the overall volume of the package. The students found this task quite engaging and commented afterwards on how revealing it had been. In particular, it had enabled them to see how mathematics might be relevant to an out-of-school practice such as shopping.

In addition to rethinking the nature and purposes of homework, schools can also do much to reconceptualise their relationships with parents and the ways in which parents can support their children’s learning. Many – if not most – parents share Ryan’s mother’s desire to help their children with their school work, in mathematics as well as other areas of the curriculum. At the same time, many parents may lack the knowledge and/or confidence to provide the most appropriate forms of support. In the numeracy strand of the Home School Knowledge Exchange Project we worked with schools to develop ways in which information about teaching methods and mathematics topics could be shared with parents. At the same time we developed activities where the exchange of knowledge between home and school was in the opposite direction, from home to school. For example, children were given disposable cameras and asked to take photographs of activities involving ‘everyday mathematics’ – such as card games, cooking
or shopping – in which they had been involved outside of school. A full account of these activities and their impact on children, parents and teachers can be found in Winter, Andrews, Greenhough, Hughes, Salway, and Yee (forthcoming).

In conclusion, we have attempted in this chapter to show how mathematics homework can be the source of tension and conflict, and that this tension and conflict tells us something important about the various practices and identities which are present in the homework event. At the same time, we have tried to demonstrate the value of looking at the relationship between home and school in terms of Wenger’s ideas about boundaries, boundary objects, brokering and the need to reconcile different aspects of identity. More generally, we have tried to show the importance of seeing the learning of mathematics as a social activity embedded in various practices which are not always in harmony. While we may not welcome such lack of harmony, we need to recognise it and learn from it.

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