Impact of Channel Estimation Errors on the Performance of DFE equalizers with Space Time Block Codes in Wideband Fading Channels

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Abstract—Multiple-Input Multiple Output (MIMO) techniques can be applied to radio systems to enhance performance in fading channels, to exploit spatial and temporal diversity, and to increase the potential transmission rate. Here we investigate the combination of Decision Feedback Equalizers (DFE’s) with Alamouti space-time block codes (STBC) in time-varying wideband channels. Most prior work considers the use of STBC in flat-fading channels. However, for many mobile applications, time and frequency selectivity in the wireless channel must be included in the analysis. As a result of delay spread and terminal mobility, the transmitted data stream can suffer from severe time varying Inter Symbol Interference (ISI) which requires a form of channel equalization.

For a quasi-static environment Alamouti’s linear combining scheme provides a reliable and low complexity MIMO solution when perfect channel state information (CSI) is available. In this paper we address the mitigation of wideband channels effects on Alamouti STBC codes. The study considers the minimum mean square error (MMSE) performance as a function of channel estimation error for Maximum Likelihood (ML) and DFE equalizers.

Index terms—Space Time Block Codes, Intersymbol Interference, delay spread, multipath fading, adaptive equalization.

I. INTRODUCTION

The significant expansion of mobile and cellular technology over the last two decades is a direct result of more robust coding techniques and powerful signal processing methods. A recent example is the exploitation of multiple antennas at both the transmitter and receiver. This arrangement, known as Multiple-Input Multiple-Output (MIMO), takes advantage of the spatial separation between antenna elements to create uncorrelated spatial channels. This translates into radio systems with improved spectral and power efficiency. Examples of MIMO systems include Space-Time Block Codes (STBC) [1], [2] and Space-Time Trellis Codes (STTC) [3]. These techniques are particularly attractive at the base station, where large antenna spacing can be easily accommodated. STBC are used in many communication systems since, compared to STTC, they provide a lower complexity and higher performance solution. The combining scheme proposed in [1] showed that it is possible to completely reconstruct the transmitted data symbols and suppress interference from other codewords given perfect knowledge of the Channel State Information (CSI) in quasi-static channels. However, this assumption is not always true, especially for high data rate systems where the radio channel is both time and frequency selective. As a result, the MIMO transceiver must tolerate degradation in signal quality as a result of ISI and Doppler spread.

In recent years there has been a tendency to combine MIMO with multi-carrier modulation; most notably Orthogonal Frequency Division Multiplexing (OFDM). Particular examples include the physical layer of high performance Wireless Local Area Networks (WLANs), and in particular the latest 802.11n standard [11]. An OFDM signal is constructed using N equally spaced sub-carriers. These are sent in parallel over the radio channel. The channel bandwidth is effectively divided into multiple flat-fading narrowband sub-channels, each of which is well suited to narrowband MIMO exploitation. Hence, in a frequency selective channel, a correctly designed OFDM system is protected from ISI. In a MIMO-OFDM system the MIMO technique is applied on a per sub-carrier basis. There are two main drawbacks to this approach. Firstly, MIMO-OFDM is unable to exploit frequency diversity at the symbol level. Secondly, OFDM suffers from a high Peak to Mean Power Ratio (PMPR) and this hinders power efficient transmission [12].

Given continued advancements in time domain equalizer theory [5], it is now possible for a single carrier system employing iterative equalization to outperform a multicarrier system using OFDM in terms of data rate and Packet Error Rate (PER) [5]. This gain is mainly a result of diversity at the symbol level and the use of optimum detectors in the form of exhaustive Maximum Likelihood. Furthermore, compared to OFDM, single carrier systems experience much lower PMPR levels, and are thus better suited to up-link transmission from power limited handsets. Unfortunately, for channels with very long Power Delay Profiles (PDP), or when large constellation alphabets are used, the computational complexity of iterative Maximum Likelihood detection becomes unfeasible, and sub-optimal solutions are required. In this work the performance of a wideband single carrier system is explored using a combination of STBC MIMO and finite length DFE detection.

The paper is structured as follows. In section II, the MIMO transmission model is given, followed by an overview of the Alamouti code in section III. In section IV a theoretic analysis of MMSE based equalization is developed and a range of simulation results are presented. Conclusions are discussed in section VI.
II. MULTIPLE INPUT MULTIPLE OUTPUT SYSTEMS

A. System Model

We consider the general case of an N_t-by-N_r MIMO system, as shown in Fig. 1, operating in a wideband multipath fading channel corrupted by Additive White Gaussian Noise. The Channel Impulse Response (CIR) of memory L, between transmitter T and receiver R, denoted $h_{lj,t}$, is assumed to have a minimum phase response and is modeled as an FIR filter of order $v = L-1$. The discrete time representation of the received signals $y_j(t)$ at antenna $j$ is given by:

$$y_j(t) = \sum_{i=1}^{N_t} \sum_{k=0}^{v} h_{ij,k} x_i(t-kt) + n_j(t)$$  \hspace{1cm} (1)

where $h_{ij,k}$ is the $k$-th multipath component of the $h_{ij}$ link:

$$h_{ij} = [h_{ij,0} \ h_{ij,1} \ h_{ij,2} \ ... \ h_{ij,L-1}]^T$$

Equivalently:

$$y_j(t) = \sum_{i=1}^{N_t} h_{ij}^T x_i(t-v) + n_j(t)$$  \hspace{1cm} (2)

The transmitted data sequence from antenna $i$ is:

$$x_i^{(t:v)} = [x_i(t-v) \ x_i(t-v+1) \ ... \ x_i(t)]^T.$$  

![Fig. 1 General setup for an N_t-by-N_r MIMO system](image)

B. Channel Modeling and Estimation

Wireless communication channels are characterized by their time and frequency selective fading. If the channel’s delay spread is negligible compared to the symbol duration, the receiver will be able to distinguish only a single multipath component. This is known as frequency-flat fading, whereby all the spectral components experience a common channel response. Alternatively, for most wideband systems, the channel’s delay spread is much greater than the symbol duration, and the system suffers from temporal spreading of the signal energy, or ISI. This is known as frequency-selective fading, since each spectral component experiences a unique channel response. For frequency-selective fading channels we assume that each resolvable multipath component fades independently. With sufficient transmit antenna spacing, the fading observed from different antennas is uncorrelated. In addition, each tap in the wideband channel model is represented as a time-varying Rayleigh fading process. The spaced-time autocorrelation function follows a zeroth-order Bessel function of the first kind $J_0(2\pi f_d t)$, which is derived from the Jakes’ Power Spectral Density, with $f_d$ representing the maximum Doppler frequency.

C. Channel Estimation

Wideband multipath fading channels degrade the quality of single carrier systems as a result of ISI in the received signal. In order to suppress the effect of ISI, receivers commonly employ adaptive equalization. The transmitted data stream is often unknown a priori at the receiver and as a result the detection scheme requires knowledge of the propagation channel. This is achieved through estimation. Channel estimation is usually based on the transmission of a preamble containing training sequences, or pilot symbols, agreed between the transmitter and receiver. For quasi-static channels, it is common for a training sequence to be inserted at the start of each data burst, and for this sequence to be used for channel estimation solely in that data block. Different channel estimation techniques exist, and these include Kalman filtering methods [9], ML based methods [9] and other blind estimation algorithms [13]. In the case of fast time-varying channels, the channel’s complex impulse response differs from symbol to symbol. Here training sequences must be sent continuously within the data burst. Pilot insertion is performed at the expense of signal bandwidth, and must therefore be kept to a strict minimum. For the sake of simplicity we assume here that the estimation errors for each multipath component of the CSI can be modeled as independent and identically distributed (i.i.d) complex White Gaussian noise.

III. ALAMOUTI’S TRANSMIT DIVERSITY CODE

STBC codes have been developed for slowly time varying channels [1-4]. The method consists of transmitting redundant information over a time window of $T$ symbols, using $N_t$ transmit antennas, without the need for extra transmit power or any prior knowledge of the channel state matrix. The degree of diversity offered by the system is equal to the number of independently detectable channels over which a symbol is transmitted. In narrowband systems, the full diversity of the system is equal to $N_t N_r$.

STBC codes are implemented in a generic matrix form regardless of the modulation scheme in use. This is not the case for STTC, where the computational complexity increases with the size of the constellation alphabet. STBC codes are designed according to the orthogonal design criterion, as explained in [3]. As a result, STBC systems offer optimal performance at low and medium SNR values, compared to STTC, which makes them more desirable for outdoor radio systems, where operating range is vitally important. In this paper, for simplicity we only consider the 2-by-1 Alamouti’s STBC code. The STBC scheme proposed by Alamouti in [1], satisfies the Orthogonal Design criterion, and consists of spreading the energy of two successive symbols over a time window of $T$, which equals two symbol periods, using two transmit antennas. Let $x_1$ and $x_2$ represent the generated symbols at times $t$ and $t+\tau$ respectively; Alamouti’s code is given by:

$$X = \begin{bmatrix} x_1 \\ -x_2^* \\ x_2 \\ -x_1^* \end{bmatrix}$$  \hspace{1cm} (3)
From (1), the received symbols \( r_i \) and \( r_x \), in a flat-fading channel at times \( t \) and \( t + \tau \) respectively are:

\[
\begin{bmatrix}
  r_1 \\
  r_2
\end{bmatrix} = \begin{bmatrix}
  h_{1,1}^i x_1 + h_{1,2}^i x_2 \\
  -h_{1,1}^{i*} x_2 + h_{1,2}^{i*} x_1
\end{bmatrix} + [n_{1,1}] + [n_{1,2}]
\]

(4)

where \( h_i^k \) denotes the channel between transmitter \( i \) and receiver 1, at time \( k \). This can be re-expressed as follows:

\[
\begin{bmatrix}
  r_1 \\
  r_2
\end{bmatrix} = \begin{bmatrix}
  h_{1,1}^i \\
  h_{2,1}^i
\end{bmatrix} \begin{bmatrix}
  x_1(t) \\
  x_2(t)
\end{bmatrix} + [n_{1,1}] + [n_{1,2}]
\]

From (1), if the channel fading coefficients at times \( t \) and \( t + \tau \) for both links are highly correlated, i.e. the channel is experiencing quasi-static fading, (5) becomes:

\[
\begin{bmatrix}
  r_1 \\
  r_2
\end{bmatrix} = H X_i + n_i
\]

(6)

where \( H \) denotes the Hermitian conjugate of \( H \). The combiner’s output is then:

\[
\hat{X}_i = H^H \begin{bmatrix}
  r_1 \\
  r_2
\end{bmatrix} = \left[ h_{1,1}^i \right]^2 + \left[ h_{1,2}^i \right]^2 \begin{bmatrix}
  X_1 \\
  X_2
\end{bmatrix} + H^H n_i
\]

(7)

The output of this combiner is used with a Maximum Likelihood detector, which provides the same diversity order as Maximum Ratio Combining with two receive antennas.

For time-varying fading channels, or for cases where the receiver produces incorrect channel estimates, the output of the combining scheme in (7) becomes:

\[
\hat{X}_i = \Re X_i + H^H n_i
\]

(8)

where the combining gain matrix \( \Re \) is given by:

\[
\Re = \begin{bmatrix}
  h_{1,1}^i (\hat{h}_{1,1}^i) + h_{1,2}^i (\hat{h}_{1,2}^i) & -h_{1,2}^i (\hat{h}_{1,1}^i) \\
  \hat{h}_{1,2}^i (\hat{h}_{1,1}^i) & h_{1,2}^i (\hat{h}_{1,2}^i) + h_{1,1}^i (\hat{h}_{1,1}^i)
\end{bmatrix}
\]

where \( \hat{h}_i^k \) represents the estimate of channel link \( h_i^k \). In the case of imperfect CSI, or a time-varying channel, the above demonstrates that the interference is not completely eliminated by the linear combiner. Under these conditions the ML symbol detector must be used together with an interference suppression scheme. Alternatively an ML space-time detector can be used [1]. The ML space-time decoder searches for the most likely pair of Alamouti encoded symbols to produce the set of received data.

In a frequency selective channel, and for a 2-by-1 antenna system, the work presented in [1] can be extended by combining equations (1) and (6). The received signal is then given by the following expression:

\[
\mathbf{y}(t) = \sum_{i=0}^{v} \begin{bmatrix}
  h_{1,1}^i(i) \\
  h_{1,2}^i(i)
\end{bmatrix} \mathbf{X}_{v-i} + \mathbf{n}(t)
\]

(9)

where \( h_i^k(i) \) represents the \( i \)-th ISI channel coefficient between transmitter \( l \) and the receiver, and \( k \) corresponds to the transmission time \( t \) or \( t + \tau \). Let \( H_i^l \) be the \( i \)-th ISI components of the channel for transmission at time \( t \). Equation (9) is now equivalent to:

\[
\mathbf{y}(t) = \sum_{i=0}^{v} \mathbf{H}_i^l \mathbf{X}_{v-i} + \mathbf{n}(t)
\]

(10)

For a transmission packet of length \( 2v+1 \), the received data samples during this packet are expressed as shown in equations (11), and more simply by (12):

\[
\begin{bmatrix}
  \mathbf{y}(v+1) \\
  \mathbf{y}(v+1) \\
  \vdots \\
  \mathbf{y}(1) \\
  \vdots \\
  \mathbf{y}(1)
\end{bmatrix} = \begin{bmatrix}
  \mathbf{H}_v \\
  \vdots \\
  \mathbf{H}_1
\end{bmatrix} \begin{bmatrix}
  \mathbf{X}_v \\
  \vdots \\
  \mathbf{X}_1
\end{bmatrix} + \begin{bmatrix}
  \mathbf{n}(v+1) \\
  \vdots \\
  \mathbf{n}(v+1) \\
  \vdots \\
  \mathbf{n}(v)
\end{bmatrix}
\]

(11)

\[
\mathbf{y}(t + L) = \mathbf{H}^L \mathbf{X}_{v+2L} + \mathbf{n}(t + L)
\]

(12)

IV. RESULTS AND DISCUSSION

The simulated results presented in this section were formed by transmitting different packets, each of which is one thousand symbols long, until a hundred packet errors were obtained. For each packet transmission, a new wideband channel is realized. Throughout the experiment it was assumed that each link is modeled as a two tap minimum phase wideband channel, and each multipath component is a Rayleigh fading process.

A. ML Detector

The ML space-time decoder is an optimal decoder since it searches for the codeword that is the most likely to have been transmitted given knowledge of the received signal and an estimate of the channel matrix [6]. As mentioned previously, we assume that the channel estimation errors are modeled as i.i.d. complex White Gaussian noise samples. The ML STBC detector is expressed in equation (13) below:

\[
\hat{X} = \arg \min \left\{ \left\| \mathbf{r} - \mathbf{H} \hat{X} \right\|^2 \right\}
\]

(13)

From equation (13), the ML detector can be characterized with high computational complexity since an exhaustive search is performed for the most likely transmitted codeword. This complexity increases for long delay profiles, or when large constellation alphabets are used. In this section we examine the effect of channel estimation errors on the performance of the ML detector. For the sake of simplicity, it assumes perfect reconstruction of previous ML decisions. The BER performance of a 2-by-1 STBC system using ML detection was simulated for different values of \( f_0 T \), and different values of channel estimation error.

In the case of perfect channel knowledge, the simulated BER is shown in Fig.2. The performance of the system degrades...
as a result of mobility, which can be seen in the high irreducible BER floors. These errors arise due to the high Doppler spread, which results in fast-fading, and cause significant amplitude and phase variations over both time and distance. For the case of imperfect channel estimation, the estimation errors were simulated by corrupting the channel state matrix with complex i.i.d. White Gaussian noise samples with a variance given by $SNR_c$.

![Fig. 2. BER performance of ML detector with perfect channel estimates](image)

**Fig. 2. BER performance of ML detector with perfect channel estimates**

The performance of the ML detector in the presence of channel estimation errors is shown in Fig. 3 for $f_{DTs} = 0.0001$ and $f_{DTs} = 0.00001$. The performance of the receiver is shown to degrade as the variance of the estimation error increases. For both scenarios, when $SNR_c = 0$ dB, the system experiences a constant high Irreducible BER (IBER) floor, which cannot be reduced by increasing the transmit power. As the value of $SNR_c$ is increased towards 30dB, the BER performance of both systems improves and approaches the noise free channel estimation curves shown in Fig.2. For low SNR values, i.e. below 8dB, the improvement in BER performance due to increases in $SNR_c$ is hardly noticeable for both cases. For higher SNR values, the performance of the two systems improves in terms of BER and error floor. To clearly observe the impact of $SNR_c$ on system performance, Fig.4 shows the IBER floors for different $SNR_c$ values.

![Fig. 3. BER performance of ML detector with noisy channel estimates](image)

**Fig. 3. BER performance of ML detector with noisy channel estimates**

The system performance converges to that of an ML detector with perfect channel knowledge for $SNR_c$ values greater than 20dB. For lower values, we find that the ML detector is sensitive to the quality of the channel estimate. Thus, as long as the $SNR_c$ of the estimation error is greater than 20 dB, the system performance converges towards the noise free estimation case.

### B. MMSE Equalizers

In order to reduce the computational complexity of the receiver it is possible to apply a sub-optimal equalization solution. Filter based equalizers aim to choose appropriate filter coefficients to combat the frequency selective nature of the channel. Minimum Mean Square Error (MMSE) filter based equalizers aim to minimize the probability of detection errors, as shown in equation (14)

$$\bar{X} = \arg \min \| e(t) \|^2 = \arg \min \| x(t) - \bar{x}(t) \|^2$$  \hspace{1cm} (14)

where $\bar{x}(t)$ represents the soft equalizer’s output sample sequence. Compared to Linear Equalizers, DFE’s are known to result in good performance for channels with severe ISI. DFE’s make use of feed-forward and feed-back filters to suppress ISI. The feed-back filter is fed by previous decisions from the equalizer output and is used to cancel post-cursor ISI. The output of the DFE equalizer, as shown in Fig. 5 below, is expressed as:

![Fig. 5. Structure of a DFE equalizer](image)
We can also define the equalizer's input auto-correlation matrix $R_{XX}$ and the noise auto-correlation matrix $R_{NN}$ by:

$$R_{XX} = E[x_{t-2+N_{r}-1}x_{t-2+N_{r}-1}^H] = I_{2(N_{r}+v)}$$

$$R_{NN} = E[n_{t-2+N_{r}-1}n_{t-2+N_{r}-1}^H] = \sigma_n^2 I_{2N_{r}}$$

We can also define the equalizer's input auto-correlation matrix $R_{YY}$ and the equalizer's input-output cross-correlation matrix $R_{YX}$ by:

$$R_{YY} = E[y_{t+2+N_{r}-1}y_{t+2+N_{r}-1}^H] = H^HR_{XX}H + R_{NN}$$

$$= H^HI_{2(N_{r}+v)}H + \sigma_n^2 I_{2N_{r}}$$

$$R_{YX} = E[y_{t+2+N_{r}-1}x_{t+2+N_{r}-1}^H] = H^HR_{XX}$$

$$= H^HI_{2(N_{r}+v)}$$

Using the analysis described in [7][8], the theoretical MMSE equation for the DFE defined above is given by:

$$E[|e(t)|^2] = B^HR_{XX} - R_{YX}R_{YX}^{-1}R_{YX}B$$

where $B^H$ represents the feed-back filter matrix.

A 2-by-1 STBC system using the symbol based Alamouti scheme was simulated over a range of SNR and $f_DT_s$ values with channel estimation errors. The results shown in Fig. 6 assume perfect reconstruction of the channel coefficients at the receiver. As expected, the performance of the system degrades with increasing $f_DT_s$. This occurs as a result of amplitude and phase variations in the time evolving channel estimate, which are due to the Doppler spread. Comparing these results with those in Fig. 2 it can be seen that the ML detector outperforms the DFE equalizer, which is partly due to error propagation in the DFE equalizer. The ML detector also results in lower IBER levels, although these gains are at the expense of system complexity. In order to compensate for this, the STBC system can be implemented with multiple receivers, which improves the diversity order of the system. Given that this single carrier system is most effective on the uplink of a cellular system, the use of multiple receive antennas is feasible since these would be located at the base-station. From Fig. 7, for $f_DT_s = 0.001$, the BER performance is enhanced significantly as the number of receivers is increased. For instance, the 2-by-1 system requires an increase of 10dB in SNR to improve the BER performance by an order of magnitude, whereas the 2-by-4 or the 2-by-8 systems only require 4 dB and 2 dB increases in the transmit signal power, respectively.

Fig. 6. BER performance of the STBC DFE equalizer with perfect channel knowledge

Fig. 7. BER performance of the DFE equalizer with multiple receivers

Fig. 8. BER performance of DFE equalizer with noisy channel estimates

Figs. 8 and 9 show the performance of the DFE equalizer in the case of channel estimation error. Comparing Fig. 8 with Fig. 2 we see that the ML detector offers a better performance at high SNR, and also when the system experiences low Doppler spreads. As the value of SNR decreases, the DFE equalizer gains an advantage and
produces better BER performance. For instance, for $\text{SNR}_c = 0\, \text{dB}$, the DFE equalizer is capable of reducing the error floor observed in Fig. 3. for the ML detector. In addition, for a low mobility scenario, i.e., $f_{DT} = 0.00001$, and for $\text{SNR}_c = 10\, \text{dB}$, the DFE equalizer reduces the BER floor seen in the ML detector to $10^{-3}$. In this case the DFE benefits from the symbol-by-symbol update of the equalizer coefficients. Besides, the optimal DFE equalizer is very sensitive to the channel estimation error, and its performance converges towards that with perfect channel knowledge for $\text{SNR}_c$ values greater than 20 dB.

Fig. 9. IBER floors for DFE equalizer with channel estimation errors

V. CONCLUSION

In this paper we have presented an evaluation of the performance of Alamouti’s STBC scheme in time-varying wideband channels. This was achieved by comparing the error performance of different equalizer structures. We conclude that the performance of the wideband STBC system degrades as a result of mobility and channel estimation error. In a time-varying channel, the transmitted symbol suffers from significant amplitude and phase variation over distance and time due to high Doppler spreads. This was shown to result in a high BER floor, particularly for larger values of $f_{DT}$. The resulting irreducible error floors, which cannot be lowered by increasing transmit power, can only be reduced by employing accurate channel tracking. Overall, the ML STBC detector was shown to outperform the symbol-by-symbol DFE equalizer for rapid channel variations in terms of BER performance, and achieved a full diversity order for small channel estimation errors. The performance of the ML STBC detector in time-varying channels is sensitive to channel estimation errors. However, when the value of $\text{SNR}_c$ was greater than 20 dB, the degradation due to channel estimation errors was negligible.

The performance of the channel estimator depended on several factors, such as the length of the training sequence, the type of estimation algorithm, and the channel conditions. Generally, ML detectors do not tolerate poor channel estimates, and this can be considered as a further drawback in addition to their high complexity. Results show that DFE based STBC equalizers can be considered as an alternative to the computationally demanding ML detector. For the single receive antenna solution the DFE performance was noticeably worse than that of the ML solution for noise free channel estimates. However, the use of multiple receive antennas at the base-station significantly improved the DFE solution with modest additional complexity. Results also showed that the DFE solution was particularly attractive for systems with high channel estimation error.

Overall, as a result of their low PMPR values, the wideband single carrier STBC schemes considered here can be considered as viable candidates for future cellular up-link standards. OFDM or OFDMA solutions are considered more suited for the down-link.

REFERENCES