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THE DESIGN OF LOW COMPLEXITY TWO-CHANNEL LATTICE-STRUCTURE PERFECT-RECONSTRUCTION FILTER BANKS USING GENETIC ALGORITHMS

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ABSTRACT

This paper considers the design of reduced complexity 2 band QMF filter banks. In order to ensure perfect reconstruction, a lattice architecture has been adopted. Genetic algorithms are used to search for an optimal set of coefficients comprising simple sums of signed power of two terms. This allows the entire filter bank to be implemented using only additions and a single multiplicative scaling factor.

Results are presented which show that the proposed method can be used to design filters with superior (in terms of minimax ripple) performance, compared to various infinite precision designs. Genetic algorithms can also be used to jointly optimise both performance and complexity in order to achieve an optimum performance/complexity trade off.

KEYWORDS

QMF filter banks, genetic algorithms.

1. INTRODUCTION

Multirate digital filter banks [1] are frequently employed in applications areas such as image and video compression, speech and audio processing and various aspects of communication systems. A range of filterbank structures have been reported, with the most common implementation being based on two channel quadrature mirror filter (QMF) designs. When implemented using standard FIR filter algorithms with coefficients derived from conventional optimisation methods, these require significant numbers of multipliers and adders even for relatively short filters.

Multipliers are widely acknowledged to be the most complex components in any implementation and, in order to reduce this computational burden, several authors have proposed their replacement by shift-add structures based on coefficients represented using constrained signed power of two (SPT) terms. In the past, this has achieved

by rounding the coefficients generated by optimal (infinite precision) design methods. However, it is well known that this approach results in a sub-optimal solution. In an attempt to optimise the set of SPT terms for a particular filter design, linear programming and simulated annealing have been employed by previous workers [7].

Recently, genetic algorithms (GAs) [2] have emerged as a powerful and robust tool for solving the real valued optimisation problems. Previously, the authors have used this approach for designing reduced complexity non-separable circularly symmetric and diamond shaped two-dimensional digital filters [3]. In this paper, GAs have been used to optimise the design of lattice structure QMF filter banks employing g SPT coefficients. Two cases are considered: in the first case, (GA1), frequency response error is used as the sole objective function. In the second case, (GA2), a multiple criterion objective is used to jointly minimise the frequency response error and the computational complexity.

A generic two-channel FIR filter bank is shown figure 1. Here $H_0(z)$ and $H_1(z)$ represent the low pass and high pass filters respectively, in the analysis bank and $G_0(z)$ and $G_1(z)$ are the corresponding synthesis filters. Assuming lossless channels and codecs, we can write the reconstructed signal as:

$$Y(z) = \frac{1}{2}\left[\, H_0(z)G_0(z) + H_1(z)G_1(z)\, \right]X(z) + \frac{1}{2}\left[\, H_0(-z)G_0(z) + H_1(-z)G_1(z)\, \right]X(-z) \tag{1}$$

The aliasing term $(X(-z))$ can be cancelled by:

$$G_0(z) = 2H_1(-z) \tag{2}$$

$$G_1(z) = -2H_0(-z) \tag{3}$$

Perfect reconstruction is obtained when $y(n)$ is a delayed replica of the input $x(n)$:
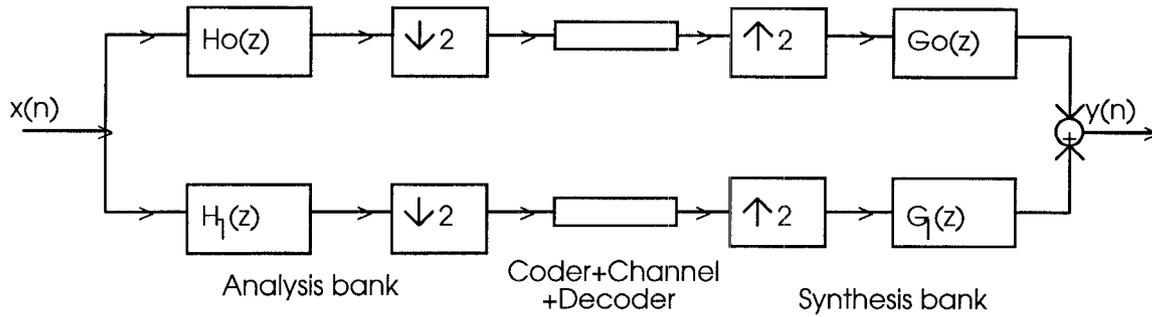
$$Y(z) = z^{-2k+1}X(z) \tag{4}$$

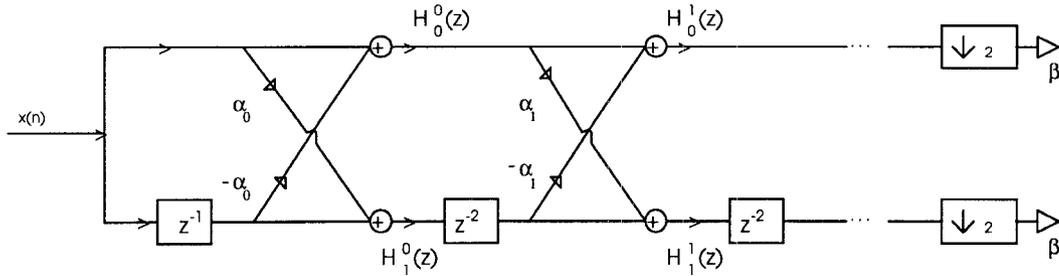*Fig 1.   Two-channel analysis/synthesis filter bank system.*

Fig 2.   Analysis Bank

$$H_0(z)H_1(-z) - H_0(-z)H_1(z) = z^{-2k+1} \qquad (5)$$

Recently several novel lattice-structure perfect-reconstruction filter banks have been reported [4, 5, 6]. These exhibit lower complexity in terms of number of multipliers and adders used, compared with other FIR perfect reconstruction filter banks. They also guarantee perfect reconstruction even when coefficients are approximated.

2   FILTER DESIGN FORMULATION

In this paper we employ the QMF lattice structure of [4] which is shown in figure 2 where:

$$H_0^0(z) = 1 - \alpha_0 z^{-1} \qquad (6)$$

$$H_1^0(z) = \alpha_0 + z^{-1} \qquad (7)$$

For an $m+1$ stage lattice filter bank

$$\begin{pmatrix} H_0^m(z) \\ H_1^m(z) \end{pmatrix} = \begin{pmatrix} 1 & -\alpha_m z^{-2} \\ \alpha_m & z^{-2} \end{pmatrix} \begin{pmatrix} H_0^{m-1}(z) \\ H_1^{m-1}(z) \end{pmatrix} \qquad (8)$$

$H_0^m(z)$ and $H_1^m(z)$ denote the low pass and high pass filters of an $m+1$ stage filter bank respectively. β is the scale factor which can be expressed as

$$\beta = \prod_m (1 + \alpha_m^2)^{0.5} \qquad (9)$$

The synthesis filters $G_0(z)$ and $G_1(z)$ can be written as

$$G_0(z) = z^{-N} H_0^m(z) \qquad (10)$$

$$G_1(z) = z^{-N} H_1^m(z) \qquad (11)$$

where N is the order of the filters.

3   REDUCED COMPLEXITY DESIGNS

In order to reduce architectural complexity, the lattice coefficients, $\alpha(n)$ were represented as sums of SPT terms with a constraint on the total number of terms used as follows:

$$\alpha(n) = \sum_{k=1}^{p} c_k . 2^{-g_k} \, \delta(b_k - n) \quad c_k \in \{-1,0,1\},$$

$$g_k \in \{-2,-1,0,1,\dots,B\} \qquad (12)$$

$$b_k \in \{0,\dots,J-1\}$$

J = number of stages
P = total number of SPT terms

As in [7], a minimax error criterion was adopted for optimisation. The objective function used for GA1 (without any constraints on the number of adders) was:

$$\delta(\alpha(n)) = \max\left| \frac{1}{G}\left|H_0^m(e^{j\omega})\right| - T(\omega)\right| \qquad (13)$$

where G is a scaling factor. and $T(\omega)$ is the desired frequency response. For GA2, the corresponding function used was:

$$\delta(\alpha(n)) = \max\left| \frac{1}{G}\left|H_0^m(e^{j\omega})\right| - T(\omega)\right| + w.\,adders \qquad (14)$$

where W is a weighting constant used to bias the influence of adders in the objective function.

4 PROBLEM REPRESENTATION

In order to reduce the search space and thereby improve the search efficiency, it was found useful to restrict the first J SPT terms to be non-zero and allocate these to the first J coefficients.(i.e: $c_k \neq 0$, $b_k = k$ for $1 \leq k < J$). Thus the first J coefficients require one bit for c_k and 3 or 4 bits for g_k (4 bits for $g_k > 8$). The remaining terms require 2 bits for c_k 3 or 4 bits for g_k and 3 or 4 bits b_k (4 bits for $b_k > 8$). In the case of filter design 24D, an average of two SPT terms per lattice coefficient was used (with wordlength, B = 13 bits) giving a chromosome length of 180 bits. In order speed up the convergence, the initial population was seeded with the SPT terms obtained after rounding the optimal infinite precision design coefficients reported in [4].

5 RESULTS

Table 1 shows the maximum frequency response ripple of both an optimal infinite precision design and GA1 for a set of filters with the same specifications as those used by Johnston in [8] (the number in the filter specification corresponds to its length and the letter indicates various transition bandwidths). The average number of SPT terms used in each design case is also reported in the same table. The maximum stopband ripple in each design using GA1 is less than or equal to the maximum stopband ripple of the optimal infinite precision design. This can be attributed to the fact that a minimax error criterion was used in the GA design, whereas a least mean squared error criterion was used in the optimal (infinite precision) case.

It is interesting to note that the maximum frequency response ripple of filter bank 32E is -29 dB which is equal to the design value reported in [7] using linear programming. Since other results are not given in [7], these cannot be compared directly. The performances of filter banks designed using GA1 and GA2 are compared in table 2, with the total number of adders used given in each case. It can be observed that, in the case of GA2, the total number of adders used has been reduced at the expense of some degradation in performance. This demonstrates the flexibility of the GA approach in trading of complexity and performance. The lattice coefficients of the filter bank, 24D obtained using both GA1 and GA2 are reported in table 3. The frequency responses of the low pass filter from filter bank 24D designed using GA1, GA2 and the optimal (infinite precision) approach are compared in figure 3.

Table 1
Performance comparisons between maximum ripple of the lowpass filter of the filter banks designed using optimal infinite precision design (Optimal) [4] and Genetic Algorithms (GA1).

Filter bank design	Maximum ripple δ /(dB)		Average number of SPT term per coefficient used
	Optimal	GA1	
16B	-52.0	-52.0	3
16C	-33.0	-33.0	2
16F	-29.0	-29.4	2
24D	-32.0	-34.0	2
32D	-40.0	-40.0	2
32E	-25.0	-29.0	2

Table 2.
The performance comparisons between GA1 and GA2

Filter bank design	Total number of adders used	Maximum stopband ripple / dB
16B (GA1)	48	-52.0
(GA2)	42	-46.7
16C(GA1)	32	-33.0
(GA2)	26	-31.2
16F(GA1)	32	-29.0
(GA2)	28	-28.6
24D(GA1)	48	-34.0
(GA2)	44	-32.2
32D(GA1)	64	-39.8
(GA2)	58	-36.8
32E(GA1)	64	-29.0
(GA2)	58	-28.2

6 CONCLUSIONS

This paper has demonstrated the capability of genetic algorithms in designing low complexity lattice structure perfect reconstruction filter banks. The design, GA1 produces superior or equal results to the optimal infinite precision design. In order to jointly minimise the number of adders and frequency response ripple, a multiple criterion optimisation has been successfully used in GA2 which significantly reduces the total number of adders used for a slight loss of performance in the frequency domain.

Table 3
Lattice coefficients of the filter bank, 24D designed using GA1 and GA2

Lattice coefficients	Values from GA1	Values from GA2
$\alpha(0)$	$-2^1-2^0-2^{-3}+2^{-8}$	$-2^1-2^0-2^{-3}$
$\alpha(1)$	2^0	2^0
$\alpha(2)$	$-2^{-1}-2^{-4}$	$-2^{-1}-2^{-4}$
$\alpha(3)$	$2^{-1}-2^{-3}$	$2^{-1}-2^{-3}$
$\alpha(4)$	$-2^{-2}-2^{-7}$	$-2^{-2}-2^{-7}$
$\alpha(5)$	$2^{-2}-2^{-4}$	$2^{-2}-2^{-4}$
$\alpha(6)$	$-2^{-3}-2^{-6}$	$-2^{-3}-2^{-6}$
$\alpha(7)$	$2^{-3}-2^{-5}$	$2^{-3}-2^{-5}$
$\alpha(8)$	-2^{-4}	-2^{-4}
$\alpha(9)$	$2^{-5}+2^{-6}-2^{-8}$	$2^{-5}+2^{-6}$
$\alpha(10)$	$-2^{-5}+2^{-7}$	$-2^{-5}+2^{-7}$
$\alpha(11)$	2^{-6}	2^{-6}
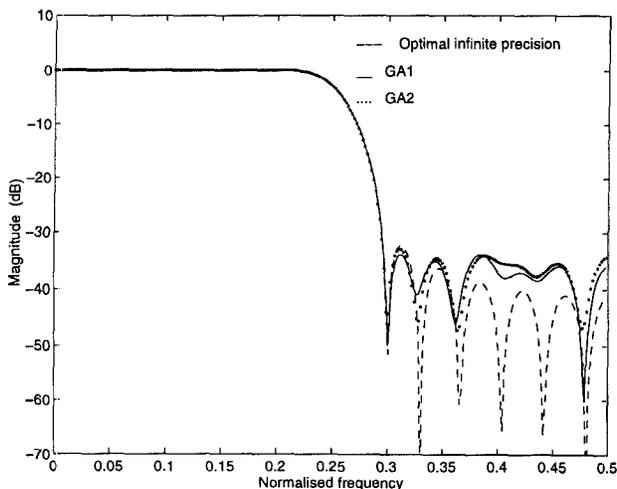
Fig 3.   Frequency response of the lowpass filter for filter bank, 24D

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