Network Quantum Steering

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The development of large-scale quantum networks promises to bring a multitude of technological applications as well as shed light on foundational topics, such as quantum nonlocality. It is particularly interesting to consider scenarios where sources within the network are statistically independent, which leads to so-called network nonlocality, even when parties perform fixed measurements. Here we promote certain parties to be trusted and introduce the notion of network steering and network local hidden state (NLHS) models within this paradigm of independent sources. In one direction, we show how the results from Bell nonlocality and quantum steering can be used to demonstrate network steering. We further show that it is a genuinely novel effect by exhibiting unsteerable states that nevertheless demonstrate network steering based upon entanglement swapping yielding a form of activation. On the other hand, we provide no-go results for network steering in a large class of scenarios by explicitly constructing NLHS models.

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The quest to deepen our understanding of quantum theory and its seemingly counterintuitive properties has lead to many fruitful avenues of research. In particular, the phenomenon of quantum correlations has enjoyed significant attention and developments; see, e.g., Refs. [1–3].

Quantum correlations expose a rich structure when considered in scenarios with many parties. A case of particular interest is that of quantum networks featuring a number of distant parties connected by several quantum sources. Significant further work is still required to reach a deeper theoretical understanding of these scenarios, while also keeping in line with experimental and technological developments toward quantum networks [4].

Recently, a generalization of the concept of Bell locality [5] was proposed to tackle the question of quantum nonlocality in networks; see Ref. [6] for a recent review. The key idea is to consider the various sources in the network to be statistically independent [7–9]. This independence leads to nonconvexity in the space of relevant correlations, undermining the use of preexisting tools and creating a need for new approaches, both analytically [10–18] and numerically [19]. The network structure offers new interesting effects, such as the possibility to certify quantum nonlocality "without inputs" (i.e., a scenario where each party performs a fixed quantum measurement) [8,9,20–22]. Also, the use of nonclassical measurements allows for novel forms of quantum nonlocal correlations that are genuine to networks [23]. In parallel, several works have explored the structure of quantum states assuming a certain underlying network structure [24–27].

In this work, motivated by the difficulty in characterizing quantum networks both conceptually and computationally, we consider quantum network scenarios in which some of the parties are trusted while the others are untrusted. This naturally connects to the notion of quantum steering [28] (see Refs. [2,3] for reviews) which captures quantum correlations in a scenario involving a trusted and an untrusted party. While the notion of multipartite steering has been previously considered [29,30], our work explores a different direction, targeting the scenario of networks with independent sources.

Our main focus here will be on the simplest setting of a linear network with trusted end points and intermediate untrusted parties who each perform a fixed measurement. We begin by formalizing the notions of network local hidden state (NLHS) models and network steering. We then leverage standard steering and nonlocality scenarios to provide simple examples of network steering. Next, we outline a surprising effect in which two-way unsteerable states can demonstrate network steering through entanglement swapping, leading to a form of activation. Finally, we characterize some natural scenarios that always admit a NLHS model by identifying properties of the sources. We conclude by listing some promising future avenues for research.

Basic concepts.—We first briefly summarize the notion of steering, as it represents the basis of what is to follow.

In a (bipartite) steering scenario, one party performs measurements on a shared state ρ^{AB}, which "steers" the quantum state of the other particle. If Alice performs a set of measurements labeled by x with outcomes a and corresponding positive operator valued measure (POVM) elements $M_{a|x}$, then the collection of subnormalized "steered states" of Bob are $\sigma^B_{a|x} \coloneqq \mathrm{Tr}_A(M^A_{a|x} \otimes \mathbb{1}^B \rho^{AB})$, where $p(a|x) = \mathrm{Tr}(\sigma_{a|x})$ are the statistics of Alice's measurements. The collection of subnormalized states $\{\sigma_{a|x}\}_{a,x}$ are commonly referred to as an *assemblage* [31]. If the assemblage can be explained by a local hidden state (LHS) model, of the form $\sigma_{a|x} = \sum_\lambda p(\lambda)\, p(a|x,\lambda)\sigma_\lambda$, where λ is a hidden variable distributed according to $p(\lambda)$, σ_λ are "hidden states" of Bob, and $p(a|x,\lambda)$ are local "response functions" of Alice, then we say that it has LHS form, or does not demonstrate steering [28]. If there exist measurements such that $\sigma_{a|x}$ does not admit such a LHS decomposition, we say that the state ρ^{AB} is *steerable* from A to B. If for all measurements we can never demonstrate steering with a given state, we say it is unsteerable (from A to B) [32].

Network steering.—We will now introduce our main new notion, that of network steering. Here, we have a collection of independent sources which distribute quantum states to a subset of parties. In the standard network nonlocality scenario, all parties are assumed to be untrusted and to perform "black-box" measurements. Here, in contrast, inspired by the steering scenario, we will consider only a subset of the parties to be untrusted and the remainder trusted. We will be interested in the (subnormalized) states that are prepared for the trusted parties by the measurements of the untrusted parties. We refer to this general setup as network steering.

We focus primarily on a simple scenario, with n parties arranged in a line, where the end point parties are trusted, and intermediate parties are untrusted and each perform a single fixed measurement. The simplest such scenario has three parties and two sources [see Fig. 1(e)], as in entanglement swapping [34]. Here the first two parties share a state ρ^{AB}, the second and third parties share a state $\rho^{B'C}$, and the central party performs a fixed measurement $M_b^{BB'}$. The subnormalized states prepared for A and C by this measurement are

$$\sigma_b^{AC} = \mathrm{Tr}_{BB'}([\mathbb{1}^A \otimes M_b^{BB'} \otimes \mathbb{1}^C]\rho^{AB} \otimes \rho^{B'C}), \qquad (1)$$

which occur with probability $p(b) = \mathrm{Tr}(\sigma_b^{AC})$. We will refer to $\{\sigma_b\}_b$ as a network assemblage.

In order to determine when this network assemblage demonstrates network steering, we need to introduce the notion of a NLHS model, which takes the form

$$\sigma_b^{AC} = \sum_{\beta,\gamma} p(\beta)p(\gamma)p(b|\beta,\gamma)\,\sigma_\beta^A \otimes \sigma_\gamma^C, \qquad (2)$$

where β and σ_β^A are the hidden variable and hidden states of the first source, γ and σ_γ^C those of the second source, and $p(b|\beta,\gamma)$ the local response function of Bob. If there is no such model that can explain the network assemblage σ_b, then we say it demonstrates network steering. Interestingly,

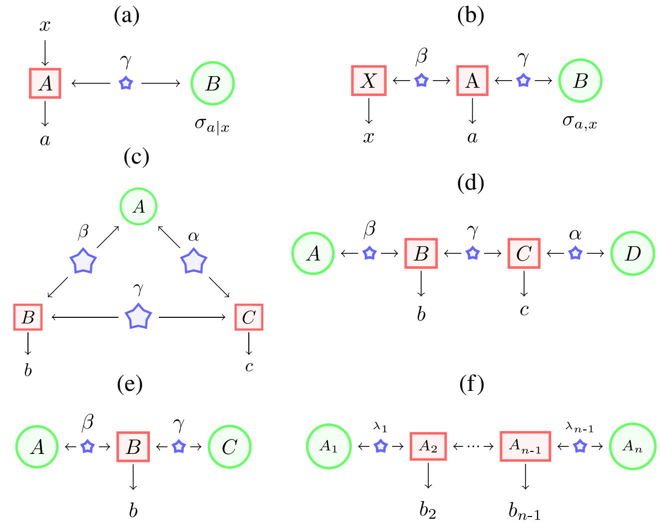

FIG. 1. Network steering scenarios. Green circles represent trusted parties, and red squares represent untrusted parties. (a) Standard steering scenario. (b) Steering scenario without inputs. (c) Triangle scenario with a trusted party. (d) Triangle scenario interpreted as a line. (e) Entanglement swapping scenario with trusted end points. (f) Generalized line scenario with trusted end points.

whereas conventional quantum steering requires multiple measurements to be performed by the untrusted party, just as with network nonlocality, we shall see here that even a fixed measurement can suffice to demonstrate network steering.

We note first in Eq. (2) that each σ_b^{AC} is, in fact, separable. Thus, the presence of entanglement in any single σ_b suffices to rule out a NLHS model and therefore demonstrates network steering.

The above generalizes in a natural way to the n-party line network depicted in Fig. 1(f), with outcomes $b_2, \ldots, b_{n-1}$. We explicitly include the straightforward generalization of Eqs. (1) and (2) in the Supplemental Material Sec. I [35], and see that the following observation holds generally.

Observation 1.—For any linear network with trusted end points, the entanglement of a single $\sigma_{b_2,\ldots,b_{n-1}}$ is sufficient to rule out a NLHS model and thus demonstrate network steering.

For more general networks, we can represent them as undirected graphs, where each node is either untrusted or trusted, and the edges represent independent sources. If all the parties are untrusted, the quantity of interest is the observed statistics $p(a,b,\ldots|x,y,\ldots)$. When at least one party is trusted, this is replaced by some network assemblage $\sigma_{a,b,\ldots|x,y,\ldots}$. A key observation that will prove useful is the following equivalence between networks, a generalization from the network nonlocality case [9].

Observation 2.—Any network with an untrusted party A that has an input x received with probability $p(x)$ and outcome a is equivalent to a network with an additional untrusted party A' who shares an additional source with A, neither of whom now has an input. In this new network, the outcome of A' is x, the old input of A. The relation between the network assemblages in the first and second scenarios is $p(x)\sigma^{A\cdots}_{a,\ldots|x,\ldots} = \sigma^{AA'\cdots}_{a,x,\ldots}$.

By virtue of the fact that quantum mechanics admits local tomography, we also note the following observation.

Observation 3.—A trusted party connected to n independent sources can without loss of generality be replaced by n end point trusted parties, each connected to a single source.

This allows us, for example, to interpret linear networks as rings with a single trusted party; e.g., the four party linear network with trusted end points can also be viewed as the triangle network where one of the parties is trusted, as in Figs. 1(c) and 1(d). This observation motivates our choice to focus our discussion on linear networks, which we understand now to be relevant for more complex, nonlinear networks. We detail further basic observations in Supplemental Material Sec. III [35].

Demonstrating network steering.—We now begin our exploration of demonstrating network steering and explain how and when steerable states will lead to network steering when placed in a network. We consider first the scenario of Fig. 1(e). If one source distributes a state which is steerable in the standard steering scenario, then Observation 2 would seem to indicate that even if the second source distributes only separable states (which we will refer to as a separable source), it should still be possible to use this to encode "the input" to the measurement, and thus demonstrate network steering. Here we make this intuition precise.

Consider the network scenario depicted in Fig. 1(b), with two untrusted parties without inputs steering a third, leading to a network assemblage $\sigma_{a,x}$. Here the NLHS condition reads

$$\sigma_{a,x} = \sum_{\beta,\gamma} p(\beta)\, p(\gamma) p(x|\beta) p(a|\beta,\gamma)\sigma_\gamma. \tag{3}$$

We can then observe the following:

Claim 1.—If $\sigma_{a,x}$ has a NLHS model, then $\sigma_{a|x} \coloneqq \sigma_{a,x}/p(x)$ has a LHS model, where $p(x) = \mathrm{Tr}\sum_a \sigma_{a,x}$.

Proof.—We can write Eq. (3) as

$$\sigma_{a,x} = p(x)\sum_\gamma p(\gamma)\, p(a|x,\gamma), \sigma_\gamma, \tag{4}$$

where $p(x) \coloneqq \mathrm{Tr}(\sum_a \sigma_{a,x}) = \sum_\beta p(\beta)p(x|\beta)$ and $p(a|x,\gamma) \coloneqq [1/p(x)]\sum_\beta p(\beta)p(x|\beta)p(a|\beta,\gamma)$. The result then follows.

This is an analogous result to that proved in Ref. [9] relating Bell scenario statistics $p(a,b|x,y)$ to network nonlocality statistics $p(a,b,x,y)$, the corresponding distribution without inputs. We link this to the scenario from Fig. 1(e) where both end points are trusted.

Claim 2.—If σ_b has a NLHS model, then $\sigma_{b,x} \coloneqq \mathrm{Tr}_A([M_x^A \otimes \mathbb{1}^C]\,\sigma_b)$ has a NLHS model, for any measurement M_x.

Proof.—When σ_b has a NLHS model of the form (2), it follows that

$$\sigma_{b,x} = \sum_{\beta,\gamma} p(\beta)p(\gamma)\,\mathrm{Tr}(M_x\sigma_\beta)p(b|\beta,\gamma)\sigma_\gamma, \tag{5}$$

which is a NLHS model of the form (3), with $p(x|\beta) \coloneqq \mathrm{Tr}(M_x\sigma_\beta)$.

Putting this together, suppose that $\rho^{B'C}$ is steerable, such that $\sigma_{b|x} \coloneqq \mathrm{Tr}([M_{b|x} \otimes \mathbb{1}]\rho^{B'C})$ demonstrates steering for some $M_{b|x}$. Let $\rho^{AB} = \sum_x (1/d)|x\rangle\langle x| \otimes |x\rangle\langle x|$ where d is the number of measurements x, and $\{|x\rangle\}_x$ form an orthonormal basis, and $M_b = \sum_{x'} |x'\rangle\langle x'| \otimes M_{b|x'}$. The resulting network assemblage σ_b from Eq. (1) is seen to be

$$\sigma_b = \sum_x \frac{1}{d}|x\rangle\langle x| \otimes \sigma_{b|x}. \tag{6}$$

Now, from the above claims we can see that this must demonstrate network steering. Indeed, if instead it had a NLHS model, then from Claim 2, $\sigma_{b,x} \coloneqq \mathrm{Tr}_A([|x\rangle\langle x| \otimes \mathbb{1}^C]\,\sigma_b) = (1/d)\sigma_{b|x}$ would have a NLHS model with $p(x) = 1/d$. Then, from Claim 1, $\sigma_{b,x}$ would have a LHS model, but by assumption it does not. This shows that all steerable states lead also to network steering when placed in a network with an appropriate separable state. Interestingly, this occurs even though σ_b is separable.

Similar arguments apply for showing that in the line with four parties from Fig. 1(d), we can always demonstrate network steering when the central state is nonlocal, and the adjacent end point sources are suitable separable states providing the inputs. That is, if $\sigma_{b,c}$ has a NLHS model, then by A and D applying measurements M_x and M_y the associated probability distributions $p(b,c,x,y)$ and $p(b,c|x,y)$ necessarily have NLHV and LHV models, respectively (see Ref. [9]). So for any nonlocal central source, we can find appropriate measurements and adjacent separable sources such that $\sigma_{b,c}$ demonstrates network steering.

Activation.—The above constructions of network steering relied on steering or nonlocality in standard scenarios. Here we show that network steering is possible even when using only (two-way) unsteerable states, which can be viewed as a form of activation. Note that this complements previous examples of activation of steering in the standard bipartite scenario [38].

We define the doubly erased Werner (DEW) state as the two-qubit Werner state after both subsystems have undergone an identical erasure channel:

$$\rho_{\mathrm{DEW}}(\eta,\omega) := \Lambda_\eta \otimes \Lambda_\eta\left(\omega|\psi^-\rangle\langle\psi^-| + (1-\omega)\frac{\mathbb{1}}{4}\right), \qquad (7)$$

where $|\psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$, and $\Lambda_\eta(\rho) = \eta\rho + (1-\eta)|2\rangle\langle 2|$, where $|2\rangle$ represents the loss of the system. $\rho_{\mathrm{DEW}}(\eta,\omega)$ is entangled when $\omega > \frac{1}{3}$ (and $\eta \neq 0$), and is unsteerable (in both directions) when $\eta \leq \frac{2}{3}(1-\omega)$. This follows from Ref. [39], as for any state ρ^{AB} unsteerable from Alice to Bob, we have that $\mathbb{1}^A \otimes \Omega^B[\rho^{AB}]$ is also unsteerable from A to B, for any channel Ω [40]. Note also that in the context of entanglement swapping, projecting two copies of $\rho_{\mathrm{DEW}}(\eta,\omega)$ onto $|\psi^-\rangle\langle\psi^-|$ leads to $\rho_{\mathrm{DEW}}(\eta,\omega^2)$ (with probability $\eta^2/4$), that is to a DEW state with squared visibility (see Supplemental Material Sec. II [35] for details).

Consider now the line network from Fig. 1(f) with each source distributing a copy of $\rho_{\mathrm{DEW}}(\eta,\omega)$, and all untrusted parties performing the fixed measurement $M_0 = |\psi^-\rangle\langle\psi^-|$, $M_1 = \mathbb{1} - |\psi^-\rangle\langle\psi^-|$, leading to the network assemblage $\sigma_{b_2,\ldots,b_{n-1}}$. Now, if we choose $\eta = \frac{2}{3}(1-\omega)$ and $1 > \omega > (\frac{1}{3})^{\frac{1}{n}}$, then each DEW is entangled but unsteerable, and we find, due to the entanglement-swapping property noted above, that the element $\sigma_{0,\ldots,0}$ (corresponding to a successful swap in each case) will be proportional to the state $\rho_{\mathrm{DEW}}(\eta,\omega')$ with $\omega' > \frac{1}{3}$, and therefore entangled. From Observation 1, this precludes a NLHS model description, and therefore demonstrates network steering, even though each DEW state was unsteerable.

Simple NLHS models.—We finish our exploration by considering to what extent the properties of the quantum sources directly affect the possibility of a NLHS model. We will refer to a source as being separable, unsteerable, or local if it is only capable of generating separable, unsteerable, or local states, respectively. As an illustrative example, in the three-party scenario of Fig. 1(e), if one source is separable and the other source is unsteerable (toward the trusted party), then for any fixed central measurement, the network assemblage σ_b will always be NLHS. Indeed, taking $\rho^{AB} = \sum_\gamma p(\gamma)\sigma^A_\gamma \otimes \sigma^B_\gamma$ and inserting into Eq. (1) gives

$$\sigma_b = \sum_\gamma p(\gamma)\,\sigma^A_\gamma \otimes \mathrm{Tr}_{BB'}([M_b \otimes \mathbb{1}^C]\sigma^B_\gamma \otimes \rho^{B'C}). \qquad (8)$$

Defining $M_{b|\gamma} := \mathrm{Tr}_B(M_b\sigma^B_\gamma \otimes \mathbb{1}^C)$ which form a set of valid measurement operators leads us to write

$$\sigma_b = \sum_\gamma p(\gamma)\sigma^A_\gamma \otimes \mathrm{Tr}_{B'}([M_{b|\gamma} \otimes \mathbb{1}^C]\rho^{B'C}). \qquad (9)$$

If $\rho^{B'C}$ is unsteerable from B' to C, this allows us to extract a LHS model yielding

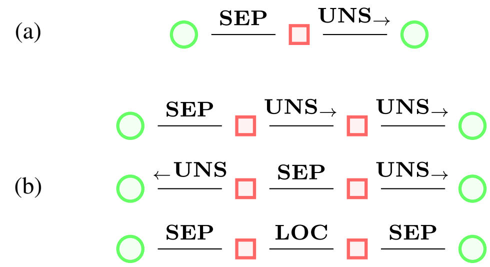

FIG. 2. Classifying the structure of some NLHS models. Green circles represent trusted parties, and red squares represent untrusted parties who perform a fixed measurement. (a) In the scenario of Fig. 1(e), when one source is separable (SEP), this acts as an input to the adjacent measurements, and by taking the second source as unsteerable (UNS$_\rightarrow$) then this always leads to a NLHS model. (b) Similar results hold in the "unwrapped" triangle scenario [Fig. 1(d)], where now sources can also be taken as local (LOC). We expand and detail this further in Supplemental Material Sec. I [35].

$$\sigma_b = \sum_\gamma p(\gamma)\,\sigma^A_\gamma \otimes \left(\sum_\lambda p(\lambda)p(b|\lambda,\gamma)\sigma^C_\lambda\right) \qquad (10)$$

$$= \sum_{\gamma,\lambda} p(\gamma)p(\lambda)\,p(b|\lambda,\gamma)\sigma^A_\gamma \otimes \sigma^C_\lambda, \qquad (11)$$

which is a NLHS model (2). Hence, the combination of a separable and unsteerable source (to the trusted party) can never lead to network steering, as shown in Fig. 2(a).

Similar results follow in more complicated scenarios. In Fig. 2(b), we give the three configurations which always lead to NLHS models in the (unwrapped) triangle scenario of Fig. 1(d), and we give further generalizations for the line scenario of Fig. 1(f) in Supplemental Material Sec. I [35]. The main concept behind all of these results is that separable and unsteerable sources provide a form of input, allowing us to write down large classes of nontrivial NLHS models.

Conclusions.—We introduced the notions of network steering and network local hidden state models. We discussed illustrative examples and showed that the network scenario leads to a form of activation of steering. Finally, we started a characterization of NLHS models based solely upon properties of the sources. There are many fascinating and novel future questions to tackle.

First, it would be interesting to determine if either NLHS assemblages or the full set of network assemblages can be characterized via techniques based on semidefinite programming, using for instance the approach of Ref. [18]. A related direction is to further classify NLHS models based on the properties of the sources. For instance, consider four parties sharing separable, local, and unsteerable sources, or five parties sharing separable, local, local, and separable sources. In neither case do we currently know if network steering can arise or not.

Here we focused primarily on the properties of the sources, but it would also be interesting to consider the measurements and understand which of their properties (e.g., entanglement or incompatibility) are relevant for network steering. Future work could also consider the significance of our work for quantum repeaters [4], explore links with superactivation of quantum steering [38], or extend recent work on postquantum steering [41] to this setting.

Finally, our initial motivation for this work was to attempt to gain clarity on network nonlocality problems, such as those in the triangle network. It is our hope that developing our framework further will lead to discovering novel nonlocal correlations unique to networks.

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[1] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Rev. Mod. Phys. **86**, 419 (2014).
[2] D. Cavalcanti and P. Skrzypczyk, Rep. Prog. Phys. **80**, 024001 (2017).
[3] R. Uola, A. C. S. Costa, H. C. Nguyen, and O. Gühne, Rev. Mod. Phys. **92**, 015001 (2020).
[4] S. Wehner, D. Elkouss, and R. Hanson, Science **362**, eaam9288 (2018).
[5] J. S. Bell, Physics **1**, 195 (1964).
[6] A. Tavakoli, A. Pozas-Kerstjens, M.-X. Luo, and M.-O. Renou, arXiv:2104.10700.
[7] C. Branciard, N. Gisin, and S. Pironio, Phys. Rev. Lett. **104**, 170401 (2010).
[8] C. Branciard, D. Rosset, N. Gisin, and S. Pironio, Phys. Rev. A **85**, 032119 (2012).
[9] T. Fritz, New J. Phys. **14**, 103001 (2012).
[10] R. Chaves and T. Fritz, Phys. Rev. A **85**, 032113 (2012).
[11] A. Tavakoli, P. Skrzypczyk, D. Cavalcanti, and A. Acín, Phys. Rev. A **90**, 062109 (2014).
[12] R. Chaves, Phys. Rev. Lett. **116**, 010402 (2016).
[13] D. Rosset, C. Branciard, T. J. Barnea, G. Pütz, N. Brunner, and N. Gisin, Phys. Rev. Lett. **116**, 010403 (2016).
[14] M. Weilenmann and R. Colbeck, Quantum **2**, 57 (2018).
[15] E. Wolfe, R. W. Spekkens, and T. Fritz, J. Causal Infer. **7**, 20170020 (2019).
[16] N. Gisin, J.-D. Bancal, Y. Cai, P. Remy, A. Tavakoli, E. Z. Cruzeiro, S. Popescu, and N. Brunner, Nat. Commun. **11**, 2378 (2020).
[17] J. Åberg, R. Nery, C. Duarte, and R. Chaves, Phys. Rev. Lett. **125**, 110505 (2020).
[18] E. Wolfe, A. Pozas-Kerstjens, M. Grinberg, D. Rosset, A. Acín, and M. Navascués, Phys. Rev. X **11**, 021043 (2021).
[19] T. Kriváchy, Y. Cai, D. Cavalcanti, A. Tavakoli, N. Gisin, and N. Brunner, npj Quantum Inf. **6**, 70 (2020).
[20] T. C. Fraser and E. Wolfe, Phys. Rev. A **98**, 022113 (2018).
[21] M.-O. Renou, E. Bäumer, S. Boreiri, N. Brunner, N. Gisin, and S. Beigi, Phys. Rev. Lett. **123**, 140401 (2019).
[22] M.-O. Renou and S. Beigi, arXiv:2011.02769.
[23] I. Šupić, J.-D. Bancal, Y. Cai, and N. Brunner, arXiv: 2105.12341.
[24] T. Kraft, S. Designolle, C. Ritz, N. Brunner, O. Gühne, and M. Huber, Phys. Rev. A **103**, 052405 (2021).
[25] M. Navascués, E. Wolfe, D. Rosset, and A. Pozas-Kerstjens, Phys. Rev. Lett. **125**, 240505 (2020).
[26] M.-X. Luo, Adv. Quantum Technol. **10**, 1002 (2020).
[27] T. Kraft, C. Spee, X.-D. Yu, and O. Gühne, Phys. Rev. A **103**, 052405 (2021).
[28] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007).
[29] D. Cavalcanti, P. Skrzypczyk, G. Aguilar, R. Nery, P. S. Ribeiro, and S. Walborn, Nat. Commun. **6**, 7941 (2015).
[30] Q. Y. He and M. D. Reid, Phys. Rev. Lett. **111**, 250403 (2013).
[31] M. F. Pusey, Phys. Rev. A **88**, 032313 (2013).
[32] Note that steering can be asymmetrical; some states are steerable from Alice to Bob, but not the other way around [33].
[33] J. Bowles, T. Vértesi, M. T. Quintino, and N. Brunner, Phys. Rev. Lett. **112**, 200402 (2014).
[34] M. Żukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).
[35] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.170405 for further observations concerning basic network structures, a discussion of NLHS models in general linear networks, and a detailed calculation of entanglement swapping between doubly erased Werner states, which includes Refs. [36,37].
[36] I. Kogias, P. Skrzypczyk, D. Cavalcanti, A. Acín, and G. Adesso, Phys. Rev. Lett. **115**, 210401 (2015).
[37] T. Moroder, O. Gittsovich, M. Huber, R. Uola, and O. Gühne, Phys. Rev. Lett. **116**, 090403 (2016).
[38] M. T. Quintino, N. Brunner, and M. Huber, Phys. Rev. A **94**, 062123 (2016).
[39] N. Tischler, F. Ghafari, T. J. Baker, S. Slussarenko, R. B. Patel, M. M. Weston, S. Wollmann, L. K. Shalm, V. B. Verma, S. W. Nam, H. C. Nguyen, H. M. Wiseman, and G. J. Pryde, Phys. Rev. Lett. **121**, 100401 (2018).
[40] M. T. Quintino, T. Vértesi, D. Cavalcanti, R. Augusiak, M. Demianowicz, A. Acín, and N. Brunner, Phys. Rev. A **92**, 032107 (2015).
[41] A. B. Sainz, N. Brunner, D. Cavalcanti, P. Skrzypczyk, and T. Vértesi, Phys. Rev. Lett. **115**, 190403 (2015).