Asset Pricing and Asymmetric Reasoning*

Elena Asparouhova†

University of Utah

Peter Bossaerts

University of Melbourne and University of Utah

Jon Eguia

University of Bristol

William Zame

UCLA

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†Corresponding author: Elena Asparouhova, Tel: +1-801-587-3975, E-mail: e.asparouhova@utah.edu, Address: David Eccles School of Business, University of Utah, SFEBB 7105, Salt Lake City, UT 84112

ABSTRACT

We present a theory and experimental evidence on pricing and portfolio choices under asymmetric reasoning. We show that under asymmetric reasoning, prices do not reflect all (types of) reasoning. Some agents who observe prices that cannot be reconciled with their reasoning switch from perceiving the environment as risky to perceiving it as ambiguous. If ambiguity averse, these agents become price-insensitive and no longer influence prices directly. We present the results of an experiment and report that consistent with the theory i) mispricing decreases as the fraction of price-sensitive agents increases, and ii) price-insensitive agents trade to more balanced portfolios.

JEL Classification: G11, G12, G14

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I. Introduction

This paper explores the extent to which asymmetric reasoning is reflected in asset prices and in the individual choices that support those prices. To do this we conduct a series of laboratory experiments in which agents trade assets whose payoffs depend (in a known way) on the (laboratory) state of the world. The prior probability distribution over states of the world and a public signal about this distribution are available to all investors. If all investors reasoned correctly, they would all use Bayes' Rule to infer the true posterior distribution over states of the world, and hence the true distribution of asset payoffs. However, we create an environment in which application of Bayes' Rule is difficult and some investors do not apply it correctly; their reasoning is incorrect.[1] In such an environment, the question we ask is whether and to what extent the correct/incorrect reasoning of various investors is reflected in asset prices. Do prices reflect both correct and incorrect reasoning?

Our experimental findings – supported by a simple theoretical model – demonstrate how prices do not reflect the opinions of all investors who reason incorrectly. The observed portfolio choices of investors and our theoretical model suggest an explanation: confronted with prices that are at odds with their view of the world, investors who do not reason correctly suspect that they may be wrong, and therefore view the financial prospects as sufficiently *ambiguous* – rather than simply *risky* – so that they choose not

[1] That many individuals make errors in Bayesian updating – even in environments much simpler than the ones we construct – has been confirmed in numerous experiments; see for instance Kahneman and Tversky (1973); Grether (1992); El-Gamal and Grether (1995); Holt and Smith (2009).

to be exposed to the perceived ambiguity. As a result, their incorrect reasoning is not reflected in prices.[2]

Ambiguity aversion matters for pricing because the behavioral consequences of ambiguity aversion are quite different from the behavioral consequences of risk aversion. In particular, ambiguity aversion may lead some investors to avoid ambiguity altogether by choosing portfolios whose payoffs are constant across the ambiguous states. Such investors make choices that are independent of prices (for a range of prices that depends on how ambiguity averse they are and how much ambiguity they perceive). If this is the case then asset prices for securities whose payoffs are not constant across the ambiguous states will be determined by investors who perceive only risk, not ambiguity, and who therefore make choices that are dependent on prices. It is important to note that risk aversion will almost never lead an investor to choose a riskless portfolio, so that the choices of investors who are simply risk averse will be reflected in prices.[3]

Ambiguity aversion, like risk aversion, is thought of as a common and immutable characteristic of individuals, perhaps even genetically pre-determined (Camerer and Weber, 1992; Cronqvist and Thaler, 2004; Kuhnen and Chiao, 2009) However, the *perception* of ambiguity may arise depending on context. Here, we propose that the perception of ambiguity is triggered endogenously when we present investors with difficult updating

[2]Ambiguity is sensed when one is not sure about the true probabilities; risk, on the other hand, is a situation where one knows the probabilities objectively (Ellsberg, 1961).

[3]A caveat must be understood here. An agent who does not purchase a particular security does not directly affect the price of that security, but might indirectly affect the price because his/her holding of other securities affects supplies; hence the prices of all securities might be different from what they would be if that agent were entirely absent from the market.

problems: while objectively the situation involves only risk, a subject who has difficulty solving the problems could have her confidence undermined, and may perceive ambiguity if her own solution to the updating problem were very different from the solution suggested by the market through the market price. Our proposal is inspired by an earlier evidence that perception of ambiguity can be induced by mere confrontation with a more authoritative source. Fox and Tversky (1995) find strong empirical support for the hypothesis that "people's confidence is undermined [...] when they compare themselves with more knowledgeable individuals" and they argue that "this contrast between states of knowledge is the predominant source of ambiguity aversion." Our conjecture is that the market plays this role of an authority with higher knowledge – whether it does is an empirical question, and our experiments speak to this question.[4]

We develop an equilibrium theory of asset pricing with agents who reason asymmetrically. Our assumptions on individual behavior are adapted from the decision-theoretic model of learning by Epstein and Schneider (2007), who explain that decision-makers' confidence about the environment can change – along with their beliefs – in light of new information. In their theory, each agent's perception of ambiguity is dynamic and contextual. Our model uses a static snapshot of this idea. Epstein and Schneider (2008) apply their theory of learning to asset pricing, using a single representative trader. This trader is initially not ambiguous (as she has a single prior) but then observes an ambigu-

[4] An alternative theory that would lead to the same behavior and would also appeal to the comparative ignorance argument is that of Chew and Sagi (2008). A trader who doubts her Bayesian inference would prefer sources of uncertainty that do not depend on the Bayesian inference in question, and hence, like an ambiguity averse agent, will attempt to avoid the ambiguity.

ous signal about the asset (in our case, a confusing price, more generally a signal drawn from a set of likelihoods), and this signal induces the trader to take into consideration multiple priors. We derive the implications for equilibrium prices as financial markets aggregate the asymmetric reasoning of multiple agents, some of whom behave as described by Epstein and Schneider (2007, 2008), while others remain standard expected utility maximizers who use a single prior and perceive only risk, not ambiguity. Our model guides both the experimental design and the empirical approach. In the choice of a difficult updating task, we take inspiration from the well-known "Monty Hall problem."[5] The updating problems used in the experiment are even more involved than the Monty Hall problem. To illustrate our experimental design, consider a setting where one state (Arrow) security is traded.[6] The security pays if a card drawn from a deck at the end of trading is red. Initially, the deck contains two black cards (one spade and one club) and two red cards (one heart and one diamond). One of the cards is randomly chosen and discarded before trade starts, but subjects are not informed about which one. Subjects are then allowed to trade one of the two securities for six minutes. Trading is halted in minute three, when one card is picked out of the three remaining cards. This pick is not

[5]For a detailed overview of the problem, see http://mathforum.org/dr.math/faq/faq.monty.hall.html. Monty Hall was the host of a popular weekly television show (aired in the 60's) called "Let's Make a Deal." In one portion of the show, Monty would present the contestant with three doors, one of which concealed a prize. Monty would ask the contestant to pick a door; after which Monty – who knew which door concealed the prize – would open one of the two remaining doors, never revealing the prize. Monty would then offer the contestant the opportunity to switch to the other unopened door. Updating correctly demonstrates that the probability that the original door conceals the prize is 1/3 – as it was initially – so that switching dramatically increases the probability of success. However, many contestants – and others – update incorrectly and believe that the probability that the original door conceals the prize is 1/2, so that switching makes no difference.

[6]This is the second setting listed in Table 2.

fully random: the heart card is never picked and randomization is only over the other suits. The picked card is revealed to the subjects and discarded, thus leaving a deck of only two cards. After this, trade resumes until the termination time, at which point one of the remaining two cards is drawn, and the color of this card determines asset payoffs. Subjects are fully informed about the details of this procedure to determine payoffs before trade starts, and this information presents them with quite a complicated Bayesian updating problem.[7]

The equilibrium implications of our conjecture that investors lose confidence and sense ambiguity when confronted with dissonant prices in the marketplace are as follows. Such investors view asset payoffs as ambiguous, and this leads them to trade to a portfolio that does not expose them to the ambiguity for any price within a range, where this range depends on the investor's ambiguity aversion and the perceived ambiguity. Because they have become price-insensitive, these investors do not contribute directly to the determination of the security prices. In contrast, agents who are confident in their ability to update behave as if they know the true probabilities over outcomes, and hence regard asset payoffs as merely risky rather than ambiguous. That is, payoffs are perceived to follow a known distribution. Risk aversion affects the choices of such investors and hence choices will be sensitive to prices. For example, they will not choose riskless portfolios unless prices are consistent with risk neutrality. Because of their sensitivity to prices,

[7]Indeed, it requires careful calculation (that may amuse the reader) to determine the prior probability before trading begins that the card drawn at the end of trading will be red (hint: it is strictly greater than 0.5), or the posterior probability after the mid-trading halt.

these investors do directly contribute to the determination of the security prices, as is standard in theories of asset pricing.

Subjects are at times unreceptive to the correct way of reasoning in Monty Hall problems even after experience that should have revealed their mistake (Friedman, 1998); some agents who are confident in their updating ability may nevertheless be wrong, and hence we do not expect prices to conform perfectly to theoretical predictions that would obtain if all investors updated correctly. We shall assume that only agents with beliefs that are sufficiently close to those reflected in prices remain confident. In addition, we assume that subjects who reason correctly believe that their reasoning is correct and do not perceive ambiguity. This seems to us to be reasonable but whether it is in fact true is an empirical question. Providing support for this assumption, Halevy (2007) finds that only 4% of agents who correctly reduce compound lotteries sense (and avoid) ambiguity in the Ellsberg experiment (whereas a full 95% of agents who incorrectly reduce compound lotteries exhibit this behavior).

In order to obtain clearer predictions, we consider two environments both in the theory and in the experiments. In the first, there is no aggregate risk; in the second, there is aggregate risk. When there is no aggregate risk, and if investors all update correctly, equilibrium prices should be risk neutral with respect to the true probabilities (independently of individual preferences). If investors do not all update correctly yet continue to react to prices then prices may be different from risk neutral prices. Hence this treatment provides a convenient test of price predictions. However, it does not provide a convenient test of portfolio predictions, because it may be impossible to distinguish between investors who update correctly and investors who are ambiguity averse: if prices equal expected payoffs at correct probabilities, then both choose portfolios that are devoid of risk. When there is aggregate risk, equilibrium prices depend on the way individuals update and on individual preferences, so we cannot readily test equilibrium price predictions. However, the presence of aggregate risk provides a useful test of individual behavior: we predict that investors who update correctly choose to hold some aggregate risk (provided prices are not risk neutral), while investors who are ambiguity averse refrain from holding any risk at all regardless of prices (within a range).

Our central predictions are: i) if there is no aggregate risk, prices should be closer to expected payoffs based on the correct Bayesian inference as the proportion of subjects who cannot make the correct Bayesian inferences decreases; the proportion of price-insensitive subjects provides a lower bound to the latter number; ii) subjects who cannot update correctly should hold ambiguity neutral portfolios (in our setting, these correspond to balanced portfolios), while agents who can update should hold increasingly diverging portfolios as the aggregate risk in the economy grows. These predictions are fully born out in the data.

Our results shed light on recent experimental findings of Kluger and Wyatt (2004) who also used a design suggested by the Monty Hall problem. Kluger and Wyatt found that if at least two among the six subjects in their experimental market updated correctly, then prices agreed with theoretical predictions. The authors explain this finding as resulting from Bertrand competition among those who update correctly. It seems to us that this explanation begs the question: surely subjects who update *incorrectly* Bertrand compete as well? And if subjects who update incorrectly Bertrand compete, why wouldn't this competition lead to the wrong prices? We provide an alternative explanation: those who cannot compute the right probabilities perceive ambiguity, and, as a result, become infra-marginal.

The idea that investors often reason incorrectly (or make cognitive mistakes) has been a cornerstone of Behavioral Finance. Among others, investors mis-calibrate their beliefs through overconfidence (Daniel et al., 1998, 2001), fall prey to gambler's and hot hand fallacies (Rabin and Vayanos, 2009), or do not fully grasp their environment, displaying representativeness bias and conservatism (Barberis et al., 1998).[8] To study how these examples of incorrect reasoning impact prices, the representative agent framework is employed, and hence, the theory effectively assumes that all agents are equal. This excludes asymmetry in reasoning. One of our contributions is to explicitly allow for such asymmetry.

There also exists a large literature that studies competitive equilibrium when agents are Bayesian (and hence, make no mistakes in updating) but employ different beliefs. One variant assumes that agents use different likelihoods to interpret the signals they receive. The differences in likelihoods could be viewed as a way to model asymmetric

[8] Both the model in Rabin and Vayanos (2009) and Barberis, Shleifer, and Vishny (1998) fall in the category of quasi-Bayesian models, whereby agents apply Bayes rule to all but one or two parameters of the model.

reasoning, and as such, the approach is relevant for our study. Harris and Raviv (1993), for instance, study the impact on equilibrium prices when investors stick to their likelihoods, and as such, never agree. But why should investors continue to disagree? Our contribution is to model what happens when investors introspect when confronted with the fact that they disagree. Another approach is to assume that all investors use the same likelihood but start from different priors. This too could be viewed as a way to model asymmetric reasoning, as long as the differences in priors cannot be explained by differences in information (Scheinkman and Xiong, 2003; Basak, 2005; Cao and Ou-Yang, 2009; Hong and Sraer, 2012). However, in a market with differing opinions, would rational investors not feel discomfort when prices (which reflect some weighted average of the beliefs) do not conform to their beliefs? We model how agents behave when they confront discordant prices, and we analyze the consequences of their behavior.

Absent shortsale restrictions or other constraints on trading, if investors persist in their beliefs, prices reflect an average belief (Williams, 1977; Diether, Malloy, and Scherbina, 2002). That is, every belief contributes to determining the prices. Since at most one set of beliefs can be correct (at times no beliefs may be correct), if errors are systematic across agents, this implies that prices must be wrong (prices will be right only if errors cancel out exactly). However, this is not satisfactory from an empirical point of view: Fama (1998) and Malkiel (2003), among others, have argued that the frequency of persistent pricing anomalies is low and on balance the evidence is in favor of approximately correct pricing. Our contribution is to show how prices are largely immune to systematic errors by a large fraction of investors. In our environment prices affect these investors' beliefs, rather than being affected by them.

Traditional game theory has long argued that it may not be reasonable for two agents to knowingly differ in their beliefs, i.e., to "agree to disagree." Instead, if there are any differences in beliefs, they must eventually be traced to differences in information – this is the "common prior assumption" of Harsanyi (1967). Hence, in traditional game theory, there is no asymmetric reasoning. Following Morris (1995), more recent game theory has allowed for deviations from the strict common prior assumption.[9] One alternative way in which game theory has embraced differences in reasoning, is by considering agents who disagree on what is the set of conceivable states (Karni and Vierø, 2013). In these attempts to accommodate disagreement in a strategic setting, at one point one must confront the fact that two agents who disagree ought to realize that only one of them (at most) can be correct. In the multi-stage speculation game of Biais and Bossaerts (1998), for instance, rational investors meet others at regular points in time, and every time they disagree (with probability 1), but no-one ever stops to wonder whether this makes sense. Our contribution is to propose a resolution if players were to wonder.

An alternative approach to tackling the difficulty of having two rational agents agreeing to disagree is to limit belief differences to events whose probabilities cannot be verified.

[9] Most work focuses on how (Bayesian) agents with different priors can end up with polarized (divergent) posterior beliefs even if they observe the same sequence of signals (Dixit and Weibull, 2007; Acemoglu, Chernozhukov, and Yildiz, 2009; Sethi and Yildiz, 2012). Both the idea that disagreement cannot occur if agents observe the same information and the idea that agents can become more polarized have been critized, but this is beyond the scope of our study. See Andreoni and Mylovanov (2012) and Baliga, Hanany, and Klibanoff (2013).

In this vain, Anderson and Sonnenschein (1985) present a model where beliefs are formed by combining models with data. If agents have access to the same data, their beliefs are allowed to deviate only to the extent that their models differ. Kurz, Jin, and Motolese (2005) assume that all agents have access to the same countably infinite, stationary time series, so that they can only disagree about the probabilities of non-ergodic events.

The remainder of this paper is organized as follows. Section II presents the theory and the empirical implications. Section III describes our experiments in detail. Section IV presents the empirical results. Sections V and VI discuss our results in light of alternative explanations and conclude.

II. Theory and Empirical Implications

In this section we present a simple asset market model that unfolds over two dates: trade takes place only at date 0; consumption takes place only at date 1. There is a single consumption good.

Let there be a continuum of agents, indexed by $i \in [0, 1]$, two assets R and B, or Red and Black stock, and two states of the world, r and b. At date 0 the realization of the state is not known to the agents. At date 1 agents learn the realization of the state, securities pay off, and consumption takes place. The two assets are Arrow securities: In state $j \in \{r, b\}$, asset $J \in \{R, B\}$ pays one unit of wealth, and the other asset pays no wealth.

Let π_r be the probability that state r occurs, and $\pi_b = 1 - \pi_r$ the probability that state b occurs (note that π_j is equal to the expected value of asset J). Public information is available with which to compute π_j. Agents use different ways to obtain this number based on the publicly available information. As a result, beliefs about π_j may not agree. Let π_j^i be the subjective probability that state j occurs, as calculated by agent i.

We assume that a proportion α of all agents use the correct reasoning, i.e., can compute the true probability. Without loss of generality, we assume that these are the agents with highest index i: $\pi_r^i = \pi_r$ for $i \in [1-\alpha, 1]$. The rest of the agents $i \in [0, 1-\alpha)$ compute the probability of state r incorrectly. Their mistake is proportional to the value of their index i, and hence, all mistaken beliefs are below the correct belief: agent i has a subjective probability

$$\pi_r^i = \underline{\pi} + \frac{\pi_r - \underline{\pi}}{1-\alpha} i,$$

where $\underline{\pi}$ is the minimal possible belief. The belief schedule is depicted in Figure 1.

Note that we have chosen the true probability to be on the boundary of the belief space, thus creating a setup where the agents with wrong beliefs have the strongest potential to influence asset prices. We could have assumed that mistaken beliefs are *above* correct beliefs, but this does not change the conclusions qualitatively. If correct beliefs are somehow *in the middle*, then wrong beliefs above and below it would cancel, and hence, wrong beliefs would not have as much an impact.

As to the total supplies of the assets R and B, we consider two treatments. In Treatment I, the aggregate endowment of assets R and B is the same, so there is no aggregate risk in the economy. In Treatment II, there is aggregate risk because supplies of R and B are different.

In Treatment I, at date 0 each agent is endowed with an equal number of R and of B, without loss of generality, one unit of each. One can think of each agent as the aggregation of heterogeneously endowed agents who share the same beliefs π_r^i. In Treatment II, there are more units of B than of R. In the experiments, for each unit of R, the aggregate agent holds 1.16 units of B.

Let w_i be the wealth of agent i at date 1, after the state of the world is revealed. For simplicity, assume a logarithmic form for the utility that agent i derives from final wealth, i.e., $u(w_i) = ln(w_i)$.

Agents may trade their endowments at date 0. Let p_R be the market prices of asset R at date 0. Absence of arbitrage dictates that the price of asset B must be $p_B = 1 - p_R$. Let (B_i, R_i) be the holdings of Black and Red securities after agents i trades.

We first derive the equilibrium choices and prices assuming that all agents maximize expected utility with their own subjective beliefs, undisturbed by prices that may suggest that their beliefs are wrong. That is, the equilibrium assumes that agents "agree to disagree." Subsequently, we determine what happens when agents with incorrect reasoning face prices that challenge their beliefs. We can compute the equilibrium explicitly if we know the distribution of agents in $[0,1]$. For convenience, we assume here that this distribution is uniform.

Agreeing To Disagree

Treatment I

In Treatment I, the initial wealth of i is $w_i^0 = p_R 1 + (1-p_R)1 = 1$, so her optimization problem is

$$\max_{\{R_i,B_i\}} \{\pi_r^i \ln(R_i) + (1-\pi_r^i)\ln(B_i)\ \} \quad s.t. \quad p_R R_i + (1-p_R)B_i = 1. \tag{1}$$

The first order conditions for optimality imply that

$$R_i = \frac{\pi_r^i}{p_R}, \qquad B_i = \frac{1-\pi_r^i}{1-p_R}.$$

Hence for any given price vector, the relative demand of agent i for asset R (as a fraction of the total demand for assets R and B) is increasing in the subjective probability π_r^i. The vector of all subjective probabilities by all agents determines the equilibrium prices.

If all agents correctly compute the true probability of state j, i.e., if α=1, in the absence of aggregate uncertainty all agents trade so as to attain a balanced portfolio of 1 unit of each security. This ensures equilibrium, and from the above first-order conditions, the equilibrium prices can be deduced to be: $p_R = \pi_r$ and $p_B = 1 - \pi_r$.

If instead $\alpha < 1$ and if all agents maximize expected utility based on their subjective beliefs then the equilibrium prices will aggregate the beliefs of all agents. Since the correct belief about the red state are the most optimistic, it follows that the equilibrium price of the Red security will be below the correct belief, $p_R < \pi_r$. As a result, $p_B > \pi_b$.

<u>Treatment II</u>

In Treatment II, there is aggregate risk because the Red asset R is scarcer than the Black asset B. If all agents correctly compute the true probabilities, the equilibrium price of Red will be above the probability of the red state (and hence, the expected payoff on the Red security), so that demand for Red is dampened: $p_R > \pi_r$. The resulting ratio $\frac{p_R}{p_B}$ will be higher than the odds ratio of the two states $\frac{\pi_r}{\pi_b}$. If instead $\alpha < 1$, the equilibrium prices aggregate the beliefs of all agents, causing p_R to be lower than it would be if all agents updated correctly. The exact equilibrium price that is determined by the magnitude $1 - \alpha$, the relative scarcity of Red and the risk attitudes of the agents (represented here by logarithmic utility).

Disagreement Leads To Doubt And To a Perception of Ambiguity

The above equilibrium assumes that, when confronted with prices that contradict their computations, agents continue to use their subjective beliefs in determining optimal demands. In what follows we drop this very assumption and hypothesize instead that from the agents who do not hold the correct belief π_r, only those whose beliefs are within ϵ of the market price continue to use their subjective probabilities. Each of the rest of the agents, confronted with the dissonance between the market price and her subjective probability, realizes that she may have made a mistake and computed the wrong probabilities. We conjecture that in these circumstances agents perceive ambiguity because they no longer trust their own computations, while doubting the belief reflected in prices (because they cannot manage to justify these beliefs either). As a result, rather than agreeing to disagree with the market, the agents completely disregard their prior and conclude that no reasonable beliefs can be established, leading them to perceive ambiguity.

We make the technical assumption that $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$. This puts an upper bound on the fraction of agents with incorrect beliefs who nevertheless agree to disagree (with the price). This assumption ensures that a strictly positive fraction of agents become price-insensitive in equilibrium. Otherwise we would be back in the above equilibrium, where all agents "agree to disagree." We model choice under ambiguity using the maxmin decision rule of Gilboa and Schmeidler (1989), which implies that agents are ambiguity averse.[10]

[10] Alternatively, one could use the theory in Ghirardato, Maccheroni, and Marinacci (2004) to model the behavior of agents who face ambiguity. Their utility representation $\alpha - \max\min$ utility function $U_i(R_i, B_i) = \alpha \min\{u(R_i), u(B_i)\} + (1-\alpha)\max\{u(R_i), u(B_i)\}$, where the coefficient α measures the degree of ambiguity aversion (not to be confused with α in this paper, which measures the fraction of agents who correctly compute probabilities); $\alpha = 1/2$ corresponds to ambiguity neutrality, and $\alpha = 1$ is the extreme degree of ambiguity aversion as in Gilboa and Schmeidler (1989). With this utility function, agents are price insensitive for prices in the range $[1-\alpha, \alpha]$. There are other types of preferences that generate ambiguity aversion. Yet the empirical findings in this paper, and those in Bossaerts et al. (2010), demonstrate that a significant fraction of subjects are price-insensitive, while their choices reveal that they avoid ambiguity. As such, the data favor a model of kinked preferences, of which the $\alpha - \max\min$ is an important member.

An agent with max min preferences who believes that π_r may take any value in $[0, 1]$ solves the optimization problem:

$$\max_{\{R_i,B_i\}} \min_{\pi\in[0,1]} \{\pi \ln(R_i) + (1-\pi)\ln(B_i)\} \; s.t. \; p_R R_i + (1-p_R)B_i = 1. \tag{2}$$

If $R_i > B_i$, then $\min_{\pi\in[0,1]}\{\pi \ln(R_i)+(1-\pi)\ln(B_i)\} = \ln(B_i)$. Similarly, if $R_i < B_i$, then $\min_{\pi\in[0,1]}\{\pi \ln(R_i) + (1-\pi)\ln(B_i)\} = \ln(R_i)$, so the optimization problem simplifies to

$$\max_{\{R_i,B_i\}} \min\{\ln(R_i), \ln(B_i)\} \; s.t. \; p_R R_i + (1-p_R)B_i = 1. \tag{3}$$

From here it immediately follows that the agent seeks a portfolio with $R_i = B_i$ under any prices p_R and $p_B = 1 - p_R$.[11]

Importantly, demands of ambiguity averse agents who perceive ambiguity do not depend on relative prices. As a result, these agents are price-insensitive.

We could consider a more general version of the optimization problem (2), assuming that the set of priors over π held by agents who perceive ambiguity is smaller than $[0, 1]$, so that there exist π_L, π_H such that $0 \le \pi_L \le \pi_H \le 1$ and agents who distrust their own computations solve the optimization problem

$$\max_{\{R_i,B_i\}} \min_{\pi\in[\pi_L,\pi_H]} \{\pi \ln(R_i) + (1-\pi)\ln(B_i)\} \; s.t. \; p_R R_i + (1-p_R)B_i = 1. \tag{4}$$

[11] This result generalizes if we drop the assumption of logarithmic utility. Let $u(w_i)$ be any strictly increasing function. Ambiguity averse agents who solve $\max\min\{u(R_i), u(B_i)\}$ seek balanced portfolios (equal number of R and B securities).

In this case, an ambiguity averse agent seeks a balance porfolio with $R_i = B_i$ under any price $p_R \in [\pi_L, \pi_H]$. As a result, the agent is price-insensitive for the range of prices $[\pi_L, \pi_H]$. For simplicity, we follow the special case $[\pi_L, \pi_H] = [0, 1]$.[12]

In contrast, knowledgeable agents continue to submit optimal demands based on their own beliefs. With a fraction α of such agents, their demands for R add up to $q_\alpha = \int_{1-\alpha}^{1} R_i = \alpha \frac{\pi_r}{p_R}$. Thus, for any $p_R < \pi_r$ the knowledgeable agents create a total excess demand equal to $\alpha \left(\frac{\pi_r}{p_R} - 1 \right)$. Notice that knowledgeable agents are price-sensitive.

There is a third category of agents, namely those with incorrect reasoning but whose beliefs are within ϵ of the market price. We assume that these agents continue to optimize given their beliefs. Because $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ and $\alpha \leq 1$, it follows $\epsilon < \frac{\pi_r - \underline{\pi}}{2}$.

An equilibrium is such that agents who stick to their beliefs (either because they are correct in their calculations, or because their calculations are close to the market price) solve the optimization problem (1); agents who perceive ambiguity maximize expression (2); and the market clears at the equilibrium price so that the aggregate demand of each asset is one. Let $q(i, p_R)$ be the quantity of Red asset demanded by agent i at price p_R.

Definition 1. *An equilibrium is a price p_R^* and an individual demand function $q : [0, 1] \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ such that*

(i) $q(i, p_R) = \arg\max_{\{R_i\}} \{\pi_r^i \ln(R_i) + (1 - \pi_r^i) \ln(B_i)\}$ *s.t.* $p_R R_i + (1 - p_R) B_i = 1$ *for any i such that* $\pi_r^i \in [p_R^* - \epsilon, p_R^* + \epsilon] \cup \pi_r$.

[12] Notice that "agreeing to disagree" corresponds to the opposite special case in which $\pi_L = \pi_H = \pi_r^i$.

(ii) $q(i,p_R) = \arg\max_{\{R_i\}} \min\{\ln(R_i), \ln(B_i)\}$ *s.t.* $p_R R_i + (1-p_R)B_i = 1$ *for any* i *such that* $\pi_r^i \notin [p_R^* - \epsilon, p_r^* + \epsilon] \cup \pi_r$.

(iii) $\int_0^1 q(i,p_r^*)di = 1$.

Treatment I

Consider Treatment I and conjecture that prices will be sufficiently close to correct beliefs (we will verify later that this is true in equilibrium) such that $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$. Let $\underline{i}$ be the agent such that $\pi_r^{\underline{i}} = p_R - \epsilon$, that is, $\underline{i} = \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \epsilon - \underline{\pi})$. Therefore, the total excess demand generated by agents with (incorrect) beliefs sufficiently close to π_r equals:

$$\int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p} - 1\right) di.$$

In the Appendix (A.3), we show that, since $\pi_r^i = \underline{\pi} + i\frac{\pi_r - \underline{\pi}}{(1-\alpha)}$,

$$\int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p_R} - 1\right) di = \frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon).$$

Because $(\pi_r - p_R - \epsilon) < 0$, the excess demand is negative, i.e., the biased agents provide excess supply to the market.

In equilibrium the aggregate excess demand must be zero:

$$\frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon) + \alpha\left(\frac{\pi_r}{p_R} - 1\right) = 0.$$

In the Appendix (A.4), we prove the following.

Proposition 1. *An equilibrium exists. If $\alpha > 0$, this equilibrium is unique and the price of the Red security is*

$$p_R = \pi_r - \left(\sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{1-\alpha} \right)^2 + \epsilon^2} - \frac{\alpha(\pi_r - \underline{\pi})}{1-\alpha} \right).$$

Notice that equilibrium prices satisfy the conjectured restriction used to derive total excess demands of agents with incorrect beliefs within ϵ of the market price, namely, $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$. See Figure 5 for a depiction of the equilibrium.

The main comparative statics properties of equilibrium prices in Treatment I are as follows. Define $|\pi_r - p_R|$ to be the mispricing in the marketplace.

Corollary 1. *Mispricing decreases in α and increases in ϵ.*

Corollary 1 states that mispricing decreases with the fraction of agents who know how to correctly compute probabilities, and increases as more agents with incorrect beliefs "agree to disagree" even if their beliefs are more distant from the market price.

Neither the fraction α nor the critical distance ϵ are directly observable, however. We need an empirically more relevant statement; one that we can verify in experiments. There, we observe not only prices, but also subjects' choices and how these choices change when prices change. Therefore, we propose to translate the corollary into the following empirically verifiable statement. Let S denote the fraction agents who remain price-sensitive (either because they know how to correctly compute probabilities or their beliefs are within ϵ of the correct beliefs). S equals:

$$S = \alpha + (\epsilon + \pi_r - p_R)\frac{1-\alpha}{\pi_r - \underline{\pi}}.$$

Note that S is readily measurable: it suffices to identify which agents change their choices as prices change.

Corollary 2. *Mispricing decreases as S increases.*

Both Corollaries 1 and 2 are proved in the Appendix (A.5).

We check that our results are robust if ϵ varies across agents. Consider the simplest case, where there are two types of agents, one that percieve ambiguity only if their (subjective) probability is more than ε away from market prices, and another that perceive ambiguity only if their probability is more than δ ($\leq \varepsilon$) away. As proved in the Appendix (A.6), mispricing continues to decrease with the fraction of agents α who know how to correctly compute probabilities. Moreover, mispricing decreases as the number of price-sensitive agents increases. Thus, both Corollaries 1 and 2 continue to hold when the individual price-sensitivity cutoff points are heterogeneous across agents.[13]

[13]One could enrich the theory even further and allow heterogeneity in perceived ambiguity. The idea would be to let the amount of perceived ambiguity increase with cognitive dissonance, while assuming that ambiguity aversion (once it emerges) remains the same across agents. Perceived ambiguity can be reduced by limiting the set of probabilities that the agent considers possible. In this version of the theory, all agents up to a certain level (of cognitive dissonance) would still remain price-sensitive, though. Those beyond that level would become entirely price-insensitive. Since the pricing and allocation predictions depend solely on dichotomous price sensitivity, the empirical implications of this richer theory would remain the same.

Treatment II

In Treatment II, equilibrium choices of price-sensitive agents (those with correct beliefs, and those with incorrect beliefs that are nevertheless within ϵ of the truth) are affected by the unbalanced supply of Red and Black securities. Price-insensitive agents (those with incorrect beliefs more than ϵ away from the truth) choose to hold balanced portfolios (equal amounts of Red and Black securities), so only price-sensitive agents are willing to accommodate the imbalance in supply of Red and Black securities. The exact equilibrium prices depend on the relative supplies of Red and Black securities, and on the number and risk preferences of price-sensitive agents. We took the latter to be logarithmic, but any other (risk averse) preference would do in order to generate the asserted portfolio effects (the choices of price-insensitive agents do not depend on the posited utility function, as mentioned before). For any market prices, the price-sensitive agents collectively absorb all the risk, acquiring more units of Black asset than of Red asset. This implies that the individual equilibrium portfolio holdings of the price-sensitive agents are (generically) unbalanced.

In Treatment II, an equilibrium exists (the proof is available from the authors),[14] but pricing predictions are ambivalent because prices are affected not only by beliefs, but also by the imbalance in relative supplies of Red and Black securities, the exact number of price-sensitive agents, and their risk aversion. Equilibrium holding predictions are, however, unequivocal: price-sensitive agents choose to invest in unbalanced portfolios,

[14] All the additional material that we mention as "available from the authors" is publicly accessible at http://uleef.business.utah.edu/SuppInfo/AsyReasonSupp/ABEZSup.pdf.

while price-insensitive agents buy balanced portfolios (with an equal number of Red and Black securities).

Thus, our theory has three main testable predictions:

Hypothesis 1. *There are price-insensitive subjects, i.e., subjects whose choices do not change as (relative) prices change.*

Hypothesis 2. *In Treatment I, mispricing* $|\pi_r - p_R|$ *decreases with the fraction of price-sensitive subjects,* S.

Hypothesis 3. *In Treatment II, price-insensitive subjects hold more balanced portfolios than price-sensitive subjects.*

A Dynamic Interpretation of Asymmetric Reasoning

We posit that some agents who observe a signal (the market price) that clashes with their prior about the state of the world (based on their calculation of the probability that the state is Red), do not engage in the cognitively challenging task of updating to generate a posterior. Rather, they become confused, they disregard their previous reasoning about the strategic environment, and thereafter, uncertain about the correct probabilities, they perceive ambiguity and they optimize as ambiguity averse agents. Definition 1 defines an equilibrium as the static outcome of this process.

Our model of individual behavior is best interpreted as an adaptation of Epstein and Schneider's (2007) dynamic theory of learning under ambiguity. According to Epstein and Schneider (2007), agents are uncertain about a parameter that represents the state of the world, and are also uncertain about the process that generates signals about the parameter. This second source of uncertainty distinguishes their model from a traditional Bayesian-updating learning model. In our application, the parameter is π_r, the probability that the state is Red. The signal is the market price.

Epstein and Schneider (2007) argue that agents simultaneously entertain various theories about the environment. A theory consists of a prior probability distribution about the parameter, and an assumed process that generates signals given a parameter value. Observing a signal, an agent calculates a posterior about the parameter, *one posterior for each theory* under consideration. Then she evaluates each of the theories according to the likelihood that it generated the observed signal. The agent uses all the theories that perform sufficiently well in the likelihood test, as follows: given a choice, the agent evaluates the expected utility for this choice under each theory that performs well in the likelihood test, then takes the minimum across theories, and assigns this value to the choice. The agent, finally, picks the choice with the highest value. In subsequent periods, new signals lead to a re-evaluation of each theory in the likelihood test, and thus they can lead to different reasoning and choices.

We can interpret the individual behavior of the agents in our model as a special case of this model of learning, in which the unknown parameter is π_r, and in which the agents who compute probabilities incorrectly consider two alternative theories:

a) $\pi_r = \pi_r^i$ and p_r is distributed uniformly in $[\pi_r - \epsilon, \pi_r + \epsilon]$.

b) π_r is drawn from an unknown distribution with positive density in $[0, 1]$, and for any π_r, p_r is drawn from a uniform distribution in $[0, 1]$, so that nothing about π_r can be inferred from from the observed p_r.

As long as $p_r \in [\pi_r - \epsilon, \pi_r + \epsilon]$, the first theory performs better in the likelihood test, and thus the agent uses it to make choices that are consistent with expected utility maximization with posterior π_r^i. On the other hand, if $p_r \notin [\pi_r - \epsilon, \pi_r + \epsilon]$, the first theory is refuted, the agent discards it, and embraces the second one. Under the second theory the agent perceives the environment to be ambiguous and being ambiguity averse, the agent makes choices that are price-insensitive. (In repetition the agent may switch theories and become price-sensitive again).

Our model omits this dynamic process on the formation of individual decision-making reasoning, to present directly the market equilibrium end result, where market prices are driven by the aggregation of the individual behavior of all agents: some agents (those who compute probabilities correctly, and those whose incorrect computations are seemingly vindicated by the signal of the market price) reason as standard expected utility maximizers with their own beliefs about the state of the world; other agents (those who compute probabilities incorrectly, and whose computations are called into question by the market price) reason as ambiguity averse agents with maximal uncertainty about the state of the world.

III. Experiments

Sessions Overview

We conducted nine experimental sessions in total, six corresponding to Treatment I (involving markets with no aggregate risk), and three corresponding to Treatment II (with aggregate risk). The participants were undergraduate and graduate students from the following universities: (i) Caltech (one session), (ii) UCLA (four sessions, including the three Treatment II sessions), (iii) University of Utah (two sessions), (iv) simultaneously at Caltech and University of Utah with equal participation from both subject pools (two sessions).[15] Subjects received a sign-up reward of \$5, which was theirs to keep no matter what happened in the experiment. Each session lasted approximately two hours in total and the average earnings from participation were \$49 per subject (including the \$5 sign-up reward).

Twenty subjects participated in each of the nine sessions. Prior experimental research[16] indicates that this is sufficient for markets like ours, organized as a continuous double auction, to be liquid enough, so that the bid-ask spread is small. In our case it rarely exceeded two or three ticks (the tick size was set at 1 U.S. cent). All accounting in the experimental sessions was done in US dollars.

[15]We recruited participants from three different campuses in order to obtain greater variability in the subject pool and observed data. The experimental laboratories where we ran our experiments are: Caltech's SSEL, UCLA's CASSEL and the University of Utah's ULEEF.

[16]See Bossaerts and Plott (2002) and Bossaerts and Plott (2004).

Upon arrival at the experimental laboratory, subjects were seated in front of computer terminals and received a set of written instructions. Each session began by the experimenter reading aloud the instructions, while also projecting them on a large screen. During the instruction period, if subjects had any questions, they were asked to raise their hands, and the experimenter would answer the questions. No oral communication between subjects was allowed. They communicated their decisions via the computer terminals.

Following the instruction period was a practice session, where subjects familiarized themselves with the rules of trading and the trading software (described in more detail below). The instruction and practice trading periods lasted approximately one hour and concluded with a questionnaire to the subjects to ascertain they understood the trading mechanism and the payoff structure of the traded securities. There was a 15-minute break between the instructional period and the actual trading session, which lasted approximately 45 minutes.

Markets

The time in the actual trading session was split into eight intervals, which we call replications. At the start of each replication subjects received endowments consisting of units of three securities. Two of them were risky and one was risk free. The risk-free security, or *Bond*, always paid $0.50 at the end of the replication. The two risky securities were referred to as *Red Stock* and *Black Stock*. The liquidation value (the amount each risky security pays at the end of a replication) of Red Stock and Black Stock was either \$0.50 or \$0. Red and Black Stock were complementary securities: when Red Stock paid \$0.50, Black Stock paid nothing, and *vice versa*. Red Stock paid \$0.50 when the "last card" (to be specified below) in a simple card game was red (hearts or diamonds); Black Stock paid \$0.50 when this "last card" was black (spades or clubs). Thus, instead of being explicitly provided with the probability distributions of the securities' payoffs, the subjects were presented with the description of Card Game Situations that determined those probabilities. The experiment used four such Card Game Situations. In the sequence of eight independent replications each of the four different Card Game Situations was implemented exactly twice.

Except for the presentation of the securities' payoff probabilities, the session setup was that of a standard experimental market.[17] Trade took place through a web-based, electronic continuous open-book system called *jMarkets*.[18] A snap shot of the trading screen is provided in Figure 2.

Within each replication, subjects were initially endowed with Red (R) and Black Stock (B) as well as cash. The Bonds were in zero net supply, and subjects started without Bonds in their endowment portfolios. Because of the presence of cash, the Bond was a redundant security. Subjects were allowed to short sell the Bond if they wished. Short

[17]See, for example, Bossaerts et al. (2010) and Meloso, Copic, and Bossaerts (2009).

[18]This open-source trading platform was developed at Caltech and is freely available under the GNU license. See http://jmarkets.ssel.caltech.edu/. The trading interface is simple and intuitive. It avoids jargon such as "book," "bid," "ask," etc. The entire trading process is point-and-click. That is, subjects do not enter numbers (quantities, prices); instead, they merely point and click to submit orders, to trade, or to cancel orders.

sales of Bonds correspond to borrowing. Subjects could use such short sales to acquire more stocks.

Subjects were given an unequal supply of the two risky securities; some started with 12 units of R and 3 of B; while others started with 9 units of B and no units of R (but with more cash). In the six sessions corresponding to Treatment I, the aggregate endowment of R and B was equal, and hence, there was no aggregate risk in the marketplace (because the payments on Red and Black Stock were complementary). In the three sessions corresponding to Treatment II, there were more subjects with endowments tilted towards B than subjects with endowments tilted towards R, so in the aggregate there were fewer units of Red Stock than Black Stock, and hence, there was aggregate risk. Table 1 provides additional details of the experimental design.[19]

While subjects could freely trade Red Stock and Bonds, they were barred from trading Black Stock. This is an important experimental design feature. In Treatment I, where there was no aggregate risk, equilibrium allocations may be indeterminate if all securities could be traded freely. First, ambiguity averse agents would like to hold balanced portfolios no matter what the prices are. There are many such balanced portfolios: one of R (Red Stock) and B (Black Stock) each, two of each; three of each; etc. Second, price-sensitive agents would want to hold balanced portfolios as well when prices equal expected payoffs (i.e., carry no risk premium), which is possible in equilibrium because there is no aggregate risk. Again, there are many portfolios that are balanced. If the

[19]The Instructions for the experiment are provided in Appendix C.

market for B is closed, however, the only way to obtain balanced portfolios is to trade to positions in R that match one's holdings of B.

We consider it an important feature of our design that subjects had a reason to trade besides "agreeing to disagree" (effectively speculating that one's reasoning is better than that of others). Specifically, subjects were initially given an unequal supply of the two securities even in Treatment I. Because subjects are generally risk averse even for the relatively low levels of risk in our experiment (Holt and Laury, 2002; Bossaerts and Zame, 2006), there are gains from trading to more balanced positions. To put this differently: we would have seen trade even if all subjects agreed on how to correctly compute probabilities.

Subjects were allowed to short sell R, in case they thought R was overpriced. To avoid bankruptcy (and in accordance with classical general equilibrium theory), our trading software constantly checked subjects' budget constraints. In particular, subjects could not submit an order such that, if it and the subject's other standing orders were to go through, the subject would generate net negative earnings in at least one state. Only orders that were within 20% of the best standing bid or ask in the marketplace were taken into account for the bankruptcy checks. Since markets were invariably thick, orders outside this 20% band were effectively non-executable, and hence, deemed irrelevant. The bankruptcy checks were effective: no agent ever ended up with negative earnings in our experiments.

The Game

Each replication of a Card Game Situation consisted of two periods. Thus, with eight replications and two periods per replications, there were sixteen periods (numbered from 0 to 15) per experimental session. The (end-of-replication) liquidation values of Red Stock and Black stock were determined through card games played by a computer and communicated to the subjects orally and through the News web page; see Figure 3 for a snapshot of the page (the table on the page is filled gradually as information comes in). The card games were inspired by the Monty Hall problem.

One of the four Card Game Situations (the second one listed on Table 2) is as follows. The computer starts a new replication with four cards (one spades, one clubs, one diamonds, and one hearts), randomly shuffled, and face down. Cards are eliminated during the game and subjects are told the rules of card elimination. The color of the *last card* to be picked determines the payoffs of the two risky securities (if red, R pays, if black, B pays). The rules of card elimination determine the probability of the last card being red. Again, subjects are *not* given this probability explicitly, they are only told of the rules of the game. Trade starts for one 3-minute period. Upon the conclusion of this first period, trading is halted. At that point, the computer discards one card *without* revealing it to the subjects. Three cards remain. Then the computer picks one card from the three remaining cards, as follows. If the discarded card was hearts, the computer picks one card at random from the three remaining cards. If the hearts card is in the three remaining cards, the computer picks randomly from the other two (non-heart) cards. The card that was picked is then revealed to the subjects, both orally and through the News web page, and then discarded. Two cards remain. Trade starts again, for another 3 minutes (period 2 of the card game replication). At the end of this second period, after markets close, the computer picks one of the two remaining cards at random. This card, called the *last card* is then revealed and determines which stock pays. If the last card is red (diamonds, hearts) then Red Stock (R) pays \$0.50. If the last card is black, then Black Stock (B) pays \$0.50. Each subject's payoff is determined by his/her holdings at the end of the second trading period and by the color of the last card.

Four variations on this game (each replicated twice), were played. Table 2 provides details of the four Card Games Situations. They differ in terms of the number of cards discarded and/or revealed after the first period, and the restriction on which cards would be revealed. This provided a rich set of equilibrium prices (in Treatment I, where prices could be identified uniquely because there was no aggregate risk) and changes of prices (or absence thereof) after the first period revelation. The probabilities of the last card being red, both before and after the elimination between the two periods are listed on Table 2.

Detailed information about the drawing of cards is in the set of experimental instructions provided to the subjects (see Appendix C). In addition, before each period, the experimenter reiterated the drawing rules to be applied in the coming period, to be sure subjects knew which card game applied.

To determine to what extent subjects understood the instructions, the (oral) questionnaire included questions such as "In the game where the computer never reveals a red card after the first trading period, will you be surprised to see a black card revealed?" Or, "If the computer initially discards one card, and then shows one black card when it could have also shown diamonds, does the chance that the last card is black decrease as a result?"

During the trading periods, the News web page was projected on the screen at the times when cards were being discarded or drawn. For the rest of the time, the large screen was projecting the development of the order book as well as the transaction prices chart.

IV. Empirical Analysis

In this section we describe the data, report the number of price-insensitive subjects, assess the level of mispricing and how this correlates with the number of price-sensitive subjects, and then present a correlation study of portfolio choice and price sensitivity.

Raw Data

The data collected during the experimental sessions consists of all posted orders and cancellations for all subjects along with their transactions and the transaction prices for the Red Stock (R) and the Bond. (Remember that subjects could not trade the Black

Stock B.) Figure 4 displays the evolution of transaction prices for R in the six sessions for Treatment I (no aggregate risk). Time is on the horizontal axis (in seconds). Solid vertical lines delineate card game replications; dashed vertical lines indicate between-period pauses when the computer revealed one or two cards. Horizontal line segments indicate predicted price levels assuming prices equal expected payoffs computed with correct probabilities. Each star is a trade. Volume is large: over 1,100 trades take place typically during an experimental session, or one transaction per 2.5 seconds.

Figures 4b and 4c display trading prices in two experiments respectively featuring low and high number of price-sensitive subjects. In Figure 4b (Utah-1), observed transaction prices appear to be rather unreactive to changes in true payoff probabilities. In contrast, in Figure 4c, which corresponds to Utah-Caltech-1 (where half of the subjects are from Caltech, and half are from the University of Utah) prices are close to expected payoffs – reflecting correct probabilities. The comparison suggests that there might be cohort effects in our data.

In Utah-1 (Figure 4b), prices appear to be insensitive to variations in the card games. There were also a very large number of price-insensitive subjects (to be discussed later), indicating that the pricing we observe in that experiment may reflect an equilibrium with only ambiguity averse subjects: when there are only ambiguity averse subjects, equilibrium prices will not react to the information provided to subjects in the different card games, and *any* price level is an equilibrium. Notice that prices in Utah-1 indeed started out around the relatively arbitrary level of \$0.45 and stayed there during the entire experiment. A notable exception is period 2 of replication 1, when it was certain that the last card would be red and hence that the Red Stock would pay, because the two revealed cards were black. Prices adjusted correctly, proving that subjects were paying attention and able to enter orders correctly, so that neither lack of understanding of the rules of the game or unfamiliarity with the trading interface can explain the information-insensitive pricing in the other periods.

The third column of Table 3 reports how far observed prices were on average from the true expected payoffs in Treatment I, stratified by experimental session and Card Game (in U.S. cents; average across transactions).[20] The data reveal that there is a wide variability in mispricing, both across experimental sessions, Utah-1 producing the worst mispricing and Caltech-Utah-1 producing the best pricing, and across card games, with Card Game 2 producing larger mispricing than the other treatments. Formally, the median mispricing in Card Game 2 is significantly higher than that of Card Game 3 (p value of 0.052; Wilcoxon signed-rank test comparing the paired absolute mean mispricing across the six experimental sessions and the two replications of each Card Game per session), and also significantly higher than that of Card Game 4 (p value of 0.005).

Since in Treatment II there is aggregate risk, equilibrium prices differ from true expected payoffs even if all agents know how to compute correct probabilities because of the presence of a risk premium. When compared to the session from Treatment I with subjects from the same cohort (UCLA), average differences between transaction prices

[20]The third column of Table 4 reports the same for Treatment II.

and correct expected payoffs (available from the authors) are indeed higher in all card games. This suggests that a risk premium affected prices, and hence, that subjects were risk averse. While expected in view of past experimentation with multi-security asset pricing (Bossaerts and Zame, 2006), it is nevertheless comforting to confirm that agents are risk averse, as we assumed.

Hypothesis 1: Presence of Price-Insensitive Subjects

Column 4 of Table 3 reports the number of price-sensitive subjects in Treatment I. Price sensitivity is obtained from OLS projections of one-minute changes in a subject's holdings of Red Stock onto the difference between, (i) the mean traded price of Red Stock (during the one-minute interval), and (ii) the expected payoff of Red Stock computed using the correct probabilities.[21] Agents who cannot update probabilities correctly from cards discarded and displayed are assumed to perceive ambiguity when their priors divert too much from market prices. Because they are ambiguity averse, they are price-insensitive, which means that their choices produce a zero slope coefficient in the above regression. To be reacting rationally to price changes, all other agents should decrease holdings when prices increase, which means that the slope coefficients for these subjects ought to be negative.

[21] We assume that agents who perceive ambiguity in a card game replication perceive it throughout the entire replication. In principle, we could have enriched our analysis by considering the possibility that some learned halfway through the replication after we provided further information about the cards, in a way that could have changed their perception of ambiguity. Specifically, some subjects could have switched from perceiving ambiguity to perceiving risk (or vice versa) between trading periods. We did not consider this refinement because we did not have sufficient data (prices; changes in portfolio allocations) to confidently determine price sensitivity over each trading period separately.

The regression with which we determine price sensitivity suffers from a well-known simultaneous-equations bias, because total changes in holdings must balance out across subjects. Because slope coefficients must sum to zero, OLS estimates will be biased upward. See Appendix B for details. Because the bias is upward, we use a generous cut-off level to categorize our subjects as price-sensitive. In particular, we use a cut-off of -1.65 for the t-statistic of the slope coefficient to determine that a subject is price-sensitive. At the same time, we applied a conservative t-statistic level of 1.9 to categorize whether a subject is price-sensitive in the other direction, namely, she increases holdings of a security when prices increase (rather perversely, as mentioned before). Subjects with t-statistics between -1.65 and 1.9 are deemed price-insensitive.

For Treatment I, Table 3, fourth column, reveals that generally only a minority of subjects was price-sensitive and reacted to price changes in the right way (reducing holdings when prices increased). In some instances only *a single*, and in one occasion *no* subject was found to react systematically and correctly to price changes. The flip side is that there always were some subjects who were price-insensitive over the range of observed prices, and their number was at times substantial.

We do observe that a small fraction of subjects were price-sensitive in a perverse way: they tended to *increase* their holdings for increasing prices. See the fifth column of Table 3. There are two possible explanations of this finding. First, these are just type II errors: the subjects at hand are really price-insensitive, but sampling error causes the t-statistic to end up above 1.9. The second possibility is that we have identified subjects who are perversely price-sensitive. We could interpret their actions as reflecting *momentum trading* or *herding*: higher prices are interpreted as signaling higher future prices (momentum) or higher expected payoffs (herding). Our theory does not account for such trading behavior. Since we cannot determine which of the two possible explanations applies, we exclude subjects with t-statistics above 1.9 from the remainder of our analysis.[22]

The results on price sensitivity for Treatment II are reported in Table 4. The numbers (of price-sensitive, both correctly and incorrectly) are similar to those for Treatment I. The relevant counterpart in Treatment I is the session UCLA, which drew participants from the same subject cohort.

Hypothesis 2: Mispricing and Number of Price-Sensitive Subjects

Absent aggregate risk (i.e., in Treatment I), our theory predicts that mispricing (defined as the absolute difference between the market price and expected payoffs computed with correct probabilities) decreases with the number of price-sensitive agents (Hypothesis 2).

The bottom of Table 3 reports two correlations (for Treatment I) when price sensitivity is measured in two ways: (i) counting all subjects whose reaction to price changes is significantly negative ($t < -1.65$); (ii) counting only those subjects whose reaction to

[22]In a previous version of the paper (available upon request from the authors) we presented our results in two parts: one where we include the entire subject pool and one where we exclude those with significantly positive slope coefficients ($t > 1.9$). None of our conclusions are affected qualitatively by the exclusion of perversely price-sensitive subjects.

price changes is significantly negative and whose average change in holdings is correct given initial holdings. Both correlations are significantly negative ($p < 0.01$), lending support to Hypothesis 2.

The bottom of Table 4 reports the same correlations for Treatment II. Here, however, there is no specific reason for the correlations to be negative, because a risk premium may interfere with pricing. Mispricing as we defined it now not only captures incorrect computation of probabilities, but also risk aversion. While the correlations are negative, they are smaller, and insignificant ($p > 0.10$).

Hypothesis 3: Portfolio Choices Depend On Price Sensitivity

With Treatment II, we can test an implication of our theory for equilibrium portfolio holdings. Specifically, price-insensitive agents should be exposed to less risk than price-sensitive ones because price-insensitive agents perceive ambiguity and avoid it trading to balanced (risk-free) portfolios, while price-sensitive ones behave as in traditional asset pricing theory, and in equilibrium, will share the aggregate risk in the economy and choose risk-exposed portfolios.

A test of this hypothesis is important, because one alternative explanation for the empirical support for Hypothesis 2 is that price-insensitive subjects are simply noise traders, and hence, not necessarily ambiguity averse. The more noise traders in the market, the worse the conformity of observed prices with theoretical predictions. So, if true, the presence of noise traders would make the data look as if they are consistent with our theory, and Hypothesis 2 in particular. Support for Hypothesis 3, however, would speak against this explanation, because the hypothesis requires that price-insensitive agents do not make arbitrary choices, unlike noise traders.

We compute for each subject i and each Card Game j ($j = 1, ..., 4$) imbalances I_{ij} separately after the first and the second trading periods in each replication (since there are 20 subjects per session , $i = 1, ..., 120$ for the treatment without risk, and $i = 1, ..., 60$ for the treatment with risk). Portfolio imbalance is defined to be the absolute difference between the number of Red Stock and Black Stock in a subject's portfolio. The goal is to determine whether more price-sensitive subjects on average have larger imbalances in their portfolio holdings.

Given that each subject participates in several Card Games, it is possible that some subjects exhibit substantial co-variation in price sensitivity and portfolio imbalances across the four Card Games, while others may show no co-variation whatsoever. If we were to just look at the average relationship between price sensitivities and portfolio imbalances pooling individuals and Card Games, we might then find a significant relationship that was due only to the within-subject variation of only a subset of subjects. Our posited relationship of the effect of price sensitivity on portfolio imbalance is not subject-specific; it is to hold *between* (across) subjects. To test this relationship, we ought to filter for variability in the average portfolio imbalances that can be predicted from within-subject variability in price sensitivity across Card Games. Technically, we hypothesize that a subject's price sensitivity (averaged across Card Games), explains the intercept in the within-subject regression (across Card Games) of portfolio imbalances onto price sensitivities. Thus, we estimate the following two-level (random effects) model:

$$\begin{aligned} I_{ij} &= I_i + \eta_{ij}, \\ T_{ij} &= T_i + \xi_{ij}, \\ \eta_{ij} &= b_{within}\xi_{ij} + \delta_{ij}, \\ I_i &= a + b_{between}T_i + \epsilon_i, \end{aligned} \tag{5}$$

where T_{ij} denotes subject i's t-statistic of the slope coefficient in the price-sensitivity regression for Card Game j and δ_{ij}, ξ_{ij}, η_{ij}, and ϵ_i are (assumed) normally distributed mean-zero errors, $i = 1, ...120$ (60) for Treatment I (II). In addition δ_{ij}, ξ_{ij}, and ϵ_i are jointly independent. Note that both I_{ij} and T_{ij} are observed, while I_i and T_i are latent variables. T_i is the mean price-sensitivity for subject i, while I_i is the intercept for subject i of the within-subjects regression. We have allowed the deviations from mean imbalance across card game situations to depend on the (deviations from mean) price-sensitivity of the subject in each of those situations, as the third of the four equations above displays. On the between-subjects level, we would expect that the less price-sensitive on average an agent is (the higher the value of T_i), the lower this agent's imbalance, i.e., $b_{between}$ should be negative. Similarly, if in a given game situation a subject displays less price sensitivity than this subject's average, then in this situation the subject should also hold a more balanced portfolio, i.e., b_{within} is expected to be negative as well.

We estimate the model in (5) twice for each treatment, once with imbalances collected after the first trading period and second time with imbalances collected after the second trading period. Throughout we focus on $b_{between}$, which provides a filtered estimate of the relationship between portfolio imbalance of a subject and price sensitivity. We do not report the within-level parameter estimates (b_{within}) as none were ever significantly different from zero.[23] Throughout, we used robust[24] maximum likelihood estimation.

As prices move away from expected payoffs computed from true probabilities, risk averse agents with correct beliefs choose more unbalanced portfolios. Therefore, our test should have more power when mispricing (defined as the difference between transaction prices and true expected payoffs) increases. Consequently, we also present estimation results for the between parameters for a specification that factors in the level of mispricing, as follows:

$$\begin{aligned} I_{ij} &= I_i + \eta_{ij}, \\ MT_{ij} &= MT_i + \xi_{ij}, \\ \eta_{ij} &= b_{within}\xi_{ij} + \delta_{ij}, \\ I_i &= a + b_{between}MT_i + \epsilon_i, \end{aligned} \tag{6}$$

[23]That b_{within} is not statistically different from zero implies that δ_{ij} and η_{ij} are independently distributed. However, within-subject co-variability of price sensitivity and portfolio imbalance may not be estimated with much precision (i.e, b_{within} may be very noisy). For example, if a subject always exhibits about the same price sensitivity, the standard error of b_{within} will be very large.

[24]We follow White (1980) and use a robust covarince matrix estimator. The models (5) and (5) were estimated using the statistical package Mplus.

where $MT_{ij} = M_{ij} * T_{ij}$, and M_{ij} is the mean absolute mispricing in the trading period (first or second) of Card Game j of the session in which subject i participated and for which the imbalance was measured, $j = 1, .., 4$, and $i = 1, ...120$ (60) for Treatment I (II). The assumptions on the error terms are identical to those of model (5). As with model (5), we estimate the model in (6) twice for each treatment, once with imbalances and mispricing collected after the first trading period and second time with imbalances and mispricing collected after the second trading period.

The results on the between-level estimates for Treatment II are displayed in boldface in Table 5. The first column within Treatment II presents the estimates when imbalance is measured after the second trading period of each replication; the second column shows the estimates when imbalance is measured after the first trading period. All estimates are highly significant. The standard errors produce t-statistics ranging from 2.8 to 4.3, and hence, p-values below 0.005. R-squared's are also high. The predicted effect is already present at the end of the first trading period. By the end of the second trading period, the magnitude of the effect increases (although not significantly). The predictions are confirmed whether we adjust for (mean) absolute mispricing (as in model (6)) or not (as in model (5)). We also obtain qualitatively similar results if we perform simple OLS regression analysis (i.e., if we do not account for repeated subject observations but treat each subject observation as an independent one instead).

For reference, Table 5 also displays results for Treatment I. In this treatment, there is no aggregate risk. As a result, price-sensitive agents hold, in the aggregate, a balanced portfolio. While individual demand functions across price-sensitive agents vary with their prior, given that we assume that price-sensitive agents are risk averse, for reasonable parameter values (simulations are available from the authors), the imbalance that price-sensitive agents would like in their portfolio is negligible.[25] Given that assets must be traded in discrete units, the approximation to the nearest integer leads to a prediction that most or all price-sensitive agents (depending on parameters) acquire a balanced portfolio, and hence, their choices are indistinguishable from those of the price-insensitive subjects, who acquire a balanced portfolio at any price. Therefore, the predicted relation between price-sensitivity and imbalance is at most a second-order effect of negligible magnitude. Indeed, all (between-level) coefficients have the predicted sign, but are statistically insignificant; t-statistics implied by displayed standard errors range from 0.5 to 1, and R^2s are a fraction of those from Treatment II.

Consequently, our theory makes the right prediction across the two Treatments. When an effect is predicted, it is present (Treatment II, aggregate risk); when the effect is predicted to be at best negligible, it is not significant in the data (Treatment I, no aggregate risk). Overall, the data therefore provide strong evidence for the conjecture that price-insensitive agents behave in an ambiguity averse manner, and against the alternative that price-insensitivity merely reflects noise trading.[26]

[25] For instance, for $\pi_r = 0.75$, $\underline{\pi} = 0.5$, $\alpha = 0.5$ and $\varepsilon = 0.02$, the predicted imbalance averaged over all price-sensitive agents is 0.07 units; for price-insensitive it is 0; if agents can only acquire assets in discrete (integer) units, the nearest approximation for almost all price-sensitive agents is to balance their portfolio. In Treatment II, the average imbalance averaged over price-sensitive agents is 1.8 units, for price-insensitive it is 0.

[26] It may be argued that price-insensitive subjects are not really ambiguity averse, but just follow a rule of thumb, investing half their wealth in each of the risky securities. The findings in Bossaerts et al.

We repeated our analysis using ordinary least square (OLS) regressions. Any difference of the results would stem from the "within" level noise in our data. While none of the qualitative conclusions change, all coefficients decrease in magnitude, sometimes significantly, as expected. The results can be obtained from the authors upon request.

V. Discussion

Camerer and Loewenstein (2004) describe how behavioral research (i) identifies "normative assumptions or models that are ubiquitously used by economists, such as Bayesian updating"; (ii) identifies "anomalies–i.e., demonstrate[s] clear violations of the assumptions or model, and painstakingly rule[s] out alternative explanations"; (iii) uses "the anomalies as inspiration to create alternative theories that generalize existing models"; and (iv) constructs "economic models of behavior using the behavioral assumptions from the third step, derive[s] fresh implications and test[s] them." We have followed (i), (iii), and (iv). In this section we suggest alternative individual behaviors within our setup and counter the resulting aggregate predictions with the outcomes from our experiments. Our theory relies on the assumption that upon being confronted with evidence that contradicts, or is not in line with their (flawed) reasoning, subjects stop perceiving the world as risky and perceive it as ambiguous instead. This assumption leads to the main insight of our model, namely that (some of) those subjects become price-insensitive. Because

(2010) reject this interpretation, though. There, choices of price-insensitive agents do reveal ambiguity aversion, but cannot be explained by the proposed rule of thumb.

agents who Bayesian update correctly remain price-sensitive, pricing improves with the number of price-sensitive agents.

We consider the following plausible alternative conjectures:

a) There are subjects who are exogenously price-insensitive, i.e., their behavior is not a results of their failure to Bayesian update correctly. This conjecture would directly explain the existence of price-insensitive agents. Price-insensitivity could come from exogenous aspirations to trade to a balanced portfolio. Such exogenously determined behavior would explain the relationship between price-insensitivity and security imbalance in the treatment with aggregate risk. However, the empirical finding that price quality correlates with the number of price-sensitive agents is not predicted under exogenous price-insensitivity. Increasing or decreasing the number of exogenously price-sensitive/insensitive agents would not change the quality of prices, as the price-sensitive group's expected (knowledge) composition would be independent of its (relative) size.[27]

b) Agent who doubt their own Bayesian updating react by not trading; instead of aspiring to a balanced portfolio, they hold on to their initial (imbalanced) endowment. This conjecture would explain the existence of price-insensitive agents, as well as the observed correlation between price quality and the number of price-sensitive agents. It would not, however, be consistent with the finding that the imbalance in a subject's final portfolio positively correlates with that subject's price sensitivity. In fact, this conjecture would lead to the opposite prediction within our experimental setup: portfolio

[27] A formalization of this claim is available from the authors.

imbalance and price sensitivity should be negatively correlated. Furthermore, if those who are price-insensitive trade little or not at all, we should observe a positive correlation between number of trades and price sensitivity. While in the right direction, however, the relationship is not statistically significant.

Within the lines of endowment-dependent portfolio choices, another possibility is that beliefs are correlated with endowments. Subjects whose endowment is skewed towards Black (Red) stock could surmise that Red stock is scarce (abundant) in the aggregate and expect a higher (lower) market price for the Red security. Thus, one could conjecture that subjects' price sensitivity correlates positively with the interaction of their endowment imbalance and signed mispricing. In the treatment with aggregate risk then, we would expect that not only we would see prices higher than expected values, but also more price-sensitive agents would come from the pool of subjects endowed with portfolios skewed towards the Black security. However, the correlation coefficient between a dummy for Black-skewed endowment and subject (average) price-sensitivity is negative (p-value of 0.06). Thus, if anything, the relationship is the opposite of the one conjectured.

c) A sophisticated alternative to the one proposed by our theory is that agents use some weighing between their belief and the price they observe in the marketplace. If this were the case, however, everyone would be price-sensitive, providing no explanation for our empirical findings.

d) Agents who doubt their own Bayesian updating react by disregarding their own calculations and always treating the market price as if it were a perfect signal of the expected return of the asset. This amounts to the degenerate case of point c), in which all of the weight is assigned to the market price. The implications are very similar to those of our explanation: under either scenario, confused subjects completely disregard their own priors and seek a balanced portfolio at any price, which generates the same three testable hypotheses. However, in the face of fluctuating market prices, the postulate that the market price is always correct is refuted as many times as the price moves. Therefore, one must assume that agents adhere to a conjecture they have seen refuted, possibly multiple times. We view such persistent inconsistency as highly implausible.

Our own theory of bounded rationality better accounts for the experimental findings by uncovering a two-way relationship between prices and beliefs, in which, importantly, prices affect beliefs. Gigerenzer and Selten (2001) argue that "models of bounded rationality describe how a judgment or decision is reached [...] rather than merely the outcome of the decision, and they describe the class of environments in which these heuristics will succeed or fail." We argue that investors who face difficult updating problems use the following heuristic when they realize that they have reasoned incorrectly: they disregard their own beliefs and hedge against the ambiguity that ensues. Since equilibrium prices stay close to their correct value, the application of this heuristic implies little loss to those who use it.[28]

VI. Concluding Summary

We have presented a theory of asset trading with symmetric information but asymmetric reasoning. The micro foundation of individual behavior in our theory is adapted from Epstein and Schneider's (2007) theory of learning under ambiguity. We assume that agents who make mistakes in their calculations of the expected return of an asset lose confidence in their calculations when confronted with a market price for the asset that is far off from their calculated expected return. The market price thereby plays the role of "authority" in the face of which agents are known to become ambiguity averse (Fox and Tversky, 1995). Confronted with an environment that they no longer perceive as risky but as ambiguous, these agents prefer to trade to portfolios that are not exposed to uncertainty, and they become insensitive to prices, at least for a range of prices.

A prediction for a market with no aggregate risk follows: in equilibrium, price quality (the proximity between price and the expected return of the asset) increases in the proportion of price-sensitive agents (Hypothesis 2). We also predict that in a market with aggregate risk, price-sensitive subjects should hold portfolios exposed to risk (imbalanced portfolios) while price-insensitive agents trade to balanced positions (Hypothesis 3); in

[28]The average payoff (participation bonus of \$5 included) to price-sensitive agents \$51.75, that of price-insensitive agents \$47.17. While the payoffs are statistically different from one another, the price-sensitivity premium is only 10%.

contrast, we do not expect to find such a difference in agents' holdings in an environment without aggregate risk – there, all agents ought to trade to balanced or near-balanced portfolios.

Experimental results are consistent with these predictions. In the experiments with no risk, price quality improves significantly with the fraction of price-sensitive subjects; in the experiments with risk, final holdings of price-sensitive subjects are more imbalanced than those of price-insensitive subjects. Importantly, when our theory predicts that the relation between portfolio imbalance and price-sensitivity will be negligible or non-existent (in the case without aggregate risk), the relation is indeed insignificant in the data.

Overall, our theory and experimental results support the claim that some agents who reason incorrectly become price-insensitive and do not contribute to set equilibrium prices in asset markets.

Appendix

A. Mathematical Details

As we describe in Section II, an agent i who perceives ambiguity maximizes the following expression:

$$\min\{\ln(R_i), \ln(B_i)\}.$$

It immediately follows that the agent seeks a portfolio with $R_i = B_i$ under any prices p_R and $p_B = 1 - p_R$.

Let the price of R be p_R. An expected utility maximizing agent i with belief π_r^i maximizes

$$U_i(R_i, B_i) = \pi_r^i \ln(R_i) + (1 - \pi_r^i)\ln(B_i).$$

The solution to this agent's optimization problem given her endowment, which by assumption is one unit of each asset, is $R_i = \frac{\pi_r^i}{p_R}$.

We calculate in turn the excess demand of knowledgeable agents, who correctly calculate their prior and use it so that they perceive risk; confused agents who mistrust their prior and do not use it, so that they perceive ambiguity; and biased agents who calculate their prior incorrectly but use it so that they perceive risk.

A.1. Excess demand of knowledgeable agents

Let q_α denote the aggregate demand of red asset by the fraction α of agents who are able to calculate the correct probabilities. Then $q_\alpha = \int_{1-\alpha}^{1} R_i = \alpha\frac{\pi_r}{p_R}$. Thus, for any $p_R < \pi_r$ the knowledgeable agents create excess demand $\alpha\left(\frac{\pi_r}{p_R} - 1\right)$.

A.2. Excess demand of agents who perceive ambiguity

For any price p_R the agents demand a risk neutral portfolio. Under the assumption of no aggregate endowment uncertainty for any subinterval of agents, the ambiguity averse agents create excess demand of 0.

A.3. Excess demand of price-sensitive biased agents

Note that from the assumption that $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ and $\alpha \leq 1$, it follows $\epsilon < \frac{\pi_r - \underline{\pi}}{2}$. Conjecture that $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$; we later check that this conjecture is satisfied in the equilibrium we construct, and that no alternative equilibrium exists if this conjecture is not satisfied.

Let $\underline{i}$ be the agent such that $\pi_r^{\underline{i}} = p_R - \epsilon$, that is, $\underline{i} = \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \epsilon - \underline{\pi})$. The excess demand generated by the biased agents is

$$\int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p_R} - 1\right) di.$$

Since $\pi_r^i = \underline{\pi} + i\frac{\pi_r - \underline{\pi}}{(1-\alpha)}$,

$$\int_{\underline{i}}^{1-\alpha} \left(\frac{\pi_r^i}{p_R} - 1\right) di = \int_{\underline{i}}^{1-\alpha} \left(\frac{\underline{\pi}}{p_R} - 1\right) di + \int_{\underline{i}}^{1-\alpha} \frac{\pi_r - \underline{\pi}}{(1-\alpha)p_R} i di = -\frac{p_R - \underline{\pi}}{p_R} \int_{\underline{i}}^{1-\alpha} 1 di + \frac{\pi_r - \underline{\pi}}{(1-\alpha)p_R} \int_{\underline{i}}^{1-\alpha} i di =$$

$$\frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} i^2|_{\underline{i}}^{1-\alpha} - \frac{p_R - \underline{\pi}}{p_R} i|_{\underline{i}}^{1-\alpha} = \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R}((1-\alpha)^2 - \underline{i}^2) - \frac{p_R - \underline{\pi}}{p_R}(1 - \alpha - \underline{i}) =$$

$$\frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R}(1 - \alpha - \underline{i})(1 - \alpha + \underline{i}) - \frac{p_R - \underline{\pi}}{p_R}(1 - \alpha - \underline{i}) =$$

$$\frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R}(1-\alpha-\frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon))(1-\alpha+\frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon)) - \frac{p_R - \underline{\pi}}{p_R}(1-\alpha-\frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon)) =$$

$$(1-\alpha)\frac{\pi_r - \underline{\pi}}{2p_R}(1-\frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon))(1+\frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon)) - (1-\alpha)\frac{p_R - \underline{\pi}}{p_R}(1-\frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon)) =$$

$$(1-\alpha)\frac{1}{2p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(\pi_r - 2\underline{\pi} + p_R - \epsilon) - (1-\alpha)\frac{p_R - \underline{\pi}}{p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon) =$$

$$\frac{1-\alpha}{p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(\frac{1}{2}(\pi_r - 2\underline{\pi} + p_R - \epsilon) - (p_R - \underline{\pi})) =$$

$$\frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon).$$

Because $(\pi_r - p_R - \epsilon) < 0$, the excess demand is negative, i.e., the biased agents provide excess supply to the market.

A.4. Equilibrium

In equilibrium the aggregate excess demand must be zero.

$$\frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon) + \alpha\left(\frac{\pi_r}{p_R} - 1\right) = 0 \Leftrightarrow$$

$$\frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(p_R + \epsilon - \pi_r) = \alpha\left(\frac{\pi_r}{p_R} - 1\right) \Leftrightarrow$$

$$\frac{1-\alpha}{2(\pi_r - \underline{\pi})}(\pi_r - p_R + \epsilon)(p_R + \epsilon - \pi_r) = \alpha(\pi_r - p_R) \Leftrightarrow$$

Denote $\pi_r - p_R$ by y. Then

$$\frac{1-\alpha}{2(\pi_r - \underline{\pi})}(y + \epsilon)(\epsilon - y) = \alpha y.$$

Denote $\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})$ by K. Then

$$y^2 + 2Ky - \epsilon^2 = 0.$$

The (positive) solution to the equation is $y = \sqrt{K^2 + \epsilon^2} - K$. Note that $\lim_{\epsilon \to 0} y = 0$, i.e. the price converges to π_r as ϵ converges to zero.

The above derived equilibrium satisfies the conjecture that $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$ as depicted in Figure 5.

To prove uniqueness, conjecture instead that $p_R > \pi_r$. Then the excess demand of all three classes of agents are negative, hence there is no equilibrium with this price.

Finally, conjecture instead that $\pi_r > p_R + \epsilon$. Then the excess demand of agents who perceive ambiguity is zero, and the excess demand of biased price-sensitive agents is also zero, while if $\alpha > 0$ the excess demand of knowledgeable agents is strictly positive. Therefore, if $\alpha > 0$, the aggregate excess demand is strictly positive, and there is no equilibrium with this price.

A.5. Comparative Statics

Since $\frac{\partial K}{\partial \alpha} = \frac{(\pi_r - \underline{\pi})}{(1-\alpha)^2} > 0$ and $\frac{\partial K}{\partial \alpha}\frac{\partial y}{\partial K} = \frac{K}{\sqrt{K^2+\epsilon^2}} - 1 < 0$, it follows that $\frac{dy}{d\alpha} = \frac{\partial K}{\partial \alpha}\frac{\partial y}{\partial K} < 0$, i.e. the difference between the price and the true probability decreases as α increases.

Let $S(\alpha, y)$ be the fraction of price-sensitive agents, as a function of the fraction of knowledgeable agents and the mispricing, $S = \alpha + (\epsilon + y)\frac{1-\alpha}{\pi_r - \underline{\pi}}$. Let $y^*(\alpha)$ be the equilibrium mispricing, as a function of α. Let $S^*(\alpha) = S(\alpha, y^*(\alpha))$ be the fraction of price-sensitive agents in equilibrium, as a function of alpha. Then,

$$\begin{aligned}
S^*(\alpha) &= \alpha + \left(\epsilon + \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} - \frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)\frac{1-\alpha}{\pi_r - \underline{\pi}} = \\
&= \alpha + \frac{1-\alpha}{\pi_r - \underline{\pi}}\epsilon - \frac{1-\alpha}{\pi_r - \underline{\pi}}\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) + \frac{1-\alpha}{\pi_r - \underline{\pi}}\sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1-\alpha}{\pi_r - \underline{\pi}}\epsilon + \frac{1-\alpha}{\pi_r - \underline{\pi}}\sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} \\
&= \frac{1-\alpha}{\pi_r - \underline{\pi}}\left(\epsilon + \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2}\right).
\end{aligned}$$

For any α then we obtain $S^*(\alpha)$ and $y^*(\alpha)$. If $S^*(\alpha)$ is strictly monotonic, we can invert it and obtain $\alpha(S)$. We are interested in $y(\alpha(S))$ and $\frac{dy}{dS} = \frac{dy}{d\alpha}\frac{d\alpha}{dS}$. We know that $\frac{dy}{d\alpha} < 0$, hence we must only determine the sign of $\frac{d\alpha}{dS}$, which, under our conjecture that $S^*(\alpha)$ is strictly monotonic, coincides with the sign of $\frac{dS^*(\alpha)}{d\alpha}$. Figure 6 presents the surface of $\frac{dS^*(\alpha)}{d\alpha}$ for any α and any ϵ, given $\pi_r - \underline{\pi} = 0.5$. It illustrates that if ϵ is not too large relative to α, the derivative is positive. We show that given our assumption that $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$, $\frac{dS^*(\alpha)}{d\alpha} > 0$ for every α, so that indeed $S^*(a)$ is strictly monotonic as assumed, and it follows $\frac{dy}{dS} < 0$.

$$\frac{dS^*(\alpha)}{d\alpha} = -\frac{\epsilon}{\pi_r - \underline{\pi}} - \frac{1}{\pi_r - \underline{\pi}}\sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} + \frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2}\left(\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2\right)^{-1/2}.$$

We want to show that given any $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$,

$$\begin{aligned}
-\frac{\epsilon}{\pi_r - \underline{\pi}} - \frac{1}{\pi_r - \underline{\pi}}\sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} + \frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2\sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2}} &> 0 \Leftrightarrow \\
\frac{\alpha(\pi_r - \underline{\pi})^2}{(1-\alpha)^2\sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2} - \epsilon &> 0 \Leftrightarrow \\
\frac{(\pi_r - \underline{\pi})}{(1-\alpha)\sqrt{1 + \left(\frac{(1-\alpha)\epsilon}{\alpha(\pi_r - \underline{\pi})}\right)^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2} - \epsilon &> 0.
\end{aligned}$$

The left hand side expression is decreasing in ϵ, hence it suffices to show that the inequality holds for $\epsilon = \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$.

$$\frac{(\pi_r - \underline{\pi})}{(1-\alpha)\sqrt{1+\left(\frac{(1-\alpha)\frac{\alpha}{1+\alpha}(\pi_r-\underline{\pi})}{\alpha(\pi_r-\underline{\pi})}\right)^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \frac{\alpha^2}{(1+\alpha)^2}(\pi_r - \underline{\pi})^2} - \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi}) > 0 \Leftrightarrow$$

$$\frac{1}{(1-\alpha)\sqrt{1+\frac{(1-\alpha)^2}{(1+\alpha)^2}}} - \alpha\sqrt{\frac{1}{(1-\alpha)^2} + \frac{1}{(1+\alpha)^2}} - \frac{\alpha}{1+\alpha} > 0 \Leftrightarrow$$

$$\frac{(1+\alpha)}{(1-\alpha)\sqrt{(1+\alpha)^2 + (1-\alpha)^2}} - \frac{\alpha}{(1-\alpha)(1+\alpha)}\sqrt{(1+\alpha)^2 + (1-\alpha)^2} - \frac{\alpha}{1+\alpha} > 0 \Leftrightarrow$$

$$(1+\alpha)^2 - \alpha((1+\alpha)^2 + (1-\alpha)^2) - \alpha(1-\alpha)\sqrt{(1+\alpha)^2 + (1-\alpha)^2} > 0 \Leftrightarrow$$

$$(1-\alpha)(1+\alpha)^2 - \alpha(1-\alpha)^2 > \alpha(1-\alpha)\sqrt{(1+\alpha)^2 + (1-\alpha)^2} \Leftrightarrow$$

$$\frac{(1+\alpha)^2}{\alpha} - (1-\alpha) > \sqrt{(1+\alpha)^2 + (1-\alpha)^2} \Leftrightarrow$$

$$\frac{(1+\alpha)^4}{\alpha^2} + (1-\alpha)^2 - 2\frac{(1+\alpha)^2}{\alpha}(1-\alpha) > (1+\alpha)^2 + (1-\alpha)^2 \Leftrightarrow$$

$$(1+\alpha)^2 - 2\alpha(1-\alpha) > \alpha^2 \Leftrightarrow$$

$$1 + 2\alpha^2 > 0.$$

A.6. Heterogeneity

Here, we consider the case where some agents perceive ambiguity when their subjective belief deviates from prices with an amount δ, whereas others perceive ambiguity when beliefs deviate more than ε, where $\delta \leq \varepsilon$. We now prove that mispricing continues to decrease in the fraction (here 2α) of agents who know how to compute probabilities (Corollary 1).

Setting

We envisage a situation where is a continuum of agents on the interval $I = [0,1]$ and a continuum of agents on the interval $J = [0,1]$. Any agent $i \in [\alpha, 1] \subset I$ and any $j \in [\alpha, 1] \subset J$ updates correctly and always makes portfolio choices that maximize her expected utility according to her (correct) subjective probability. Any agent $i \in [0, \alpha) \subset I$ makes portfolio choices that maximize her expected utility according to her (incorrect) subjective probability if and only if $|\pi_r^i - p_R| < \varepsilon$, otherwise she perceives ambiguity and maximizes maxmin utility. Any agent $j \in [0, \alpha) \subset J$ makes portfolio choices that maximize her expected utility according her (incorrect) subjective probability if and only if $|\pi_r^j - p_R| < \delta$, otherwise she perceives ambiguity and optimizes maxmin utility.

An agent who perceives ambiguity maximizes the following expression:

$$\min \{\ln(R_i), \ln(B_i)\}.$$

From here it immediately follows that the agent seeks a portfolio with $R_i = B_i$ under any prices p_R and $p_B = 1 - p_R$.

Let the price of R be p_R. An expected utility maximizing agent i with belief π_r^i maximizes

$$U_i(R_i, B_i) = \pi_r^i \ln(R_i) + (1 - \pi_r^i) \ln(B_i).$$

The solution to this agent's optimization problem given her endowment, which by assumption is one unit of each asset, is $R_i{=}\frac{\pi_r^i}{p_R}$.

Excess demand of knowledgeable agents

Let q_α denote the aggregate demand of red asset by the fraction α of agents who are able to calculate the correct probabilities. Then $q_\alpha{=}\int_{1-\alpha}^{1} R_i{=}\alpha\frac{\pi_r}{p_R}$. Thus, for any $p_R{<}\ \pi_r$ the knowledgeable agents create excess demand $\alpha(\frac{\pi_r}{p_R}{-}1)$.

Excess demand of agents who perceive ambiguity

For any price p_R the agents demand risk neutral portfolio. Because of the assumption of no aggregate endowment uncertainty for any subinterval of agents, the ambiguity averse agents create excess demand of 0.

Excess demand of price-sensitive biased agents

Note that from the assumption that $\varepsilon <\frac{\alpha}{1+\alpha}(\pi_r{-}\underline{\pi})$ and $\alpha \leq 1$, it follows $\varepsilon <\frac{\pi_r-\underline{\pi}}{2}$.

Case A) Conjecture that $p_R{+}\varepsilon > \pi_r{>}\ p_R{+}\delta > p_R{-}\varepsilon >\underline{\pi}$.

The aggregate excess demand of biased agents in J is $\int_{p_R-\delta}^{p_R+\delta} (\frac{x}{p_R}{-}1)dx = (\frac{x^2}{2p_R}{-}x)|_{p_R-\delta}^{p_R+\delta}{=}\ 0$.

The aggregate excess demand of biased agents in I follows the same calculation as in the benchmark model, with the only difference that the overall size of the group is one half of what it was in that benchmark case, so the aggregate excess demand is halved.

$$\frac{1-\alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(\pi_r - p_R - \varepsilon).$$

Because $(\pi_r - p_R - \varepsilon) < 0$, the excess demand is negative, i.e., the biased agents provide excess supply to the market.

Case B) Conjecture instead that $p_R + \delta > \pi_r > p_R - \varepsilon > \underline{\pi}$. Following analogous calculations to those for the I group, but substituting δ for ε at every step, the excess demand of biased agents in group J is

$$\frac{1-\alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \delta)(\pi_r - p_R - \delta).$$

Equilibrium

Case A) In equilibrium the aggregate excess demand must be zero.

$$\frac{1-\alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(\pi_r - p_R - \varepsilon) + \alpha(\frac{\pi_r}{p_R} - 1) = 0 \Leftrightarrow$$

$$\frac{1-\alpha}{4p_R(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(p_R + \varepsilon - \pi_r) = \alpha(\frac{\pi_r}{p_R} - 1) \Leftrightarrow$$

$$\frac{1-\alpha}{4(\pi_r - \underline{\pi})}(\pi_r - p_R + \varepsilon)(p_R + \varepsilon - \pi_r) = \alpha(\pi_r - p_R).$$

Denote $\pi_r - p_R$ by y. Then

$$\frac{1-\alpha}{4(\pi_r - \underline{\pi})}(y + \varepsilon)(\varepsilon - y) = \alpha y.$$

Denote $\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})$ by K. Then

$$\frac{1}{4}(\varepsilon + y)(\varepsilon - y) = Ky.$$

The (positive) solution to the equation is

$$y = \sqrt{4K^2 + \varepsilon^2} - 2K.$$

Note that $\lim_{\varepsilon \to 0} y = 0$, i.e. the price converges to π_r as ε converges to zero.

The above derived equilibrium satisfies the conjecture that $p_R + \varepsilon > \pi_r > p_R + \delta > p_R - \varepsilon > \underline{\pi}$

if and only if

$$\delta \leq \sqrt{4K^2 + \varepsilon^2} - 2K.$$

Otherwise we are on case B).

Case B). In equilibrium aggregate excess demand must be zero.

$$\frac{1-\alpha}{4p_R(\pi_r - \underline{\pi})}[(\pi_r - p_R + \varepsilon)(\pi_r - p_R - \varepsilon) + (\pi_r - p_R + \delta)(\pi_r - p_R - \delta)] + \alpha(\frac{\pi_r}{p_R} - 1) = 0 \Leftrightarrow$$

$$\frac{1-\alpha}{4p_R(\pi_r - \underline{\pi})}[2(\pi_r - p_R)^2 - \varepsilon^2 - \delta^2] = -\alpha(\frac{\pi_r}{p_R} - 1) \Leftrightarrow$$

$$\begin{aligned}
\frac{1-\alpha}{4(\pi_r-\underline{\pi})}(2y^2-\varepsilon^2-\delta^2) &= -\alpha y \Leftrightarrow \\
2y^2-\varepsilon^2-\delta^2 &= -4Ky \\
2y^2+4Ky-(\varepsilon^2+\delta^2) &= 0 \\
y &= \frac{-4K+\sqrt{16K^2+8(\varepsilon^2+\delta^2)}}{4} \\
y &= \sqrt{K^2+\frac{\varepsilon^2+\delta^2}{2}}-K.
\end{aligned}$$

Therefore, the equilibrium with the two populations I and J is identical to the equilibrium with a unique population with parameter $\varepsilon_{avg}=\sqrt{\frac{\epsilon^2+\delta^2}{2}}$.

Comparative Statics

Case A) Since $\frac{\partial K}{\partial \alpha}=\frac{(\pi_r-\underline{\pi})}{(1-\alpha)^2}>0$ and $\frac{\partial y}{\partial K}=\frac{K}{\sqrt{4K^2+\epsilon^2}}-2<0$, it follows that $\frac{dy}{d\alpha}=\frac{\partial y}{\partial K}\frac{\partial K}{\partial \alpha}<0$, i.e. the difference between the price and the true probability decreases as α increases.

Case B) Since $\frac{\partial K}{\partial \alpha}=\frac{(\pi_r-\underline{\pi})}{(1-\alpha)^2}>0$ and $\frac{\partial y}{\partial K}=\frac{K}{\sqrt{K^2+\varepsilon_{avg}^2}}-1<0$, it follows that $\frac{dy}{d\alpha}=\frac{\partial y}{\partial K}\frac{\partial K}{\partial \alpha}<0$, i.e. the difference between the price and the true probability decreases as α increases.

In both cases, mispricing decreases in the fraction of agents who know how to compute probabilities, confirming Corollary 1.

In regard to Corollary 2, first note that for Case B the proof for a homogeneous epsilon applies by taking the value of epsilon to be $\varepsilon_{avg}=\sqrt{\frac{\epsilon^2+\delta^2}{2}}$.

For Case A, on the other hand, let $S(\alpha, y)$ be the fraction of price-sensitive agents, as a function of the fraction of knowledgeable agents and the mispricing, $S=\alpha+(\delta+\frac{\varepsilon+y}{2})\frac{1-\alpha}{\pi_r-\underline{\pi}}$. Let

$y^*(\alpha)$ be the equilibrium mispricing, as a function of α. Let $S^*(\alpha) = S(\alpha, y^*(\alpha))$ be the fraction of price-sensitive agents in equilibrium, as a function of alpha. Recall $y = \sqrt{4K^2 + \varepsilon^2} - 2K$ and $K = \frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})$. Then,

$$\begin{aligned} S^*(\alpha) &= \alpha + \left(\delta + \frac{\varepsilon}{2} + \frac{1}{2}\left(\sqrt{4K^2+\varepsilon^2} - 2\frac{\alpha}{1-\alpha}\right)(\pi_r - \underline{\pi})\right)\frac{1-\alpha}{\pi_r - \underline{\pi}} = \\ &= \frac{1-\alpha}{\pi_r - \underline{\pi}}\left(\delta + \frac{\varepsilon}{2}\right) + \frac{1-\alpha}{2(\pi_r - \underline{\pi})}\sqrt{4K^2+\varepsilon^2}. \end{aligned}$$

For any α we obtain $S^*(\alpha)$ and $y^*(\alpha)$. If $S^*(a)$ is strictly monotonic, we can invert it and obtain $\alpha(S)$. We are interested in $y(\alpha(S))$ and $\frac{dy}{dS} = \frac{dy}{d\alpha}\frac{d\alpha}{dS}$. We know that $\frac{dy}{d\alpha} < 0$, hence we must only determine the sign of $\frac{d\alpha}{dS}$, which, under our conjecture that $S^*(a)$ is strictly monotonic, coincides with the sign of $\frac{dS^*(\alpha)}{d\alpha}$.

$$\frac{dS^*(\alpha)}{d\alpha} = \frac{1}{2}\left(-\frac{2\delta+\varepsilon}{(\pi_r - \underline{\pi})} - \frac{\sqrt{4K^2+\varepsilon^2}}{(\pi_r - \underline{\pi})} + \left(\frac{4\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2\sqrt{4K^2+\varepsilon^2}}\right)\right).$$

We want to show that $\frac{dS^*(a))}{d\alpha} > 0$. Because $\frac{dS^*(\alpha)}{d\alpha}$ is decreasing in both ε and δ, it suffices to show that $\frac{dS^*(\alpha)}{d\alpha} > 0$ given $\varepsilon = \overline{\varepsilon} = \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ and $\delta = \overline{\delta} = \sqrt{4K^2 - \varepsilon^2} - 2K$.

$$
\begin{aligned}
&\frac{dS^*(\alpha)}{d\alpha}\Big|_{\varepsilon=\overline{\varepsilon},\delta=\overline{\delta}} \\
= \; & \frac{1}{2}\left(-\frac{2(\sqrt{4K^2+\varepsilon^2}-2K)+\varepsilon}{(\pi_r-\underline{\pi})}-\frac{\sqrt{4K^2+\varepsilon^2}}{(\pi_r-\underline{\pi})}+\frac{4\alpha(\pi_r-\underline{\pi})}{(1-\alpha)^2\sqrt{4K^2+\varepsilon^2}}\right) \\
= \; & \frac{1}{2}\left(-\frac{3(\sqrt{4K^2+\varepsilon^2})}{(\pi_r-\underline{\pi})}-\frac{-4K+\varepsilon}{(\pi_r-\underline{\pi})}+\frac{4\alpha(\pi_r-\underline{\pi})}{(1-\alpha)^2\sqrt{4K^2+\varepsilon^2}}\right) \\
= \; & \frac{1}{2}\left(-3\sqrt{4\left(\frac{\alpha}{1-\alpha}\right)^2+\left(\frac{\alpha}{1+\alpha}\right)^2}+4\frac{\alpha}{1-\alpha}-\frac{\alpha}{1+\alpha}+\frac{4\alpha}{(1-\alpha)^2\sqrt{4\left(\frac{\alpha}{1-\alpha}\right)^2+\left(\frac{\alpha}{1+\alpha}\right)^2}}\right) \\
= \; & \frac{\alpha}{2(1-\alpha^2)}\left(-3\sqrt{4(1+\alpha)^2+(1-\alpha)^2}+5\alpha+3+\frac{4(1+\alpha)^2}{\alpha\sqrt{4(1+\alpha)^2+(1-\alpha)^2}}\right).
\end{aligned}
$$

Figure 7 displays the graph of $\frac{dS^*(\alpha)}{d\alpha}$ given $\varepsilon = \overline{\varepsilon}$ and $\delta = \overline{\delta}$, and demonstrates that it is always positive, i.e., the number of price-sensitive agents is positively related to α.

B. Biased Slope Coefficients

To determine whether there is any simultaneous-equation bias on the estimated slope coefficients induced by overall balance in the changes in positions, we translate our setting into a more familiar framework, namely, that of a simple demand-supply setting. In particular, we are going to interpret (minus) the changes in endowments of the price-insensitive subjects as the supply in a demand-supply system with exogenous, price-insensitive supply, while the changes in endowments of the price-sensitive subjects correspond to the (price-sensitive) demands in a demand-supply system. The requirement that changes in holdings balance then corresponds to the usual restriction that demand equals supply.

We will consider only the case where price-sensitive subjects reduce their holdings when prices increase; translated into the usual demand-supply setting, this means that we assume that the slope of the demand equation is negative.

Assume there are only two subjects. One is price-sensitive, the other is price-insensitive. The former's changes in holdings corresponds to the demand $\tilde{D}$ in the traditional demand-supply system; the latter's changes corresponds to the (exogenous) supply $\tilde{S}$. The usual assumptions are as follows:

$$\tilde{D} = A + BP + \epsilon,$$

with $B < 0$, and

$$\tilde{S} = \eta,$$

where ϵ is mean zero, and is independent of η. P denotes price.

We want to know the properties of the OLS estimate of B. Assume that P is determined by equating demand and supply (equivalent to balance between changes in holdings), i.e., from

$$\tilde{D} = \tilde{S}.$$

Then:

$$\mathrm{cov}(P, \epsilon) = -\frac{1}{B}\mathrm{var}(\epsilon) > 0.$$

Because of this, standard arguments show that the OLS estimate of B is inconsistent, with an upward bias. As such, the nominal size of the usual t-test under-estimates the true size, and one should apply a generous cut-off in order to determine whether B is significantly negative.

In our case, however, we only need to identify who is price-sensitive (i.e., whose holdings changes correspond to D in the demand-supply setting?) and who is not (whose holdings changes correspond to $\tilde{S}$?). For this, we just run an OLS projection of changes in endowments on prices. The subjects with significantly negative slope coefficients are price-sensitive and hence, map into the demand $\tilde{D}$ of the traditional demand-supply system. The argument above, however, indicated that this test is biased. Therefore, a generous cut-off should be chosen; we chose a cut-off equal to 1.6.

While we did not need this for our study, one can obtain an improved estimate of the price sensitivity once subjects are categorized as either price-sensitive or price-insensitive. Indeed, the changes in the holdings of the price-insensitive subjects can be used as instrument to re-estimate the price-sensitivity of the price-sensitive subjects. This is equivalent to using $\tilde{S}$ as an instrument to estimate B. Indeed, $\tilde{S}$ $(= \eta)$ and ϵ are uncorrelated, while $\tilde{S}$ and P are correlated $(\mathrm{cov}(\tilde{S}, P) = \mathrm{var}(\tilde{S})/B)$, so $\tilde{S}$ is a valid instrument to estimate B in standard instrumental-variables analysis.

C. Experiment Instructions

I. THE EXPERIMENT

1. Situation The experiment consists of a sequence of trading sessions, referred to as *periods*. At the beginning of *even-numbered* periods, you will be given a fresh supply of *securities* and *cash*; in *odd-numbered periods*, you carry over securities and cash from the previous period. Markets open and you are free to trade some of your securities. You buy securities with cash and you get cash if you sell securities.

At the end of *odd-numbered* periods, the securities expire, after paying *dividends* that will be specified below. These dividends, together with your cash balance, constitute your *period earnings*. Securities do not pay dividends at the end of even-numbered periods and cash is carried over to the subsequent period, so your period earnings in even-numbered periods will be zero.

Period earnings are *cumulative* across periods. At the end of the experiment, the cumulative earnings are yours to keep, in addition to a standard sign-up reward.

During the experiment, accounting is done in real dollars.

2. The Securities You will be given two types of securities, *stocks* and *bonds*. Bonds pay a fixed dividend at the end of a period, namely, $0.50. Stocks pay a random dividend. There are two types of stocks, referred to as Red and Black. Their payoff depends on the drawing from a deck of 4 cards, as explained later. The payoff is either $0.50 or nothing. When Red stock pays $0.50, Black stock pays nothing; when Red stock pays nothing, Black stock pays $0.50.

You will be able to trade Red stock as well as bonds, but not Black stock.

You won't be able to buy Red stock or bonds unless you have the cash. You will be able to sell Red stock and bonds (and get cash) even if you do not own any. This is called short selling. If you sell, say, one Red stock, then you get to keep the sales price, but $0.50 will be subtracted from your period earnings after the market closes and if the payoff on Red stock is $0.50. If at the end of a period you are holding, say, -1 bonds, $0.50 will be subtracted from your period earnings.

The trading system checks your orders against bankruptcy: you will not be able to submit orders which, if executed, are likely to generate negative period earnings.

3. How Payoffs Are Determined Each period, we start with a deck of 4 cards: one hearts (♡), one diamonds (♢), one clubs (♣) and one spades (♠). The cards are shuffled and put in a row, face down.

Our computer takes randomly one or two cards and it discards them.

From the remaining cards, our computer randomly picks one or two cards. If one of these cards is hearts (♡), then the computer puts it back and picks another one. Sometimes, the computer will even put back diamonds (♢) and pick another one. The computer then reveals the card(s) it picked and we will announce this in the News Page at the end of the period (after that, another period starts with the same securities in which you can trade again). *Note that the revealed card(s) will never be hearts, and sometimes may not even be diamonds.*

Before each period, the News Page will provide all the information that you need to make the right inferences: (i) whether one or two cards are going to be discarded initially, (ii) whether one or two cards are going to be picked from the remaining cards and whether diamonds will ever be shown.

After we show the revealed cards, one or two cards remain in the deck. Our computer randomly picks a card and *this last card determines the payoff on the securities.*

Red stock pays \$0.50 when the last card is either hearts ($\heartsuit$) or diamonds ($\diamondsuit$). In those cases, the Black stock pays nothing. This is shown in the following Payoff Table.

	Red Stock	Black Stock
If last card is $\heartsuit$ or $\diamondsuit$	\$0.50	0
If last card is $\clubsuit$ or $\spadesuit$	0	\$0.50

Here is an example. Initially there are 4 cards in the deck, randomly shuffled. They are put in a row, face down, like this:

$$\square \quad \square \quad \square \quad \square$$

Our computer randomly discards one card (the third one in this case):

$$\square \quad \square \qquad\quad \square$$

Our computer then randomly picks one card (the fourth one in this case), and reveals it, provided it is not hearts or diamonds (in this case; if it is hearts or diamonds, it replaces it with another card from the deck that is neither):

$$\square \quad \square \qquad\quad \spadesuit$$

From the remaining two cards, our computer picks one at random that determines the payoffs on the stocks.

$$\diamondsuit \quad \square \qquad \spadesuit$$

In this case, the last card picked is diamonds. As a result, you would be paid $0.50 for each unit of Red stock you're holding, and nothing for the Black stock you're holding.

Here is another example. Initially there are 4 cards in the deck, randomly shuffled. They are put in a row, face down:

$$\square \quad \square \quad \square \quad \square$$

Our computer randomly discards two cards (the second and third ones in this case):

$$\square \qquad\qquad \square$$

Our computer then randomly picks one card and reveals it, provided it is not hearts (if it is hearts, it replaces it with another card from the deck):

$$\diamondsuit \qquad\qquad \square$$

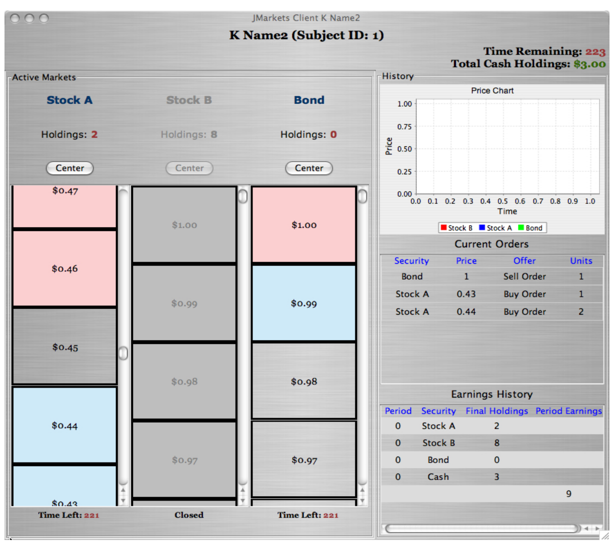
Our computer then picks the remaining card, which determines the payoffs on the stocks.

$$\diamondsuit \qquad\qquad \heartsuit$$

In this case, the last card picked is hearts. As a result, you would be paid $0.50 for each unit of Red stock you're holding, and nothing for the Black stock you're holding.

Again, the announcements of the number of cards that will be discarded initially and revealed at the end of the period can be found in the News page. This page will also display the card(s) that are turned over at the end of the period, and, at the end of the subsequent period, the final card that determines the payoff on the Stocks.

II. THE MARKETS INTERFACE, jMARKETS Once you click on the Participate link to the left, you will be asked to log into the markets, and you will be connected to the jMarkets server. After everybody has logged in and the experiment is launched, a markets interface like the one below will appear



1. Active Markets

The Active Markets panel is renewed each period. In it, you'll see several scroll-down columns. Each column corresponds to a market in one of the securities. The security name is indicated on top. At the bottom, you can see whether the market is open, and if so, how long it will remain open. The time left in a period is indicated on the right hand side above the Active Markets panel.

At the top of a column, you can also find your current holdings of the corresponding security. Your current cash holdings are given on the right hand side above the Active Markets panel.

Each column consists of a number of price levels at which you and others enter offers to trade. Current offers to sell are indicated in red; offers to buy are indicated in blue. When pressing the Center button on top of a column, you will be positioned halfway between the best offer to buy (i.e., the highest price at which somebody offers to buy) and the best offer to sell (i.e., the lowest price that anybody offers to sell at).

When you move your cursor to a particular price level box, you get specifics about the available offers. On top, at the left hand side, you'll see the number of units requested for purchase. Each time you click on it, you send an order to buy one unit yourself. On top, at the right hand side, the number of units offered for sale is given. You send an order to sell one unit each time you yourself click on it. At the bottom, you'll see how many units you offered. (Your offers are also listed under Current Orders to the right of the Active Markets panel.) Each time you hit cancel, you reduce your offer by one unit.

If you click on the price level, a small window appears that allows you to offer multiple units to buy or to sell, or to cancel offers for multiple units at once.

2. History

The History panel shows a chart of past transaction prices for each of the securities. Like the Active Markets panel, it refreshes every period. jMarkets randomly assigns colors to each of the securities. E.g., it may be that the price of the Red Stock is shown in blue. Make sure that this does not confuse you.

3. Current Orders

The Current Orders panel lists your offers. If you click on one of them, the corresponding price level box in the Active Markets panel is highlighted so that you can easily modify the offer.

4. Earnings History

The Earnings History table shows, for each period, your final holdings for each of the securities (and cash), as well as the resulting period earnings.

5. How Trade Takes Place

Whenever you enter an offer to sell at a price below or equal to that of the best available buy order, a sale takes place. You receive the price of the buy order in cash. Whenever you enter an offer to buy at a price above or equal to that of the best available sell order, a purchase takes place. You will be charged the price of the sell order.

The system imposes strict price-time priority: buy orders at high prices will be executed first; if there are several buy orders at the same price level, the oldest orders will be executed first. Analogously, sell orders at low prices will be executed first, and if there are several sell orders at a given price level, the oldest ones will be executed first.

6. Restrictions On Offers

Before you send in an offer, jMarkets will check two things: the cash constraint, and the bankruptcy constraint.

The cash constraint concerns whether you have enough cash to buy securities. If you send in an offer to buy, you need to have enough cash. To allow you to trade fast, jMarkets has an automatic cancellation feature. When you submit a buy order that violates the cash constraint, the system will automatically attempt to cancel buy orders you may have at lower prices, until the cash constraint is satisfied and your new order can be placed.

The bankruptcy constraint concerns your ability to deliver on promises that you implicitly make by trading securities. We may not allow you to trade to holdings that generate losses in some state(s). A message appears if that is the case and your order will not go through.

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Tables and Figures

TABLE 1

PARAMETERS OF THE EXPERIMENTAL DESIGN

Experiment[a]	Subject Category (Number)	Signup Reward (Dollar)	Initial Allocations[b] Red Stock (Units)	Black Stock (Units)	Bonds (Units)	Cash (Dollar)
Treatment I: Sessions With No Aggregate Risk						
Caltech	10	5	0	9	0	4
	10	5	12	3	0	1
Utah-1	10	5	0	9	0	4
	10	5	12	3	0	1
Caltech-Utah-1	10	5	0	9	0	4
	10	5	12	3	0	1
UCLA	10	5	0	9	0	4
	10	5	12	3	0	1
Utah-2	10	5	0	9	0	4
	10	5	12	3	0	1
Caltech-Utah-2	10	5	0	9	0	4
	10	5	12	3	0	1
Treatment II: Sessions With Aggregate Risk						
UCLA-R1	11	5	0	9	0	4
	9	5	12	3	0	1
UCLA-R2	11	5	0	9	0	4
	9	5	12	3	0	1
UCLA-R3	11	5	0	9	0	4
	9	5	12	3	0	1

[a]Indicates affiliation of subjects. “Utah” refers to the University of Utah; “Utah-Caltech” refers to: 50% of subjects were Caltech-affiliated; the remainder were students from the University of Utah. Experiments are listed in chronological order of occurrence.

[b]Renewed each period.

TABLE 2

CARD GAME SITUATIONS

Card Game Situation	Used In Repli-cations	# of Cards Discarded Before Revelation	Number of Cards Revealed Half-Time	Cards Never Revealed	Probability of Last Card being Red: Before Half-Time Revelation	After Black Card	After Red Card
1	1, 5	1	2	♡	5/6	1[a]	3/4
2	2, 7	1	1	♡	7/12	11/16	3/8
3	3, 6	2	1	♡	2/3	3/4	1/2
4	4, 8	1	1	♡, ♢	3/4	3/4	3/4

[a] In Card Game Situation 1 two cards are discarded and hearts is never revealed. As a result, one of the cards revealed after the half-time is necessarily a black card. Consequently, it is the color of the second card that matters for the posterior probabilities of the last card being red.

TABLE 3

NUMBER OF PRICE-SENSITIVE AGENTS AND MISPRICING IN TREATMENT I

Experiment	Card Game	Mean Abs. [a] Mispricing M	$N_{(T_b<-1.65)}$ [b]	$N_{(T_b>1.9)}$ [c]	$N_{(T_b<-1.65, T_a \text{correct})}$ [d]	Signed M
Caltech	1	3.13	7	2	6	-2.74
	2	5.54	3	2	2	5.24
	3	3.40	2	2	1	3.24
	4	1.25	4	6	2	1.17
Utah-1	1	3.50	3	1	2	3.50
	2	11.79	1	2	0	11.79
	3	11.28	0	1	0	11.28
	4	7.24	1	0	0	7.24
Caltech-Utah-1	1	3.35	5	1	1	-3.35
	2	4.75	2	0	0	4.71
	3	1.62	2	0	2	1.49
	4	1.89	5	3	3	1.62
UCLA	1	4.86	2	1	1	-4.73
	2	5.07	3	0	1	5.01
	3	2.53	3	3	3	-0.66
	4	2.9	3	0	2	-1.51
Utah-2	1	7.68	2	0	0	-7.39
	2	8.07	3	2	2	5.63
	3	5.56	6	2	3	5.63
	4	3.99	2	2	2	1.56
Caltech-Utah-2	1	8.44	1	0	0	-8.44
	2	2.51	2	0	2	-0.66
	3	2.66	3	1	1	-2.44
	4	3.24	3	0	2	-2.44

$\mathbf{Corr(M, N_{(T_b<-1.65)}) = -0.528}$ (St. Error[e] = 0.147)

$\mathbf{Corr(M, N_{(T_b<-1.65, T_a \text{correct})}) = -0.554}$ (St. Error = 0.141)

[a] M is the (absolute) difference between average transaction price and expected payoffs computed with correct probabilities, expressed in U.S. cents.

[b] Number of subjects for which $\{T_b < -1.65\}$. T_b is the t-statistic of the slope coefficient estimate in projections of one-minute changes in individual holdings of Red Stock (R) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

[c] Number of subjects for which $\{T_b > 1.9\}$.

[d] Number of subjects for which $\{T_b < -1.65\}$ and T_a has the correct sign. T_a is the t-statistic of the intercept in projections of one-minute changes in individual holdings of Red Stock (R) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

[e] All standard errors of the correlation estimate $\hat{\rho}$ obtained using a sample size of n are computed as $\frac{1-\hat{\rho}^2}{\sqrt{n}}$.

TABLE 4

NUMBER OF PRICE-SENSITIVE AGENTS AND MISPRICING IN TREATMENT II

Experiment	Card Game	Mean Abs.[a] Mispricing M	$N_{(T_b<-1.65)}$[b]	$N_{(T_b>1.9)}$[c]	$N_{(T_b<-1.65, T_a \text{correct})}$[d]	Signed M
UCLA-1R	1	2.02	0	1	0	0.49
	2	10.91	0	1	0	10.91
	3	4.87	4	1	4	4.16
	4	3.64	0	1	0	3.61
UCLA-2R	1	3.91	3	2	1	3.85
	2	12.30	1	0	0	12.30
	3	8.79	5	4	2	8.75
	4	6.60	2	2	1	6.59
UCLA- 3R	1	6.43	2	0	1	-6.38
	2	6.06	3	2	0	6.06
	3	2.66	3	2	3	0.53
	4	3.30	1	0	1	-2.58

$\mathbf{Corr(M, N_{(T_b<-1.65)}) = 0.000}$ (St. Error = 0.289) $\mathbf{Corr(M, N_{(T_b<-1.65, T_a correct)}) = -0.258}$ (St. Error = 0.269)

[a] M is the (absolute) difference between average transaction price and expected payoffs computed with correct probabilities, expressed in U.S. cents.

[b] Number of subjects for which $\{T_b < -1.65\}$. T_b is the t-statistic of the slope coefficient estimate in projections of one-minute changes in individual holdings of Red Stock (R) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

[c] Number of subjects for which $\{T_b > 1.9\}$.

[d] Number of subjects for which $\{T_b < -1.65\}$ and T_a has the correct sign. T_a is the t-statistic of the intercept in projections of one-minute changes in individual holdings of Red Stock (R) onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

TABLE 5

PRICE SENSITIVITY AND PORTFOLIO IMBALANCES

Independent Variable	Treatment I (No Aggregate Risk)		Treatment II (Aggregate Risk)	
	Second Trading Period	First Trading Period	Second Trading Period	First Trading Period
T	-13.182[a]	-11.363	**-10.674**	**-9.836**
	(23.279)[b]	(17.669)	(3.833)	(3.123)
	[0.066][c]	[0.078]	[0.247]	[0.293]
MT	-0.907	-2.562	**-2.189**	**-1.964**
	(1.521)	(2.624)	(0.56)	(0.453)
	[0.02]	[0.144]	[0.352]	[0.457]

[a]Slope coefficient

[b]Standard error, corrected for heteroskedasticity and subject clustering.

[c]R^2.

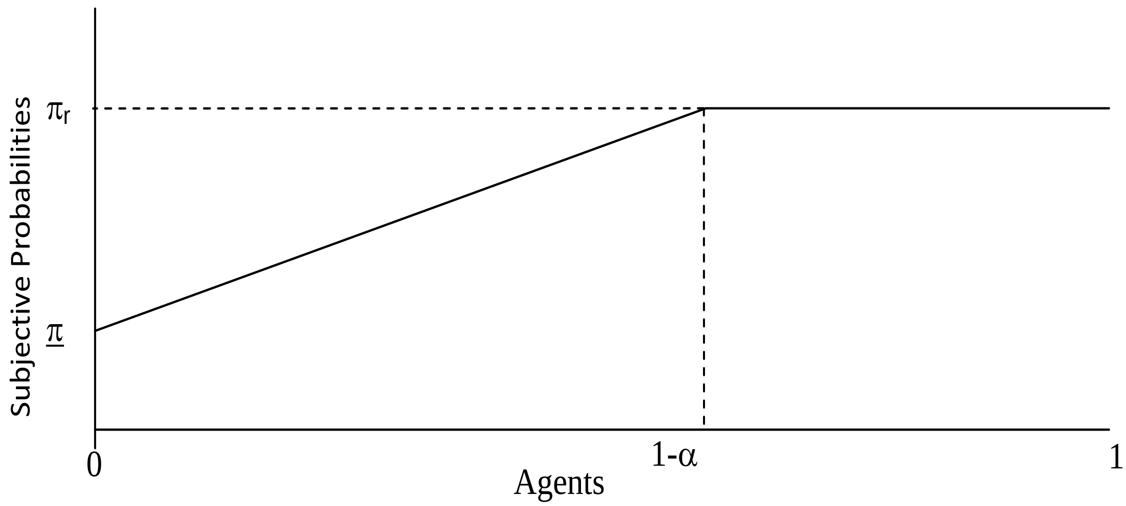

Figure 1. Beliefs. Graphical display of the relation between agent index (horizontal axis) and belief that security R will pay (vertical axis). The correct belief equals π_r. Incorrect beliefs are assumed to be below the correct belief, with a minimum equal to $\underline{\pi}$. Agents are indexed continuously from 0 to 1. A fraction α holds correct beliefs.

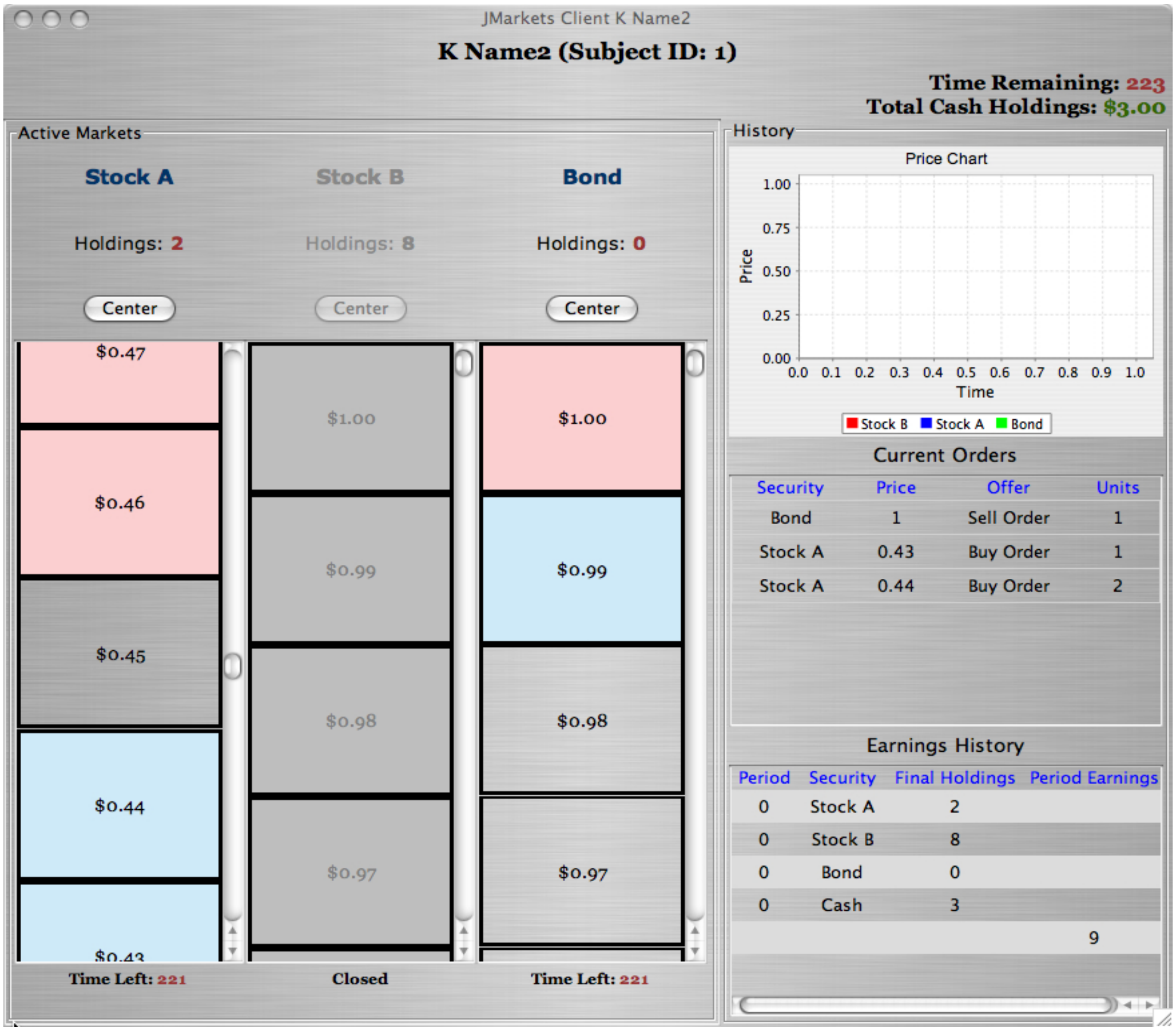

Figure 2. jMarkets Trading Interface Used For Practice Sessions. Books with limit buy orders (shown in blue) and sell orders (shown in pink) for three markets, Stock A, Stock B and Bond, are represented as scrollable columns of price levels. The "Center" button allows traders to scroll down to the price level halfway between the best buy and sell orders. The book for Stock B is grey because no trade is allowed (the market is "closed" – see Status underneath the book). To the right are a number of useful aids, such as a chart of past trade prices, a list of current offers, and earnings history. jMarkets is described in more detail at http://jmarkets.ssel.caltech.edu/.

Period	Duration	Number of Cards Discarded Initially	Number of Cards Revealed At End	Cards Never Revealed At End	Cards Revealed	Last Card	Which Stock Pays Dividend?
0	3'	1	2	♥	♠ ♣		
1	3'					♥	Red
2	3'	1	1	♥	♣		
3	3'					♥	Red
4	3'	2	1	♥	♣		
5	3'					♥	Red
6	3'	1	1	♥ ♦	♠		
7	3'					♦	Red
8	3'	1	2	♥	♦ ♠		
9	3'					♥	Red
10	3'	2	1	♥	♦		
11	3'					♠	Black

Figure 3. News Web Page. The page is filled gradually as cards are discarded and/or revealed (only the first six replications are included above).

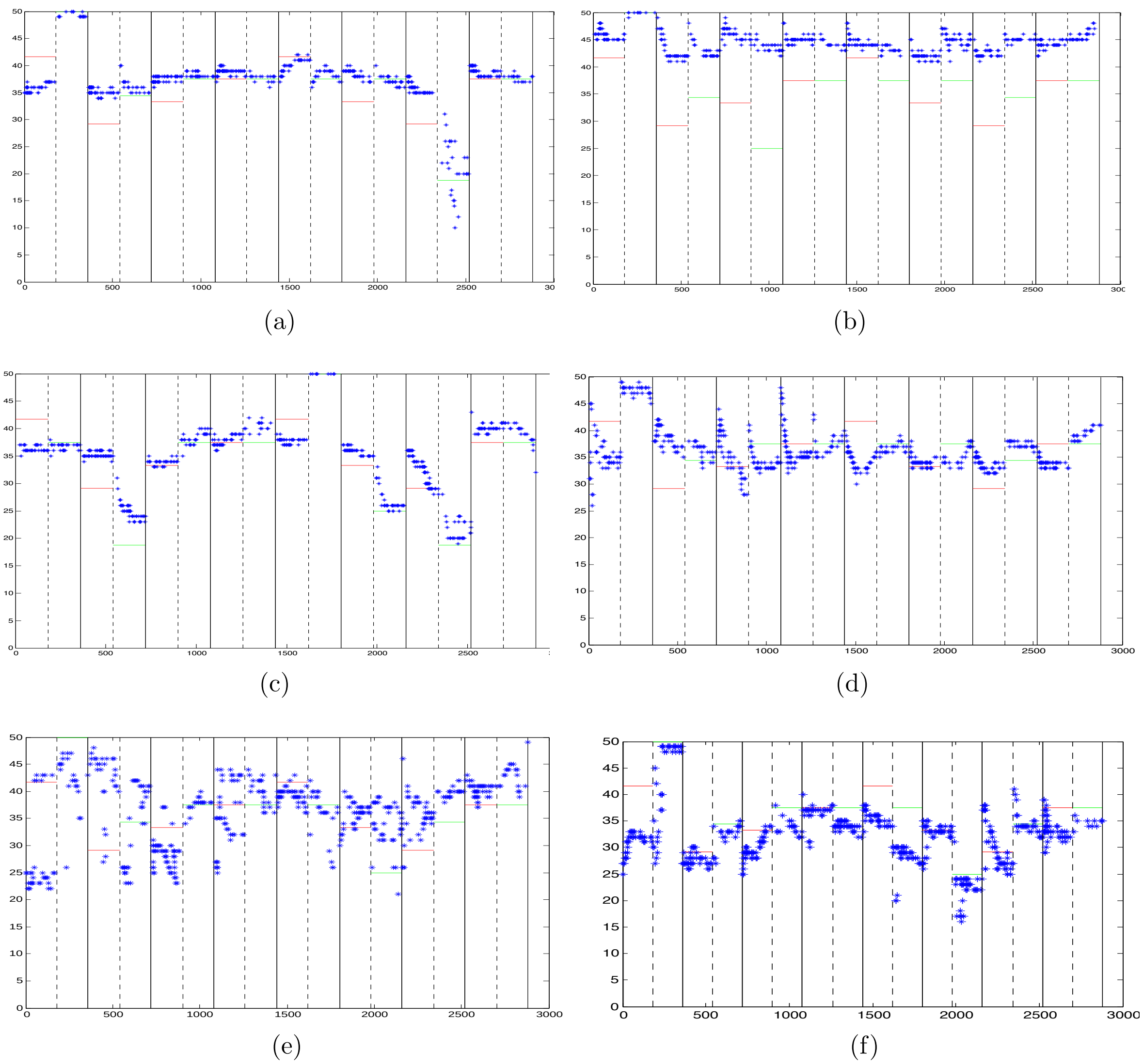

Figure 4. Evolution of Transaction Prices of Stock R in Treatment I. Blue crosses show transaction prices. Black vertical lines delineate periods. Dashed vertical lines indicate mid-period point in time when cards are partially revealed. Red horizontal line segments show true values of Stock R in the first half of the period. Green horizontal line segments indicate true values after mid-period revelation of cards. (a) Caltech; (b) Utah-1; (c) Caltech-Utah-1; (d) UCLA; (e) Utah-2; and, (f) Caltech-Utah-2.

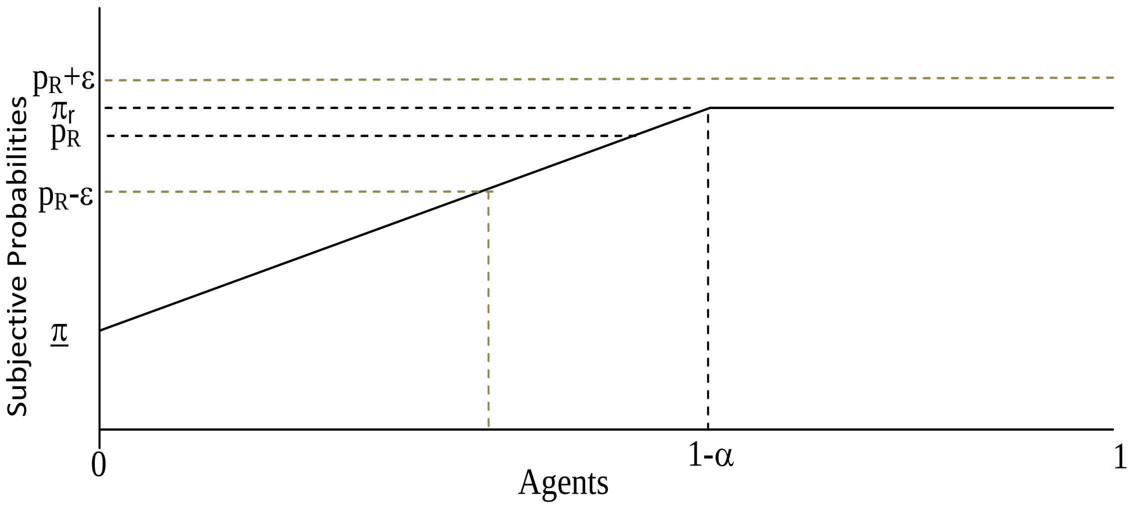

Figure 5. Equilibrium Price p_R. Agents with subjective beliefs within ϵ of the price p_R stick to their beliefs (they "agree to disagree" with the market). Those with subjective beliefs below $p_R - \epsilon$ become ambiguity averse because their beliefs are too much at odds with the market (price). The equilibrium price will be such that some agents with beliefs sufficiently close to the correct belief (π_R) continue to affect the price. As α, the proportion of agents with correct beliefs, increases, mispricing ($|p - \pi_R|$) decreases. The opposite obtains as ϵ, the maximum dissonance between subjective beliefs and market prices for which agents agree to disagree, increases.

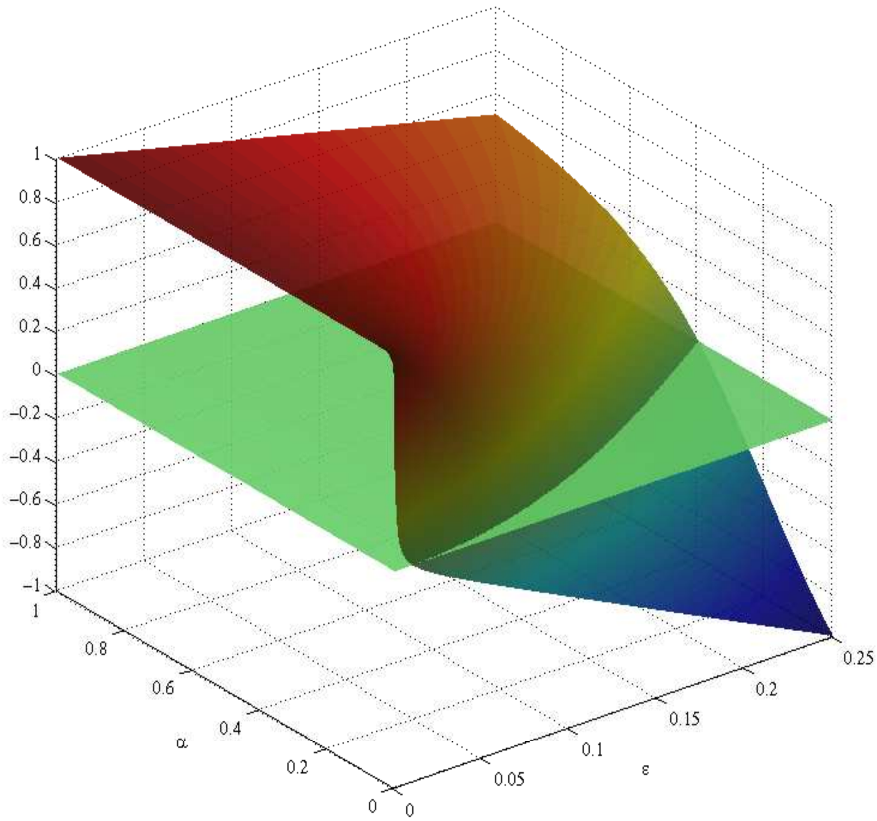

Figure 6. Change in Fraction of Price-Insensitive Agents with Changes Fraction of Agents with Correct Beliefs $\frac{\partial S}{\partial \alpha}$, as a Function of α (Fraction of Agents with Correct Beliefs) and ϵ (Maximum Dissonance between Subjective Beliefs and Market Prices for which Agents Agree to Disagree. Assumed: $\pi_r - \underline{\pi} = 0.5$.

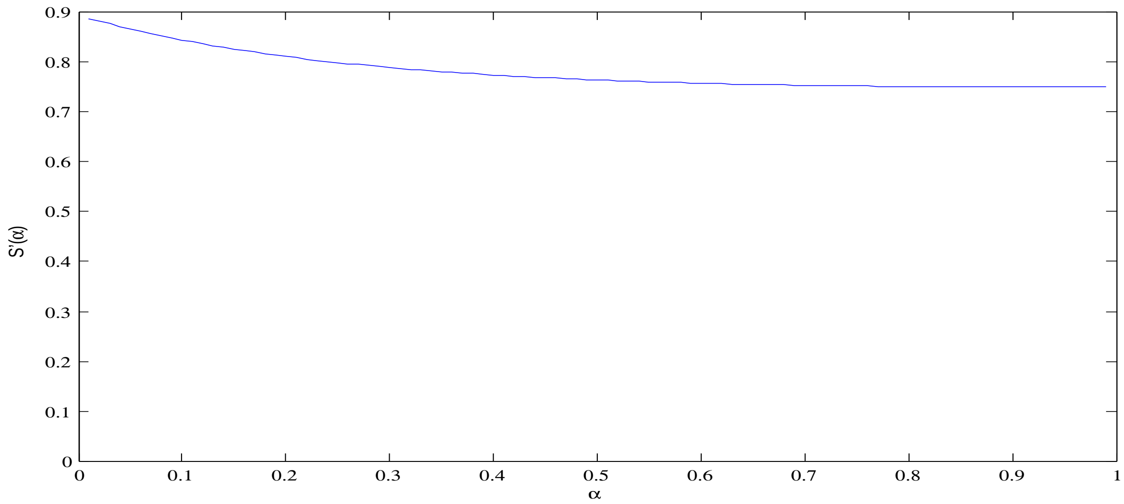

Figure 7. $\frac{dS(\alpha)}{d\alpha}$ **evaluated at** $\overline{\varepsilon} = \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ **and** $\overline{\delta}$=$\sqrt{4K^2 - \varepsilon^2}$$-2K$**.**
The figure plots the lower bound on the derivative of the fraction of price-sensitive agents as a function of the fraction of agents with correct beliefs. Since this lower bound is everywhere strictly positive, the derivative is strictly positive for any α, ε and δ.