
Peer reviewed version

Link to published version (if available):
10.3233/978-1-60750-801-4-487

Link to publication record in Explore Bristol Research
PDF-document
Predicting shear strength mobilization of London clay

Prévision de la mobilization de résistance au cisaillement d'argile de Londres

P. J. Vardanega & M. D. Bolton
Department of Engineering, University of Cambridge

ABSTRACT

When designing geotechnical structures engineers need to assign soil parameters. Soil design parameters are often inferred through correlations with basic site investigation data. The objective of this work is to determine the shape of the undrained stress-strain curve of a heavily overconsolidated Eocene clay in such a way that it may conveniently be used in simplified deformation mechanisms to predict ground movements due to construction. A database of London clay triaxial test data is presented. Use of a power model to predict strength mobilization is demonstrated for 17 previously published triaxial tests on high quality cores of London clay. A novel method of normalising these mobilization curves is demonstrated (using a reference strain at 50% mobilization of shear strength), and different relations are shown to apply to different magnitudes of strain. The parameters that influence the variation of the reference strain are studied.

RÉSUMÉ

La connaissance des caractéristiques du sol est nécessaire à la conception de structures géotechniques. Ces caractéristiques sont souvent déduites de corrélations avec des mesures sur site. L’objectif de ce travail est de déterminer la courbe contrainte-déformation d’argile sur-consolidée de façon à ce qu’elle soit facile à utiliser dans des analyses de déformation pour prédire les mouvements du sol dus à la construction. Une banque de données sur l’argile de Londres est présentée. Un modèle logarithmique est appliqué à 17 résultats de tests triaxiaux sur des blocs d’argile de Londres de haute qualité. Une méthode innovante de normalisation de ces courbes de mobilisation est présentée, basée sur une déformation de référence à 50% de la résistance au cisaillement. Les paramètres qui influencent la déformation de référence associée sont étudiés.

Keywords: shear strength, mobilization, stiffness, correlations, triaxial tests

1 INTRODUCTION

A large amount of research has been undertaken into the stress-strain behaviour of London clay. London clay is a highly fissured, Eocene clay that is very stiff and of reasonably high plasticity (e.g. Hight et al., 2003) [1]. This paper will examine the available stress-strain data of London clay and will suggest simple mathematical models to describe the response. Simpson (2010) reviews [2] the recent symposium in print on ‘Engineering in stiff clays’ and provides references to many other sources describing the engineering properties of this highly studied deposit.

2 LONDON CLAY DATABASE

High quality tests on London clay samples are available in the literature. Yimsiri (2001) performed [3] triaxial testing on London clay cores from Kennington Park near a single tunnel that is

1 corresponding author, pjv27@cam.ac.uk
part of the Northern Line. More details about the site can be found in Gourvenec et al. (1999, 2005) [4] [5].


Figure 1 shows the raw triaxial data from the two studies re-plotted. Shear strain ($\gamma$) is taken as 1.5 times the axial strain. Mobilized shear strength is taken as the product of secant shear modulus $G$ and shear strain.

2.1 Power laws

Vardanega & Bolton have suggested [9] that shear strength mobilization data can be characterized by power-laws:

$$\frac{\tau_{mob}}{c_u} = Ay^b$$

BS8002 (1994) describes [10] the quantity $c_u/\tau_{mob}$ as the Mobilization Factor $M$ which is equivalent to a factor of safety on shear strength. Plots were made of $\tau_{mob}/c_u$ ($= 1/M$) versus shear strain for the 15 tests being considered. The correlations were restricted to the range $5 \geq M \geq 1.25$ representing typical design conditions and excluding those parts of the stress-strain curves that were found to be less reliable. The authors refer to this strain region as the moderate strain region. Figure 2 shows the power curves fitted to the data. Table 1 summarises the curve fitting parameters $A$ & $b$ in equation (1), the number of data points in the correlation ($n$), and the mobilization strain ($\gamma_M=2$).

<table>
<thead>
<tr>
<th>ID</th>
<th>$A$</th>
<th>$b$</th>
<th>$n$</th>
<th>$\gamma_M=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>t36</td>
<td>5.41</td>
<td>0.49</td>
<td>134</td>
<td>0.0078</td>
</tr>
<tr>
<td>t42</td>
<td>3.69</td>
<td>0.45</td>
<td>87</td>
<td>0.0118</td>
</tr>
<tr>
<td>t52</td>
<td>3.39</td>
<td>0.41</td>
<td>139</td>
<td>0.0094</td>
</tr>
<tr>
<td>t33</td>
<td>26.45</td>
<td>0.83</td>
<td>64</td>
<td>0.0084</td>
</tr>
<tr>
<td>th2</td>
<td>5.86</td>
<td>0.47</td>
<td>94</td>
<td>0.0053</td>
</tr>
<tr>
<td>t13</td>
<td>9.98</td>
<td>0.58</td>
<td>101</td>
<td>0.0057</td>
</tr>
<tr>
<td>t19</td>
<td>15.78</td>
<td>0.66</td>
<td>119</td>
<td>0.0054</td>
</tr>
<tr>
<td>B1</td>
<td>7.39</td>
<td>0.53</td>
<td>219</td>
<td>0.0062</td>
</tr>
<tr>
<td>B2</td>
<td>7.17</td>
<td>0.50</td>
<td>110</td>
<td>0.0049</td>
</tr>
<tr>
<td>C1</td>
<td>8.33</td>
<td>0.60</td>
<td>86</td>
<td>0.0092</td>
</tr>
<tr>
<td>C2</td>
<td>7.18</td>
<td>0.54</td>
<td>78</td>
<td>0.0072</td>
</tr>
<tr>
<td>D1</td>
<td>14.69</td>
<td>0.62</td>
<td>112</td>
<td>0.0043</td>
</tr>
<tr>
<td>D2</td>
<td>11.05</td>
<td>0.64</td>
<td>125</td>
<td>0.0079</td>
</tr>
<tr>
<td>E1</td>
<td>14.93</td>
<td>0.67</td>
<td>94</td>
<td>0.0063</td>
</tr>
<tr>
<td>E2</td>
<td>25.43</td>
<td>0.75</td>
<td>151</td>
<td>0.0053</td>
</tr>
<tr>
<td>Average</td>
<td>11.12</td>
<td>0.58</td>
<td>114</td>
<td>0.0070</td>
</tr>
</tbody>
</table>

Fig. 1: Original test data from Yimsiri (2001) and Gasparre (2005).
0.1

1

0.001 0.01 0.1

Shear strain ($\gamma$)

$\tau_{mob}/C_u$

0.1

1

0.01

Normalised shear strain ($\gamma/\gamma_{M=2}$)

Fig. 2 Power laws fitted to the data range $5 \geq M \geq 1.25$.

2.2 Mobilization strain

The strain to half mobilization can be used to normalize the strain axis in Figure 2. Equation (1) can now be re-written as:

$$\frac{\tau_{mob}}{C_u} = 0.5 \left( \frac{\gamma}{\gamma_{M=2}} \right)^b$$

(2)

The average exponent ‘$b$’ to represent London clay is 0.58 (see Table 1). This is identical to the weighted average exponent – when taking into account the number of data-points to generate each curve.

Equation (2) therefore becomes:

$$\frac{\tau_{mob}}{C_u} = 0.5 \left[ \frac{\gamma}{\gamma_{M=2}} \right]^{0.58}$$

(3)

Figure 3 shows the design stress-strain curve for London clay. The moderate strain region is described by equation (3).

The relevant statistical measures quoted for the correlations in this paper are: the coefficient of determination ($R^2$), the number of data-points in the correlation ($n$), the standard error of the correlation ($S.E.$) and the probability of a correlation not existing ($p$).

Figure 4 shows the predicted values from (3) plotted against the measured values. The prediction function is generally able to predict values of $\tau_{mob}/C_u$ at any strain level to within ±20%. The goodness of fit with equation 3 can also be represented using equation (4) which is the linear regression in Figure 4. This means that for the selected range of mobilization values ($5 \geq M \geq 1.25$) a single power equation can successfully represent the response of the soil deposit.
\[
\frac{\tau_{\text{mob}}}{c_u} = 0.485 \left[ \frac{\gamma}{\gamma_{M=2}} \right]^{0.58}
\]

\( R^2=0.94, \ n=1713, \ p<0.001, \ S.E.=0.039 \) \hspace{1cm} (4)

\(\gamma_{M=2} = 0.011\)

Figure 4 Measured \(\tau_{\text{mob}}/c_u\) vs predicted \(\tau_{\text{mob}}/c_u\) plot (accuracy of equation (3)).

2.3 Prior study of reconstituted London Clay

Jardine et al (1984) tested intact London clay in triaxial tests from initially isotropic states \[11\]. The samples were taken from Canon’s Park in North London at depths of 5.3m and 7.5m. Their stress-strain data was observed by Jardine et al (1986) \[12\] to be capable of fitting the function:

\[
\frac{E_u}{c_u} = A + B \cos \left[ \alpha \left( \log \frac{e_a}{C} \right) \right]
\]

where \(E_u\) is the strain-dependent Young’s modulus (taken as \(3G\) in the current paper), \(e_a\) is the axial strain in the triaxial test (taken as \(2/3\gamma\) in the present paper), and where \(A, B, C, \alpha\) and \(T\) are curve fitting parameters.

Figure 5 compares the goodness of fit of the same triaxial data with equation (3) using \(c_u\) and one curve fitting parameter \(\gamma_{M=2}\). It is seen that equation (3) fits the data reasonably well.

\[1000\gamma_{M=2} = -2.84 \ln(d) + 15.42\]

\(R^2=0.46, \ r=-0.67, \ n=17, \ p=0.003, \ S.E.=1.79\) \hspace{1cm} (6)

While the coefficient of determination is only moderately significant a trend does exist at the 1% level of significance. Deeper samples have a smaller overconsolidation ratio and apparently exhibit lower strains to half-mobilization. It must also be acknowledged that the trend in Figure 6 could simply arise from random fluctuations and the small number of sites and samples that were investigated.

Decision makers may prefer a reasonable fit using a single physically meaningful parameter to a closely tuned function with five parameters whose physical significance is unclear. A further advantage of using \(\gamma_{M=2}\) as the sole parameter of equation (3) is that statistical correlations can be sought for it.

3 PREDICTING REFERENCE STRAIN

3.1 Predicting mobilization strain

Various soil or site characterization parameters can be investigated for their possible influence on the mobilization strain \(\gamma_{M=2}\) parameter. Fig. 6 shows a correlation (6) between \(\gamma_{M=2}\) and the sample depth \(d\) as reported in Yimsiri (2001) \[3\] and Gasparre (2005) \[6\].

\(1000\gamma_{M=2} = -2.84 \ln(d) + 15.42\)

\(R^2=0.46, \ r=-0.67, \ n=17, \ p=0.003, \ S.E.=1.79\) \hspace{1cm} (6)
Figures 7 to 9 show $\gamma_{m-2}$ plotted against confining stress ($p'_0$), plasticity index ($I_p$) and undrained shear strength ($c_u$). No significant trend is observed. The observed variation of $\gamma_{m-2}$ within a three-fold range for these London clay samples suggests that random variations are arising not from the inherent variability of the intrinsic soil properties (e.g. $I_p$), but variability in the extrinsic nature of the samples that were tested (especially from fissuring). Gasparre (2005) emphasized [6] the possible influence of fissures on test results.

\[ 1000 \times \gamma_{m-2} = -2.84 \ln(d) + 15.42 \]
\[ R^2 = 0.46 \]
\[ r = -0.67 \]
\[ n = 17 \]
\[ p = 0.003 \]
\[ S.E. = 1.79 \]

3.2 Predicting maximum shear modulus ($G_0$)

An interesting correlation is observed in the data between undrained shear strength ($c_u$) and maximum shear modulus ($G_0$). The relatively strong trend is plotted on Figure 10 and given as:

\[ G_0 = 320.7(c_u) \]
\[ R^2 = 0.735, \ r = 0.857, \ n = 17, \ p < 0.001, \]
\[ S.E. = 21.7 \]

The fact that maximum shear modulus is related to undrained shear strength gives validity to the use of $E/c_u$ ratios for settlement characteristics in London clay (e.g. Butler, 1975 & Hewitt, 1989) [13] [14]. However it should be remembered that equation (7) applies only to London clay and not other materials.
4 CONCLUSIONS

This paper has shown that the strain to 50% mobilization can be used to normalize the test results from 17 high quality triaxial tests on London clay from Kennington Park, Heathrow Terminal 5 and Canon’s Park. Values of \( \gamma_{m2} \) from 4x10^{-3} to 12x10^{-3} have been recorded. The mobilization strain varies with sample depth which may indicate a relationship to OCR.

The more significant finding is that \( \gamma_{m2} \) appears to suffer a random variation up to about ±30% around the trend line with depth. This may be due to the different impact of fissuring on the various samples.

Engineers have to compute settlements in clay using some value of stiffness at an appropriate strain level. The approach validated in this paper is to use the point of 50% mobilization as the pivot for the whole stress-strain curve for London clay plotted on log-log axes. This allows the designer either to select an appropriate linear elastic modulus at any given strain level, or to use the power curve given in (3) to perform a full non-linear analysis.

ACKNOWLEDGEMENTS

Thanks are given to Dr Apollonia Gasparre for providing her test data for analysis. Thanks are also due to Ove Arup and Partners and the Cambridge Commonwealth Trust for their financial support of the first author. The authors also thank Mr A. Bizard for his help with some French translation.

REFERENCES