A Fast and Fair Algorithm for Distributed Subcarrier Allocation Using Coalitions and the Nash Bargaining Solution

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***Abstract*—In this paper we propose a distributed, low-complexity, fast and fair resource allocation algorithm for a multiuser, wireless LTE OFDMA channel. Based on previous work by the authors [1], this algorithm partitions the users of the wireless network into coalitions and, using the game theoretic concept of the Nash Bargaining Solution, offers a cooperative solution to the problem of subcarrier allocation. The use of the NBS ensures that the fairness provided by the proposed algorithm matches that offered by the widely accepted Proportional Fair (PF) scheduler. Simulation results show that the new algorithm achieves a sum rate that can be tuned to be almost equivalent to the sum rate achieved by the PF scheduler, while only requiring limited resources and exchange of information between nodes. At the same time, extensive efficiency enhancements and its distributed nature render it fast and low-complexity enough to be implemented in a real-time wireless system.**

I. Introduction

In this paper we propose a distributed resource allocation algorithm for a multiuser, frequency selective wireless channel, where the existence of independent communication links between the users and the BS and their instantaneous fluctuation allows for the benefits of multiuser diversity to be harvested. The majority of work in the existing literature makes the assumption that a master device takes responsibility for dictating the resource allocation process. This simplifies protocol issues, but the various costs associated with this approach cannot be ignored; the central controller needs information about the channel quality of all wireless devices it is responsible for allocating resources to. Furthermore, the allocation decisions have to be sent back to wireless devices, thus altogether generating an overhead that impedes system performance.

The aforementioned costs suggest that there are potential benefits to be achieved if the resource allocation process takes place in a more distributed fashion. Coalition formation is a game theoretic concept that can be used to achieve this and it can either be centrally- or self-organized. Expensive algorithms exist for optimal solutions for the former; the latter is robust, scales well, and uses simple heuristics to form beneficial coalitions at low complexity.

We propose a fast, low-complexity distributed algorithm for subcarrier allocation in an LTE wireless channel. Our approach utilizes the Nash Bargaining Solution (NBS) and groups users into coalitions to propose a fast and efficient allocation process that does not require a central controller.

The rest of the paper is organized as follows: section II presents related work, section III outlines the system model, section IV briefly describes the NBS and section V presents the algorithm. Simulation results are presented in section VI and conclusions are drawn in section VII.

II. Related Work

A thorough presentation of game theory application in wireless channel resource allocation can be found in [4]. Authors in [5] argue that cooperative game theory is not well suited to the distributed nature of wireless networking; something that the results presented in our earlier work [1] contradict. NBS and coalitions are used in [7] to achieve fair resource allocation for multiuser, OFDMA-based wireless networks. A partially distributed scheme for the allocation of subcarriers is proposed, where the network's base station simply acts like a market place where the bargaining of subcarriers among the users takes place. A scheme similar to the above but with reduced complexity, is proposed in [8]. Finally, a distributed approach for fair resource allocation in wireless networks using NBS is presented in [9], as well as in [10].

The novelty of our work is that it considers low complexity as a key element. Most of the relevant literature in this research area discusses the resource allocation problem mainly in terms of optimal power, rate and fairness performance and largely neglects complexity issues. Our work proposes an efficient and low-complexity algorithm that is capable of being implemented in a distributed way in a real-time system.

III. System Model and Description

We focus on the downlink scenario of the SCM LTE channel [11]. The simulated network consists of a single Base Station (BS) with users randomly scattered around it, in a radius of 150 meters. The propagation scenario is the 'SCM Urban Macro', users are moving at a speed of 10m/s, the carrier frequency is 2Ghz and the transmission bandwidth 10MHz. the time slot duration is 0.5ms, the BS transmit power is 43dBm (20W) and the noise power is -104dBm. All 1024 channel subcarriers are used and no subcarrier sharing is allowed. The channel quality each individual user experiences is strongly dependent on its distance from the BS and therefore users' SNR values vary accordingly. Finally, we assume that there exists a reliable and fast feedback channel, so that users can communicate with each other and exchange the information required for the subcarrier allocation process. This channel is also used for the allocation decisions to be fed back from the mobile devices to the BS. Our performance metric is the rate achieved by each user over the whole duration of the channel simulation. We calculate the theoretical rate of user *k* in subcarrier *s*, using the following equation:

$$R_{k,s} = \frac{W}{S} \times \log_2 \left(1 + SNR \times \|H_{k,s}\|^2\right), \text{bits/s}, \quad (1)$$

where *W* is the channel's bandwidth, *S* is the number of subcarriers in the channel and $H_{k,s}$ is the channel gain for user *k* in subcarrier *s*. Finally, the sum rate achieved by user *k* for the whole channel simulation is calculated by adding up the rates for all the subcarriers the user was allocated:

$$R_k = \sum_{i=1}^{y} \sum_{S_n} R_{k,s} \ , \quad (2)$$

where S_n is the vector of subcarriers allocated to user *k* and *y* is the number of simulated channel realizations (i.e. 2000).

IV. Nash Bargaining Solution

In this section we briefly present the basics of the NBS [6]. As detailed in [7], the *game* in this case is the subcarrier allocation problem, the *players* are the users participating in the wireless network and the *goal* is to maximize the chosen utility function for all users simultaneously. Similarly to the aforementioned reference and to the vast majority of the relevant literature, we choose the data rate (eq. 3) achieved by the user in a single channel realization to serve as the utility function:

$$U = \sum_{S_n} R_{k,s} \quad (3)$$

In order to determine the NBS, we need to find the subcarrier allocation matrix that maximizes the product of data rates (i.e. the Nash function) for the users-members of each coalition:

$$\prod_{i=1}^{n} (R_i - R_{min}^i), \quad (4)$$

where *n* is the number of users in each coalition, R_i is the sum rate achieved by user *i* over the allocated subcarrier groups (i.e. subchannels) and R_{min}^i is the minimal rate requirement of user *i*. However, depending on each user's choice of R_{min}^i, eq. 4 might not converge to a solution. To avoid this complication we set R_{min}^i equal to zero for every user. This choice also improves the execution speed of the algorithm and provides [7] a proportionally fair behavior to the allocation process.

As described in [7], the unique NBS (i.e. the subcarrier allocation matrix in our system) that maximizes eq. 4 satisfies the following axioms:

1) *Individual Rationality*
2) *Feasibility*
3) *Pareto Optimality*
4) *Independence of Irrelevant Alternatives*
5) *Independence of Linear Transformations*
6) *Symmetry*

These axioms ensure that the NBS maximizes all R_i simultaneously and also provides a fair operation point for all participating *players*.

V. The Allocation Algorithm

A. Coalition Formation

Aiming to avoid the computational complexity of finding the optimal partitioning of the users into coalitions, and similar to [1] and to [7], we form equally-sized coalitions. In contrast to our previous work in [1], where the coalition size is always 2 and the number of users 5, in this work we also examine the allocation algorithm's behavior and performance with coalition sizes that range from 2 to 5 and with users' number up to 10. The reason for setting these limits is the significantly increased computational and memory requirements that larger values would impose. Additionally, we discuss the case of a significantly higher number of users in the network and propose modifications that will allow the algorithm to run sufficiently fast even in that case.

Our approach to coalition formation is straightforward; all possible partitions of the users into equally-sized coalitions are generated and, after bargaining for subcarriers within each coalition has finished, the appropriate coalition structure (i.e. partitioning of users into coalitions) has to be selected. The number of coalitions formed is:

$$c = \begin{cases} \frac{n}{m} & \text{, if } n \equiv 0 \pmod{m} \\ \lceil \frac{n}{m} \rceil + 1 & \text{, otherwise} \end{cases} \quad (5)$$

where *n* is the number of users in the network and *m* is the coalition size. If the users cannot be exactly split into equally-sized coalitions, the remaining users form a single, smaller-sized coalition. In the case of a single remaining user, this user is part of no coalition and simply gets the remaining unallocated subcarriers. The allocation of equal number of subcarriers to every user (see section V-B *'Bargaining for Subcarriers'*) guarantees fairness to all users.

B. Bargaining for Subcarriers

After forming all possible coalition structures, the utility that each structure can yield has to be calculated through bargaining within each coalition. The array of all possible subcarrier-user permutations is generated and all permutations are examined to determine which one generates the highest utility (i.e product of rates for all coalition members). Since these computations take place within each coalition, limited signalling is required between coalition members in order

TABLE I
THE PROPOSED ALLOCATION ALGORITHM

The subcarrier allocation process
1. Initialization - Create array of all possible coalition structures - Create array of all possible subcarrier group permutations for each coalition - Create array that holds achievable rates for all users
2. For each coalition structure - For each subcarrier group permutation - For each coalition of the coalition structure - perform Nash Bargaining among the coalition members - after bargaining, update array that holds users' rates
3. Choose appropriate coalition structure - calculate Jain's fairness index for each coalition structure - for PF fairness, choose coalition structure with lowest Jain's index - OR choose according to protocol requirements (e.g. existence of 'willing' master users)
4. Go to 1. and repeat allocation process as necessary

to exchange the information (i.e. achievable rates for each subcarrier) necessary. However, the very large number of permutations makes the exhaustive search for the optimal permutation simply infeasible. Therefore, we choose to make some major modifications to this approach, so as to achieve significantly improved efficiency:

1) Subcarrier Grouping: Allocating subcarriers one by one is a complex process, requiring the testing of n^k subcarrier permutations, where *n* is the number of users in each coalition and *k* is the number of subcarriers tested. Our approach, similar to that of many practical systems, is to group consecutive subcarriers into groups and allocate them as a single unit. This incurs a slight loss of sum rate performance, as a degree of fine-grained control over the allocation process is lost. Our simulations show that 10-sized subcarrier groups offer an excellent trade-off between sum rate performance and algorithm efficiency. Generally, forming subcarrier groups of size *s* provides a $n^{k\times\frac{(s-1)}{s}}$- fold increase in speed. An additional benefit is the similarly reduced memory requirements, something very crucial in devices with limited resources.

2) Equal number of subcarriers: As shown in [1], the allocation of the same number of subcarrier groups to every user maintains fairness almost identical to proportional fairness. The difference in individual user rate performance stems from the fact that subcarriers differ in 'quality' for each user. This allocation strategy hugely reduces the number of subcarrier permutations that need to be tested and makes the algorithm significantly faster, while only slightly (i.e. up to 5% according to simulation results) impacting achieved rates.

C. Selection of Coalition structure

The last stage of the allocation process is the selection of the 'winning' coalition structure. Our simulations indicate that almost identical fairness and sum rates are achieved across all structures and that the choice of a specific structure only marginally (i.e. up to 1% in terms of sum rate) changes the outcome of the allocation process. This behavior stems from the choice of equal number of subcarriers per user and from setting R^i_{min} equal to zero for every user. It also offers the opportunity to further enhance the algorithm's efficiency by reducing the number of coalition structures tested.

D. Efficiency Enhancements

1) Permutations sampling: Despite eliminating the subcarrier-user permutations that do not provide an equal number of subcarrier to all users, the number of permutations to be tested is still very large. Based on the observation that no significant differences from permutation to permutation exist, we only test a sample of them. Sampling at a 'rate' of $1/m$ (i.e. test 1 out of *m* permutations) increases the algorithm execution speed by a factor of *m*. According to our simulations, for values of *m* up to 10^3 the loss of sum rate performance does not exceed 7% and for *m* up to 10^4 loss does not exceed 15%.

2) Realizations step: Repeating the allocation process less often than every channel realization yields a great increase in algorithm efficiency, maintains fairness and only marginally reduces the sum rate, as proved during our simulations. Repetition of the allocation process every *l* realizations provides an *l*-fold increase in algorithm speed. For example, setting *l* equal to 10 produces a 10-fold increase in speed, while only a 4% loss in sum rate incurs.

3) Number of coalition structures tested: Since the effect of selecting a specific coalition structure is marginal, we can make the algorithm faster by reducing the number of coalition structures tested before allocating the subcarrier groups. The total number of the different structures is:

$$st = \frac{\prod_{i=0,1,2,\ldots}\binom{n-i*s}{s}}{(\frac{n}{s})!}, \qquad \mathrm{n-i*s\geq s} \tag{6}$$

where *n* is the number of users and *s* is the coalition size. As *n* increases to values larger than 10, *st* quickly becomes prohibitively large for the algorithm to be implementable in a real-time system; sampling at a 'rate' of $1/p$ (i.e. test 1 out of *p* permutations) increases the algorithm execution speed by a factor of *p*. Simulation results show that, even for large values of *p*, the loss in sum rate rarely exceeds 2%. As a result, the application of the proposed algorithm can be extended to networks with large number of users by allowing *p* to take large values relative to *st*, and by keeping coalition size to small values (i.e. up to 5).

E. Protocol analysis

The distributed nature of the algorithm stems from the fact that allocation decisions are made at a local level, as each coalition decides with subcarrier-user permutation will be used. The utility computation requires that each coalition member transmits its achievable data rate (for every subcarrier group) to the 'master' user of the coalition. After performing the necessary calculations and determining with permutation provides the highest utility, the master user transmits the result

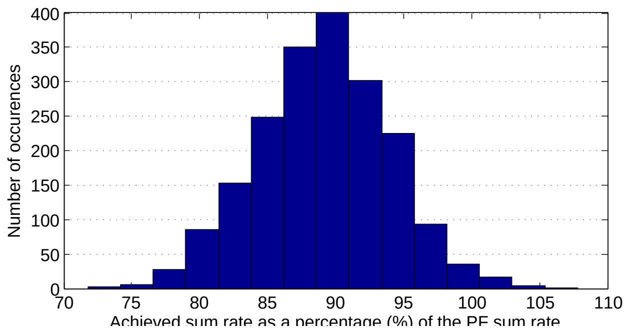

Fig. 1. Rate & Time comparison

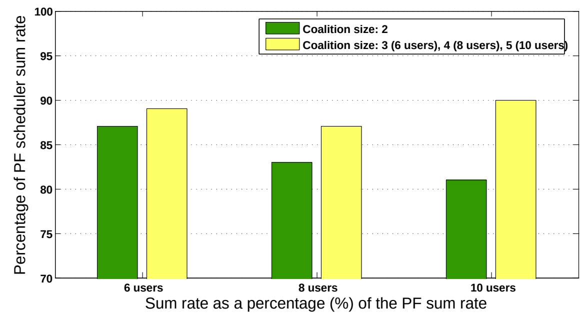

Fig. 2. Sum rate comparison for different coalition sizes

(i.e. total data rate for all members) to the 'leader' user of the network, who in turn makes the selection of the appropriate coalition structure (the leader remains the same throughout the whole allocation process) and transmits the allocation decision back to the BS. It is also the leader's responsibility to randomly pre-assign an equal number of subcarrier groups to each coalition at the start of the process. The roles of 'master' and 'leader' are randomly assigned to 'willing' users, who declare their 'willingness' by appropriate beaconing at the start of the allocation process. There must be one master user within each coalition and one leader user in the network. A master user can also be the leader. If no users are willing to assume these roles, they are randomly assigned, with a preference towards more powerful users (e.g. laptop preferred over smartphone). Additionally, the existence of willing users in each coalition can favor the selection of a specific coalition structure over another one.

Periodic beaconing ensures that the algorithm adapts as users join and leave the network. This generates only minimum overhead, as the repetition of the allocation process is significantly more frequent than that of the beaconing process since the time-scale of changes in the network form (i.e. users leaving/joining) is much slower than than instantaneous fluctuation in signal quality. As a failsafe strategy, when a master or leader user leaves the network, the BS continues with the previous allocation until beaconing takes place again and the algorithm adapts to the new network form.

VI. Results

We compare the proposed algorithm against the PF scheduler [12], due to its wide acceptance both in literature, as well as in actual products. Additionally, many of the game theoretic schedulers proposed in the literature are not implementable in practice due to their complexity. The time window we use for the PF scheduler is 500-subcarriers long. According to our simulations, this a value provides a good balance between sum rate and short-term fairness. Simulation was performed for 2000 different channels (i.e. different users' locations), with each channel being simulated for 2000 realizations (i.e. uncorrelated instances of small scale fading effects). The results presented in this section are averaged over the 2000 different channels.

A. *Sum rate & Fairness*

We use the sum rate achieved by the users as the basic performance metric. The proposed algorithm yields a sum rate that can be tuned (by choice of parameter values) to be equivalent to up to 90% of the PF sum rate on average between different simulation sets. This averaged value varies between 69% and 108% of the PF sum rate, as the users' long-term channel qualities vary significantly between the different channels that are simulated, due to the random location of the users around the BS. This is an excellent result, given the speed improvement over the PF scheduler that our algorithm achieves, and is illustrated in Fig. 1, where a histogram of the sum rate as a percentage of the PF sum rate is presented, for the case of a network of 10 users.

The effect of coalition size is also investigated. Results show that sum rate increases as the size of the coalition increases, since a larger coalition size offers a wider range of subcarrier group permutations among coalition members and allows for multiuser diversity benefits to be harvested in a more efficient way. This is illustrated in Fig. 2, for the case of three networks consisting of 6, 8 and 10 users respectively. It should be noted that increasing the coalition size to values larger than 5, even for networks with a large number of users, carries a significant penalty, as a much greater number of permutations needs to be tested. If *permutation step* is increased to counter this effect, the benefits of choosing a larger coalition size will be diminished.

To investigate the fairness achieved by the proposed algorithm we calculate Jain's fairness index (eq. 7):

$$Fairness = \frac{(\sum_{i=1}^{n} R_i)^2}{(n \times \sum_{i=1}^{n} {R_i}^2)} \tag{7}$$

R_i represents the data rate achieved by user *i* over the whole simulation (i.e. 2000 realizations or 2 seconds) of the channel and *n* is the number of users in the channel. The value for *Fairness* ranges between 0 and 1, with a result equal to 1 indicating that all users achieve the same data rate. Results show that Jain's index achieved by the proposed algorithm (when the coalition structure with the lowest Jain's index value is selected) is consistently between 90% and 105% of Jain's index of the PF scheduler, with the average value being 99.5%.

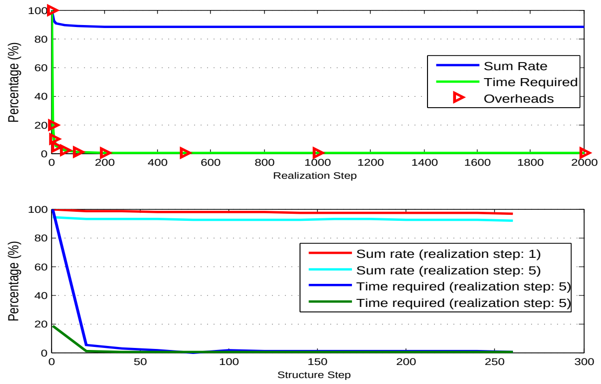

Fig. 3. Effect of 'Realization' and 'Structure' step

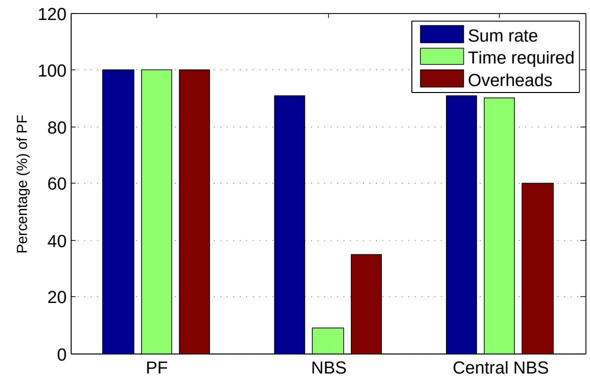

Fig. 4. Time & overheads comparison

B. Algorithm Efficiency & Overheads

A key benefit of the approach used in this paper is the ability to select parameter values (i.e. permutation, realization and structure step) that significantly reduce time and overhead requirements, while retaining a very large part of the achievable sum rate. A sample of this trade-off is presented in Fig. 3, where results from a network with 10 users are presented. Similar gains can be observed when compared against the PF scheduler and against the same, but centralized, NBS scheduler. This is illustrated in Fig. 4, again for a network consisting of 10 users. Time measurements have been made during simulations and results reflect the expected values, as calculated for NBS and PF respectively:

$$
\begin{aligned}
t_{NBS} &= R \times st \times s \times \\
&\quad [P \times (N_{group} \times add. + mult.) + compar.] \\
t_{PF} &= R \times N_{sub} \times N \times \\
&\quad [(ws + 1)\, P \times add. + div + compar.]
\end{aligned}
$$

These equations calculate the number of operations needed for the allocation process and *ws* represents the window size (measured in subcarriers) used for the PF scheduler. The difference between the distributed and centralized NBS is that in the distributed case the operations are off-loaded to the users (instead of the BS) and so the process is significantly sped up.

Overheads have been calculated using eq. 8, considering the achievable data rate of a single user over a single subcarrier group as the 'overhead unit':

$$
o = \begin{cases} P \times (s-1) \times st \times p \times R, & \text{for dist. NBS} \\ N \times N_{groups} \times R, & \text{for centr. NBS} \\ N \times N_{sub} \times R, & \text{for PF} \end{cases} \tag{8}
$$

P is the number of permutations tested, *s* the coalition size, *st* the number of coalitions formed, *p* the number of coalition structures tested, *R* the number of times the algorithm was repeated in a single simulation (i.e. 2000 realizations), *N* the total number of users, N_{sub} the number of subcarriers and N_{groups} the number of subcarrier groups. In these calculations we choose to ignore the beaconing overheads, as beaconing takes place significantly less often than the allocation process and also requires much less information to be transmitted.

VII. Conclusions

A fast and fair distributed subcarrier allocation algorithm was presented in this paper. By utilizing the concept of coalitions and bargaining, this algorithm achieves a sum rate that can be tuned to be a close match (up to 90%) to the rate offered by the widely used PF scheduler. The achieved fairness almost replicates proportional fairness, providing a fair operating point for all users. The ability to tune parameters such as *permutation*, *realization* and *structure step* minimizes execution time and overheads and, therefore, makes the proposed algorithm suitable for implementation in real-time systems and allows it to scale up well for networks with a large number of users.

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