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A New FDTD Model in the Study of Hollow Conducting Elliptical Waveguides And Cylindrical Cavity Resonator

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Abstract –A new FDTD model is investigated to analyse electromagnetic wave propagation in a hollow conducting elliptical waveguide and a 3D cavity resonator using a Cartesian co-ordinate system. Modifications based on a nonorthogonal FDTD method are presented which are more efficient than general nonorthogonal FDTD schemes in terms of computer resources such as memory and CPU time.

Introduction

Traditionally, the propagation of electromagnetic waves in a hollow perfectly conducting waveguide has been investigated by an analytical method which involves the use of the orthogonal elliptic co-ordinate system and Mathieu functions. Some modes are easily overlooked when we determine the cut-off wavelength by finding out the parametric zeros of Mathieu function and its derivative [1],[2]. The traditional Finite-Difference Time-Domain Method (FDTD) algorithm is based on a Cartesian co-ordinate system, and it is difficult to generate meshes exactly for those curved surfaces. The nonorthogonal FDTD algorithm [5]-[6] is a rigorous scheme for the study of curved electromagnetic structures, but two additional equations are needed in each iteration step in order to realise the transform between the contravariant and covariant components of electric and magnetic fields. Moreover extra computer memory is needed to store the metric tensor of both E and H nodes. In this contribution, we demonstrate a new FDTD model for both 2D and 3D structures in which the theory of the nonorthogonal FDTD scheme will be used within an underlying Cartesian co-ordinate system. Most of the grid is in the Cartesian co-ordinate system and only those cells near the curved boundaries are treated as nonorthogonal cells. Therefore, a Cartesian grid is used for the majority of the problem space and less CPU time and memory is needed than FDTD in nonorthogonal grids. Compared with CPFDTD [3], [4], there is less approximation and in addition, method is more readily applied to dielectric boundaries.

Generation of the Meshes

Let an arbitrary curve pass through a standard FDTD cell, the original cell is bisected. The two basic types of cells near the curved boundary are defined as either 'flag' and 'triangle'. In 'flag' cells, the original cell is split, by the material boundary, into two cells which are still quadrilateral. We extend the neighbouring cell as shown in Figure 1 and replace the E_x or E_z node by a E_t node on the material boundary. The second type of cell, called 'triangle', can not be treated as FDTD cells directly, the mesh needs to be reconstructed such that all the cells are quadrilateral. To do this, an additional point is defined on the material boundary such as C' in figure 2 and the edges of the intercepted cell and its neighbours are modified as shown in figure 2(b). Then the new nonorthogonal FDTD meshes on the curved boundary for those

triangular and quadrilateral meshes are obtained. It is easy to set E and H nodes in each cell and those nodes which are near to the curved boundary are moved exactly onto the boundaries. The components of the electric field (for TE modes) and the magnetic field (for TM modes) for the nodes on the material interface are chosen to be tangential to the boundary.

FDTD Iteration Formula in New Meshes

Consider the structure shown in Figure 2(b). Node (i,k) is a Cartesian node and the traditional FDTD iteration formula is used to obtain the E and H components. During iteration, we need not introduce covariant and contravariant components, and therefore the memory requirements are the same as for standard FDTD. Node (i+1, k) is a nonorthogonal node, so we use nonorthogonal FDTD method to find value of E and H components. The equations for this node are shown as (1):

$$\begin{aligned}
 E^x(i+1,k)^{n+1} &= E^x(i+1,k)^n + \frac{\delta t}{c\epsilon_r} [H_y(i+1,k-1) - H_y(i+1,k)]^{n+\frac{1}{2}} \cdot \sqrt{\frac{1}{g_{yy}}} \cdot \sqrt{\frac{g_{xx}}{g}} \\
 E^y(i+1,k)^{n+1} &= E^y(i+1,k)^n + \frac{\delta t}{c\epsilon_r} [-H_x(i,k) + H_x(i+1,k)]^{n+\frac{1}{2}} \cdot \sqrt{\frac{1}{g_{xx}}} \cdot \sqrt{\frac{g_{yy}}{g}} \\
 H^z(i+1,k)^{n+\frac{1}{2}} &= H^z(i+1,k)^{n+\frac{1}{2}} + \frac{\delta t}{c\mu_r} \cdot \sqrt{\frac{g_{yy}}{g}} \\
 &\quad \cdot \left[\frac{E_z(i+2,k)}{\sqrt{g_{xx}(i+2,k)}} - \frac{E_z(i+1,k)}{\sqrt{g_{xx}(i+1,k)}} - \frac{E_x(i+1,k+1)}{\sqrt{g_{xx}(i+1,k+1)}} + \frac{E_x(i+1,k)}{\sqrt{g_{xx}(i+1,k)}} \right]^{n+\frac{1}{2}}
 \end{aligned} \tag{1}$$

In above equations, the contravariant components in node (i, k) and (i+2,k) need not to be transformed into covariant components because in Cartesian cells they are identical. In nonorthogonal cells, transformation is necessary by use of equation (2).

$$\begin{aligned}
 E_x(i,k) &= G_{xx} E^x(i,k) + \frac{G_{xy}}{4} [E^x(i+\frac{1}{2},k-\frac{1}{2}) + E^x(i-\frac{1}{2},k-\frac{1}{2}) \\
 &\quad + E^x(i+\frac{1}{2},k+\frac{1}{2}) + E^x(i-\frac{1}{2},k+\frac{1}{2})] \\
 H_y(i,k) &= H^y(i,k) \\
 G_{xx} &= \sqrt{\frac{g_{xx}}{g_{zz}}} \cdot g_{zz}
 \end{aligned} \tag{2}$$

where x, y, z denote generalised co-ordinates in nonorthogonal co-ordinate system, they will reduce to Cartesian co-ordinates in those Cartesian cells, and g is the metric tensor. The iteration formula for TM modes can be easily obtained by making the substitution $\epsilon \leftrightarrow -\mu$ and $E \leftrightarrow H$.

Numerical Results

The cut-off frequencies of the hollow conducting elliptical waveguide with its eccentricity varying from 0.15 to 0.85 are calculated and shown in Figure 4. The solid lines in two figures present the results by analytical formulas[1], and crosses, diamonds and circles stand for results obtained using the new FDTD model, they agree with each other very well with the maximum error not exceeding 5%. Respectively, s (sin-type), c(cos-type) are the odd and even modes of TE and TM in an elliptical waveguide. The major semi-axes of elliptical waveguides are all 0.1m, and the computational region is 0.27x0.27m² meshed using 12x12 nodes. Figure 3 shows the grids of elliptical disk generated by the computer software. Only those nodes outside the metal are given. The boundary condition for a PEC dictates that those E nodes on

and outside boundaries are zero. A three dimensional cylindrical cavity has also been analysed, the simulation results for TE modes are given with those results from CPFDTD, staircase approximation and theoretical analysis. They are shown in Table 1. The radius of cavity is 0.15m, height of it is 0.30m and the computational region is $0.5 \times 0.5 \times 0.5 \text{m}^3$ meshed by $12 \times 12 \times 12$ nodes.

Conclusions

A new FDTD model which is based on nonorthogonal FDTD algorithm was proposed so as to deal with an arbitrary electromagnetic structure on an underlying Cartesian grid. This has benefit of reduced computer resources and easily generated meshes. Numerical simulation shows the new model is very efficient and results agree with theoretical ones very well. The method may be readily extended to 3D dielectric structures and preliminary work has shown very encouraging results. We believe that it will be more powerful in computation of electromagnetic scattering problems.

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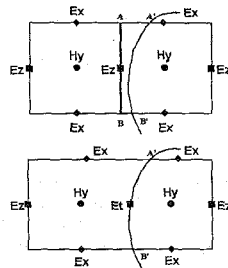


Figure 1 A 'flag' cell extend to the neighbour cells

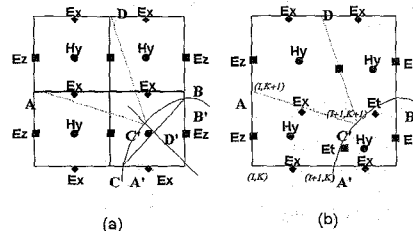


Figure 2 Reconstruction of a 'triangle' cell

PERFECT CONDUCTING ELLIPTICAL WAVEGUIDE
MESHED IN CARTESIAN CO-ORDINATE SYSTEM

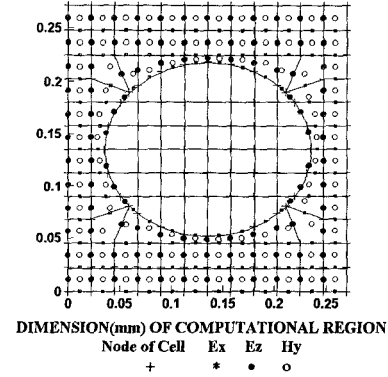


Figure 3. An elliptical waveguide meshed in a Modified Cartesian grid

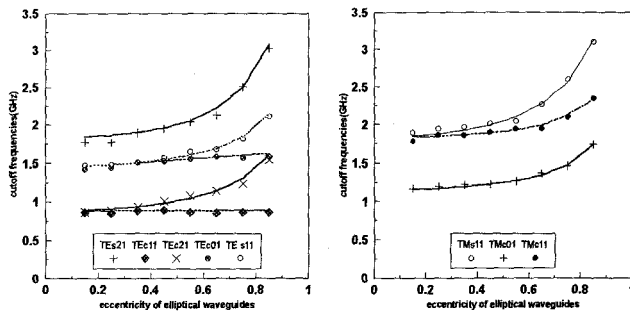


Figure 4. Cut-off frequencies of TE and TM modes in elliptical waveguides with its eccentricity varying (curves show the results from[1], discrete points show results from the new FDTD model)

Table 1 The resonant frequencies of TE modes in a Cylindrical Cavity Resonator

modes	new FDTD	staircase	CPFDTD	theoretical
TE ₁₁₁	0.7691	-	0.7707	0.7703
TE ₂₁₁	1.0670	-	-	1.0932
TE ₁₁₂	1.1473	1.1291	1.1426	1.1591
TE ₀₁₁	1.3691	1.2441	1.2610	1.3183
TE ₂₁₂	1.4114	1.3691	1.4000	1.3946
TE ₃₁₁	1.4917	1.4705	-	1.4277
TE ₀₁₂	1.5868	1.5753	1.5382	1.5773
TE ₁₁₃	1.6206	-	1.6024	1.6104
TE ₃₁₂	1.6480	1.6734	1.6294	1.6698
TE ₄₁₁	1.7557	1.7613	-	1.7646
TE ₁₂₁	1.7621	-	-	1.7691
TE ₂₁₃	1.8001	-	1.8492	1.7875
TE ₀₁₃	1.8889	1.9303	1.9000	1.9334
TE ₄₁₂	1.9248	1.9810	1.9540	1.9657