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# An Initial Bifurcation Analysis of an EV Pickup Truck

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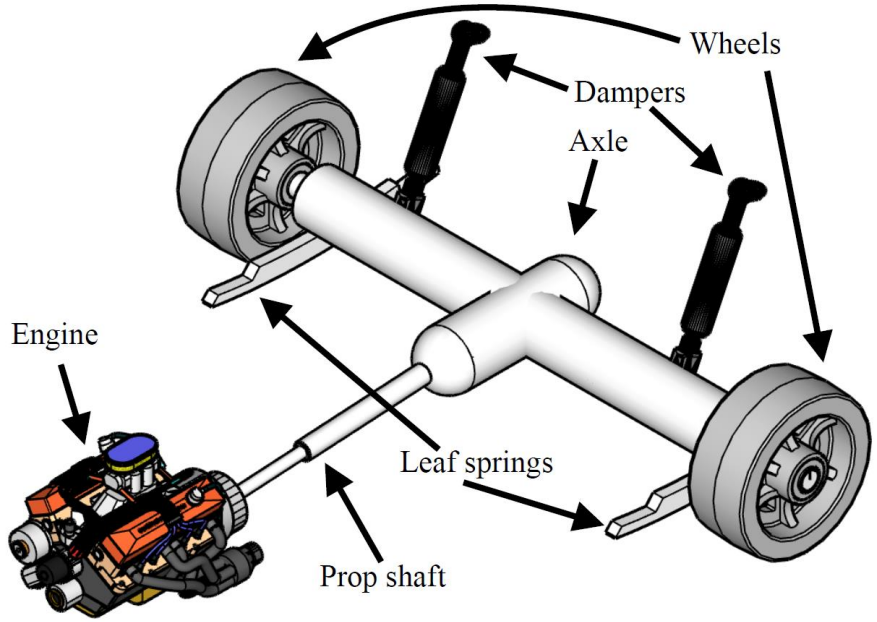
# Introduction



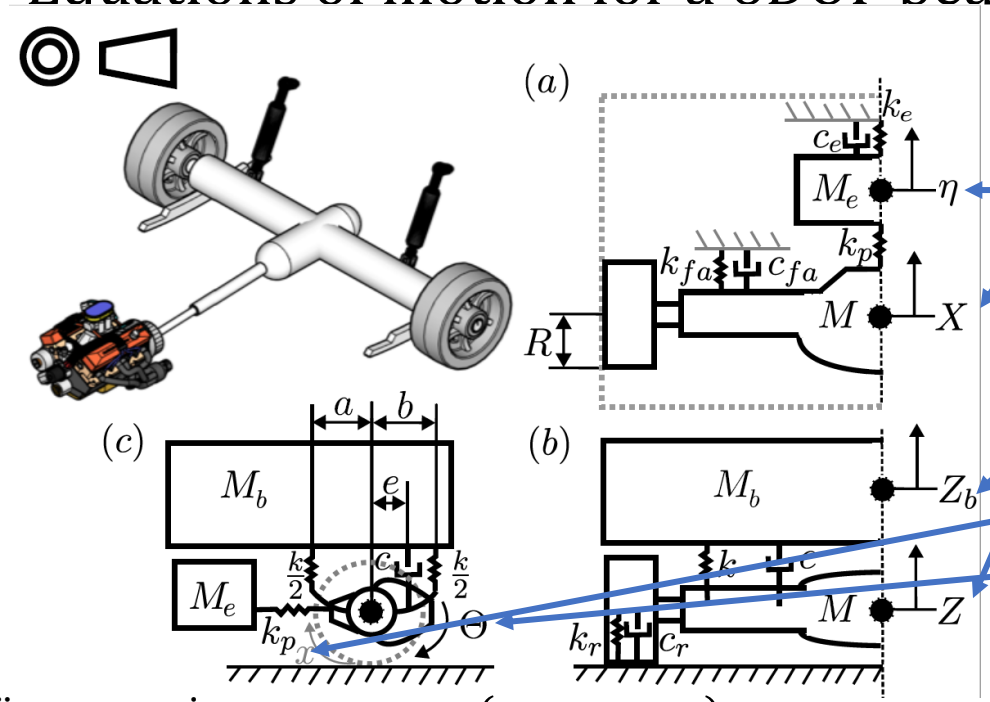
- Vehicles with a beam axle setup can undergo a problem known as “axle tramp”.




- Electrification requires changes to the powertrain which can alter the problem.
- Study compares bifurcations in a car and electric truck model.

# Beam Axle Model

- Equations of motion for a 6DOF beam axle;




Longitudinal;

- **Axle & wheel longitudinal displacement, *X***
- Engine longitudinal displacement, $\eta$

Vertical;

- **Axle & wheel vertical displacement, *Z***
- Body vertical displacement, $Z_B$

Angular;

- **Wheel angular displacement, $\delta x$**
- Axle pitch, $\theta$

$$\ddot{X} = (-c_{fa}\dot{X} - k_{fa}X + F + (k_{fa}n + kph)\theta + k_p h\eta)/(m + M)$$

$$\ddot{\eta} = (-C_e\dot{\eta} - kp\eta - ke\eta + kpX - kph\theta)/(M_e)$$

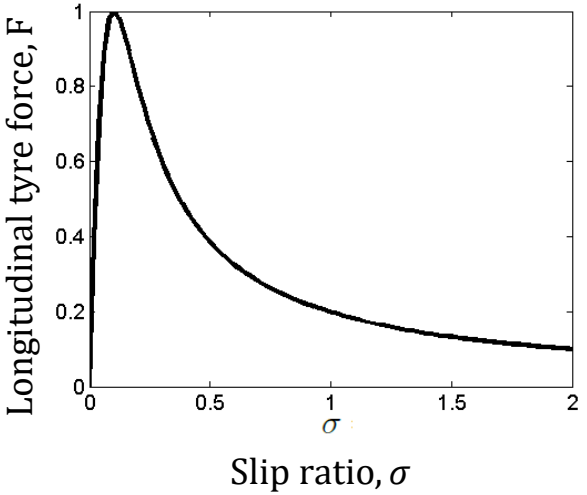
$$\ddot{Z} = (C(\dot{Z} - \dot{Z}_b) + k(Z_b - (Z - R)) - k_r(Z - R) - c_r\dot{Z} - mg - Ce\dot{\theta} - k(b - a)\theta)/(m + M)$$

$$\ddot{Z}_b = (-\mathrm{C}(\dot{Z}_b - \dot{Z}) - k(Z_b - (Z - R)) - (M_b)g)/(M_b)$$

$$\delta\ddot{x} = (-c_1\dot{x} - k_1 x - F)/i_y$$

$$\ddot{\theta} = (-Ce^2\dot{\theta} - (k(a^2 + b^2) + k_{fa}n^2 + kph^2)\theta - Ce\dot{Z} - k(b - a)(Z - R) + (kn + kph)X - k_p h\eta)$$

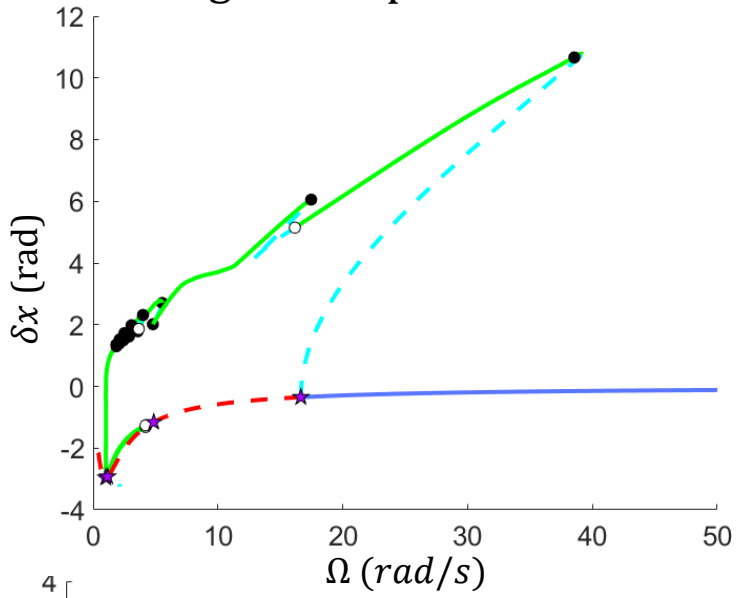
k=stiffness, c=damping, m=masses, F=tyre force, i=inertia, g=gravity, other=dimensions



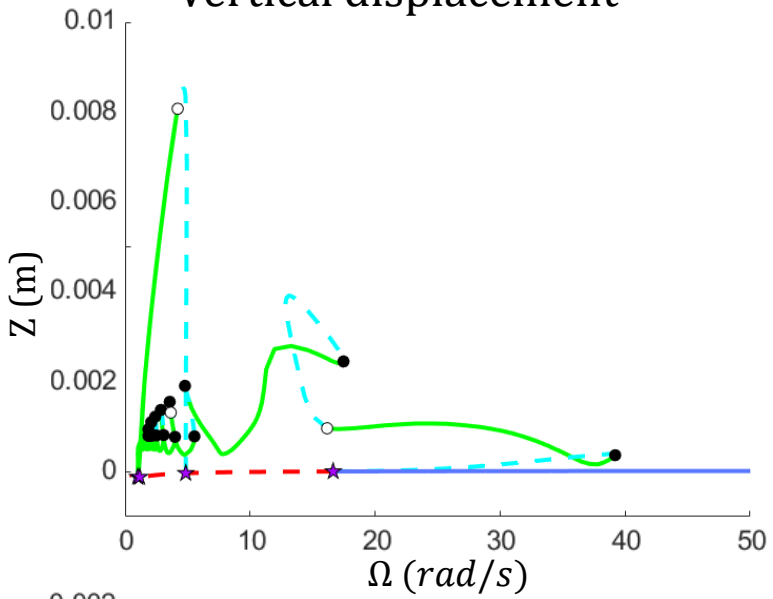
# Car Results



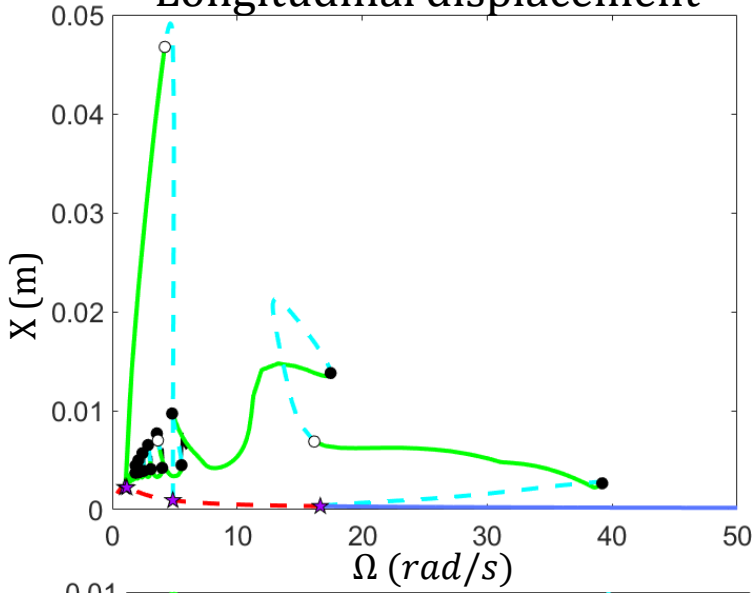
Angular displacement

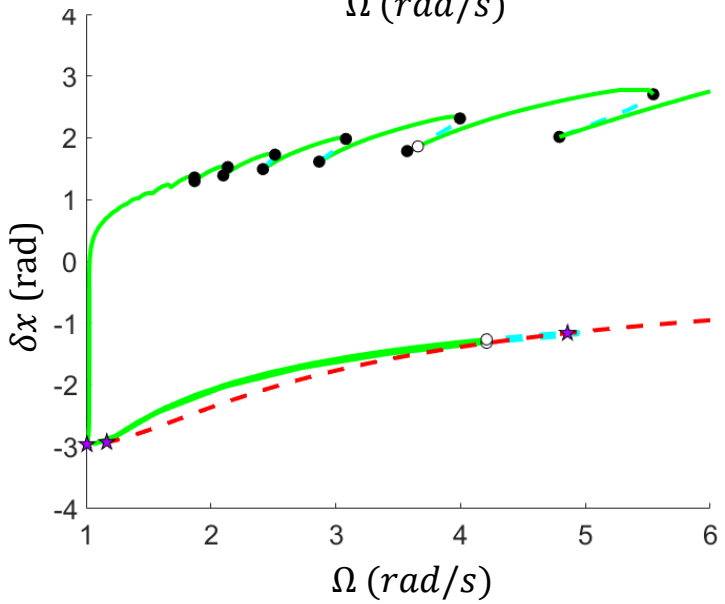


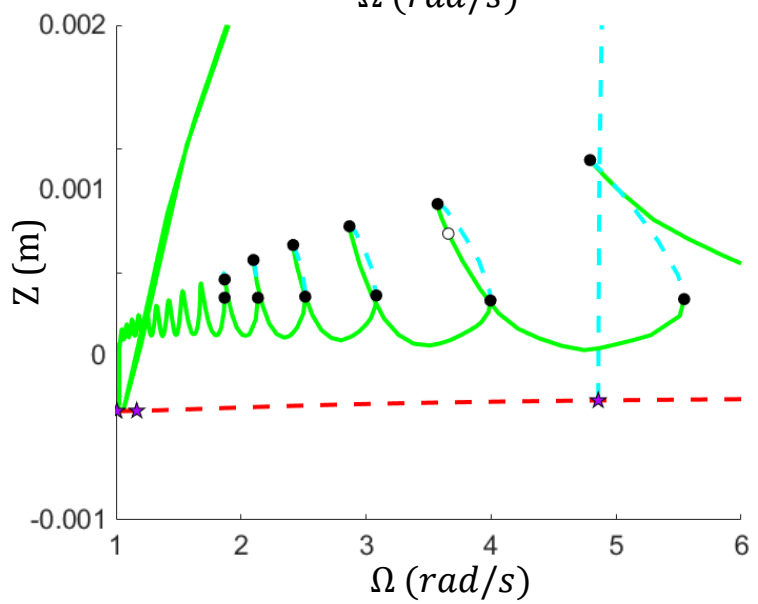

Vertical displacement

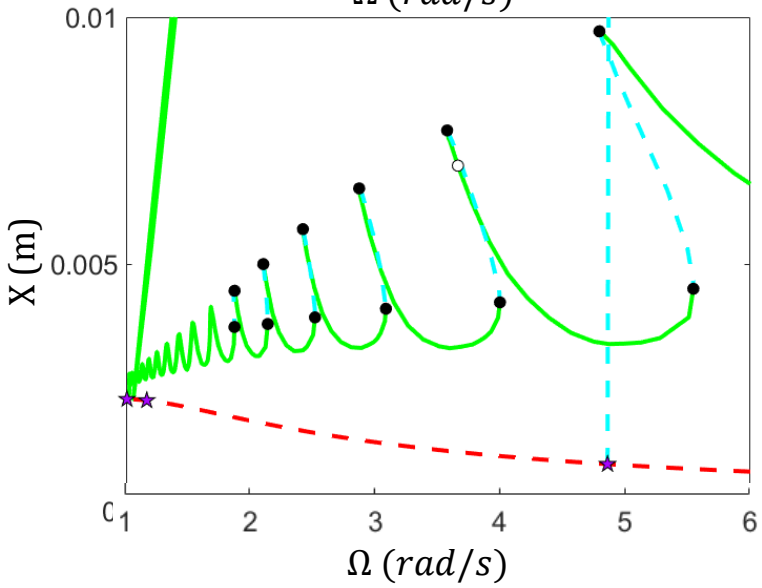



Longitudinal displacement



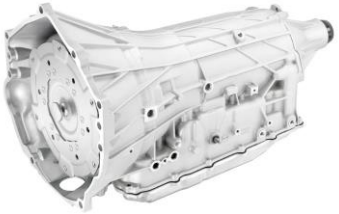
# From Petrol Car to Electric Truck

- Internal Combustion Engine Car


Crank


Dual Mass Flywheel


Clutch


Transmission

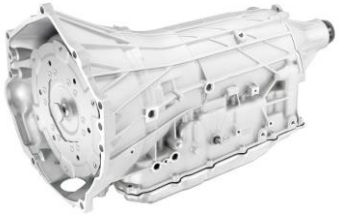

Halfshaft


Wheel


Tyre

- Electric Truck


Electric Motor


Transmission


Halfshaft


Wheel


Tyre

# From Petrol Car to Electric Truck



- Key Parameter Changes From Car Model

| Parameter | Car | EV Truck | Unit | Approx change |
|---|---|---|---|---|
| Wheel radius | 0.28 | 0.45 | m | 1.5x |
| Wheel mass | 18 | 54 | Kg | 3x |
| Mass of axle assembly | 31 | 92 | Kg | 3x |
| Mass of body | 120 | 360 | Kg | 3x |
| Axle pitch inertia | 0.27 | 0.81 | Kgm^2 | 3x |
| Suspension stiffness | 11000 | 33000 | N/m | 3x |
| Longitudinal engine stiffness | 500000 | 1530000 | N/m | 3x |
| Engine/Motor mass | 160 | 40 | Kg | 0.25x |
| **Torsional wheel stiffness (referred)** | **165** | **3850** | **N/m** | **20x** |
| Torsional wheel inertia (referred) | 1.2 | 2 | Kgm^2 | 1.5x |

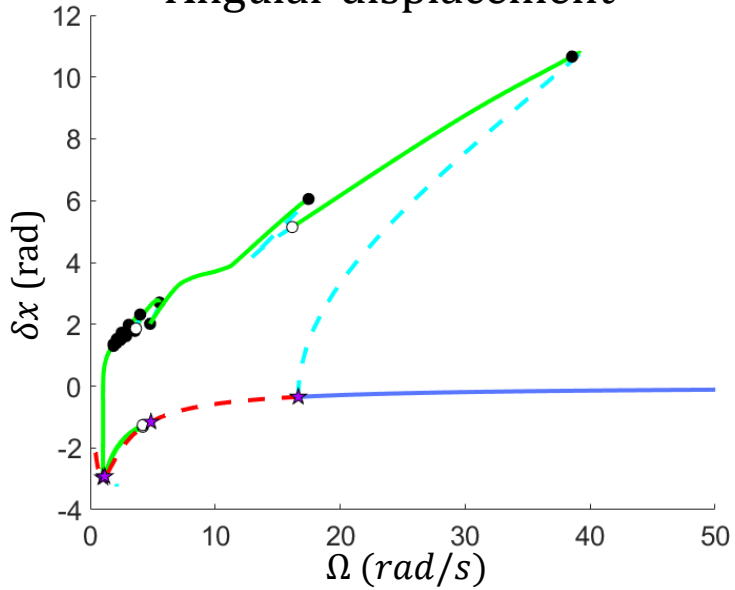
# Results

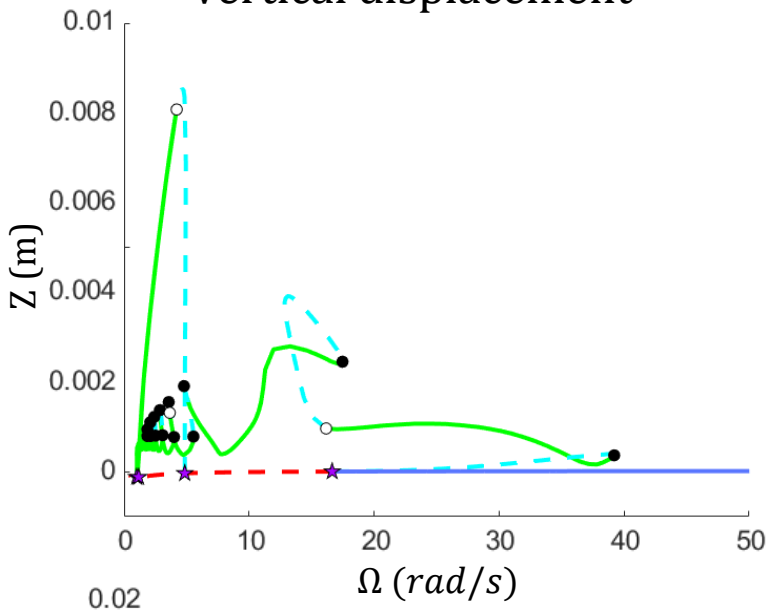
Angular displacement

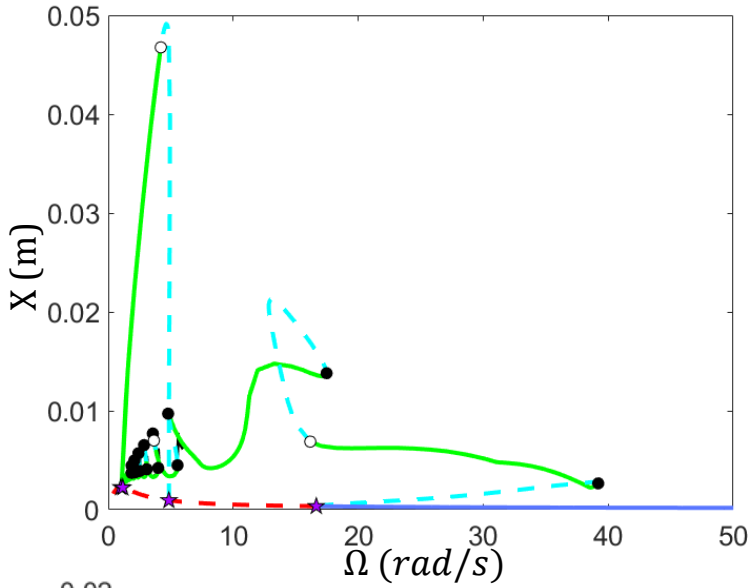
Vertical displacement

Longitudinal displacement

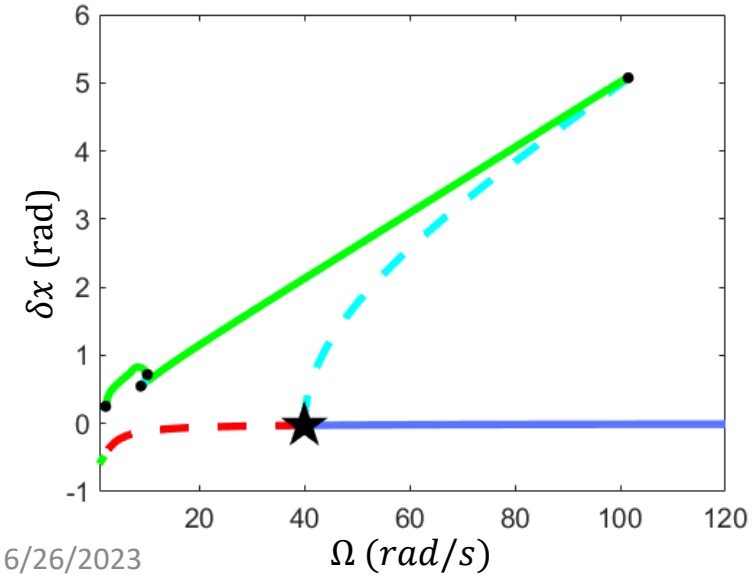
ICE Car

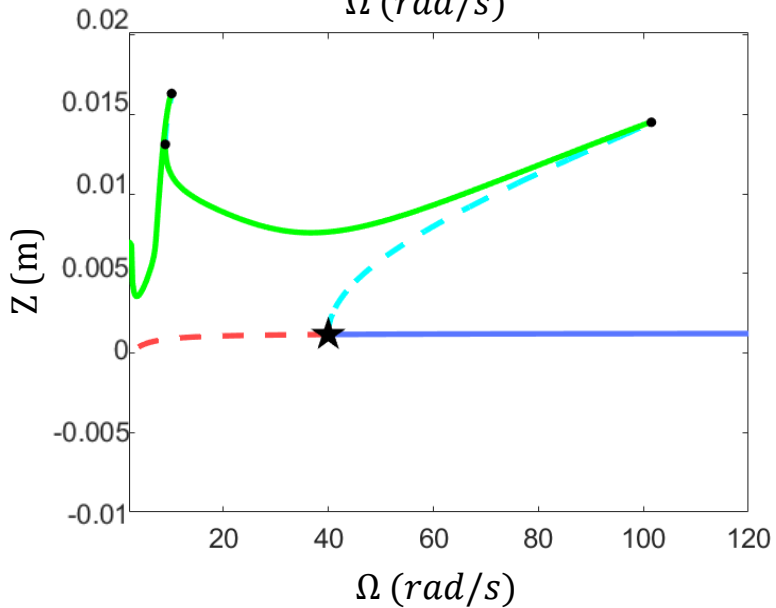


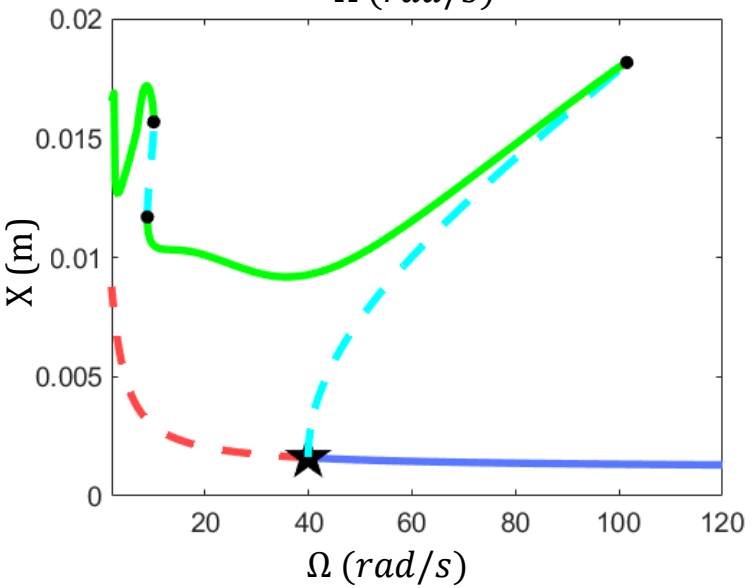




EV Truck



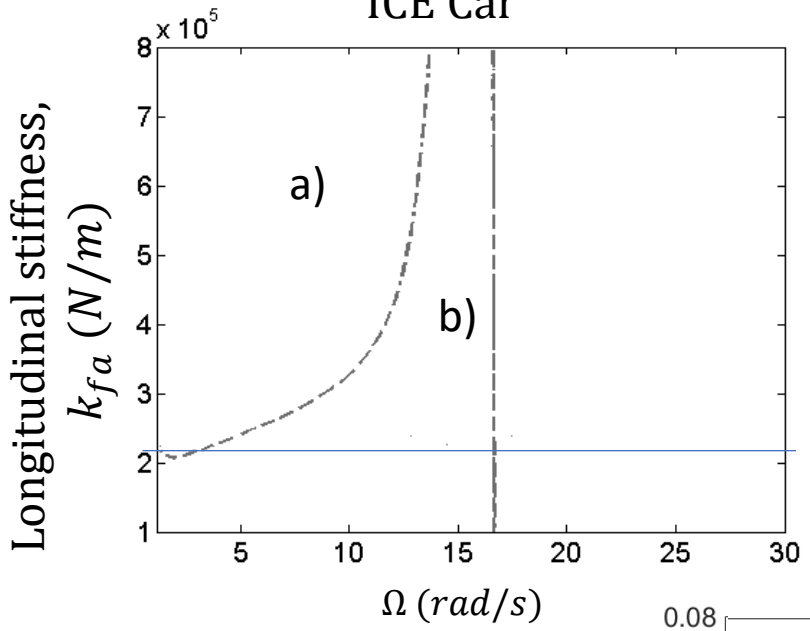



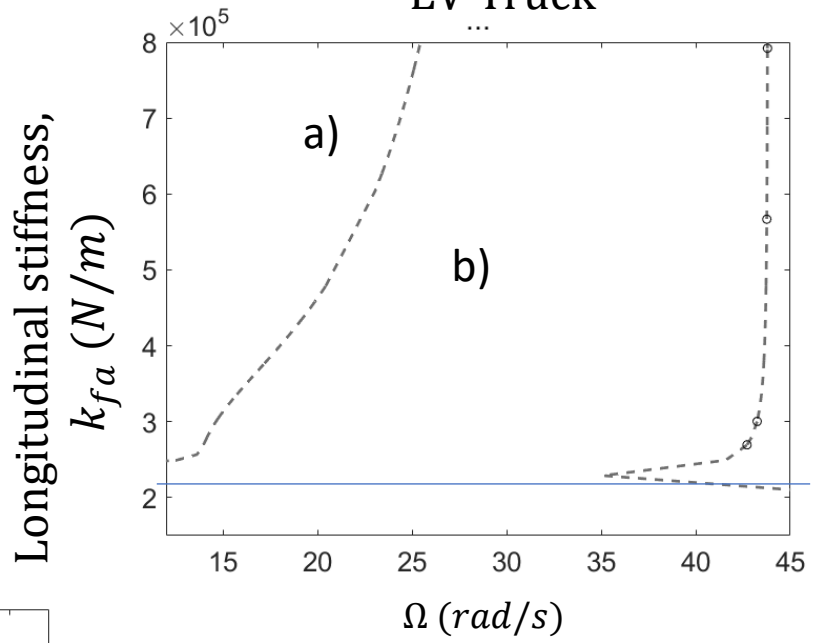
# Results

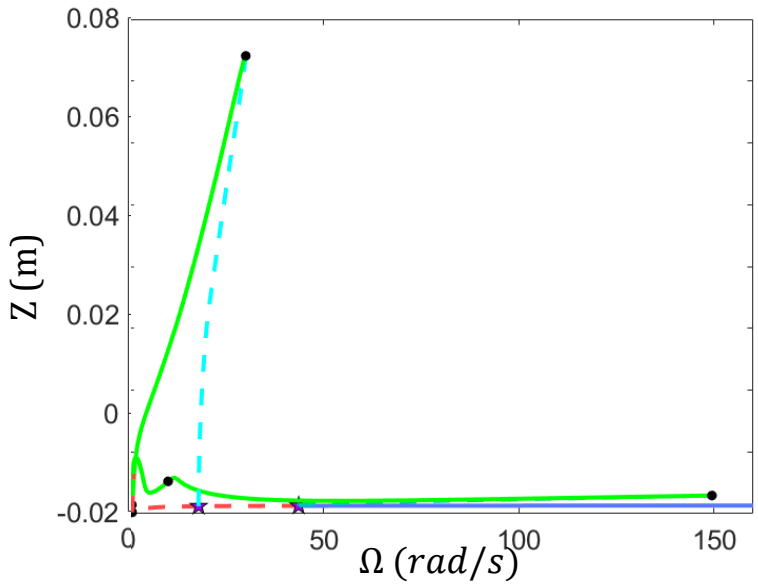


ICE Car




EV Truck



# Harmonic Forcing

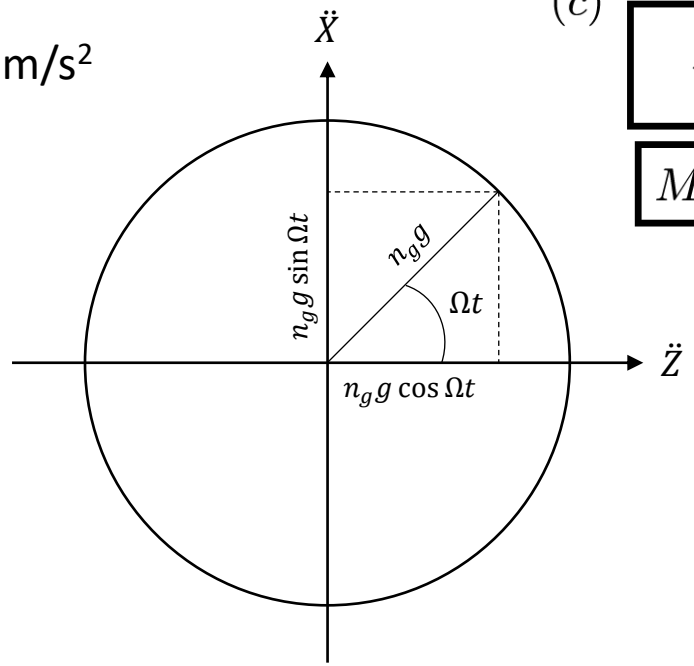


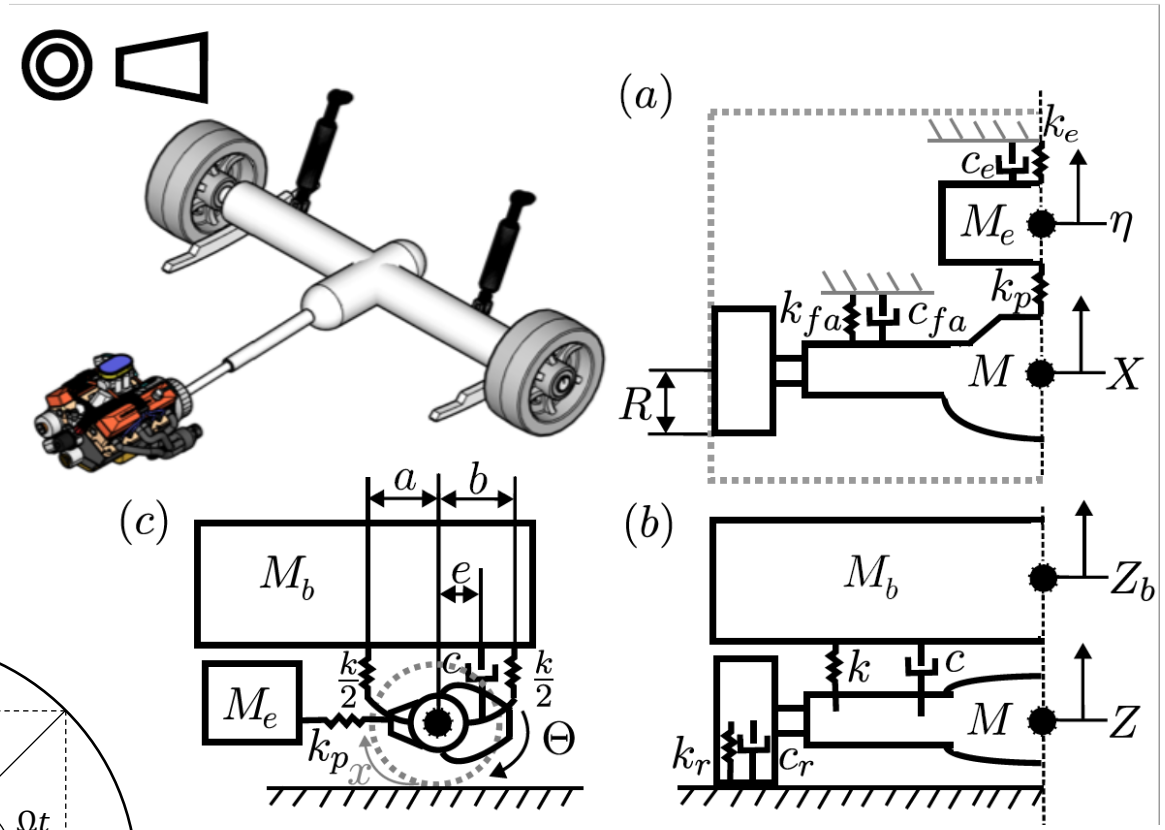
Additional vibration due to harmonic forcing from powertrain:

$$\ddot{Z} = \cdots + n_g g \cos \Omega t$$

$$\ddot{X} = \cdots + n_g g \sin \Omega t$$

$n_g g$: acceleration due to vibration in m/s²
$\Omega$: vibration frequency (rad/s)

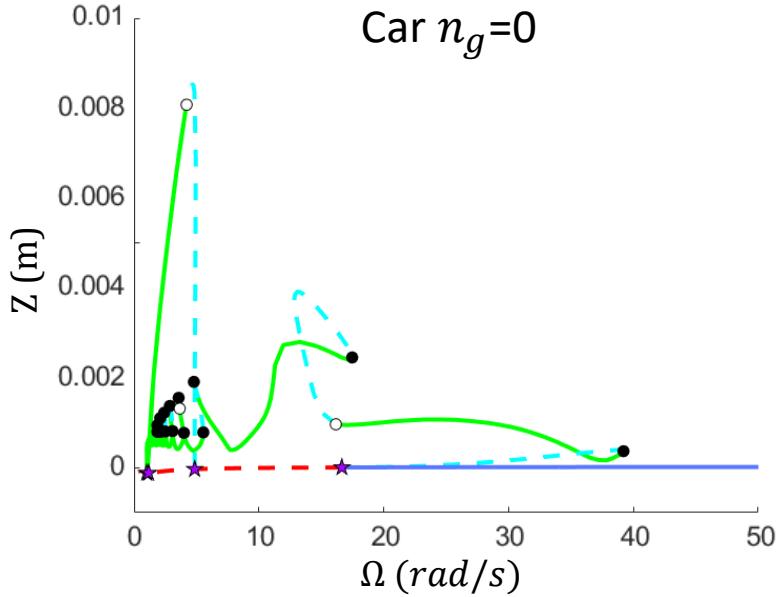


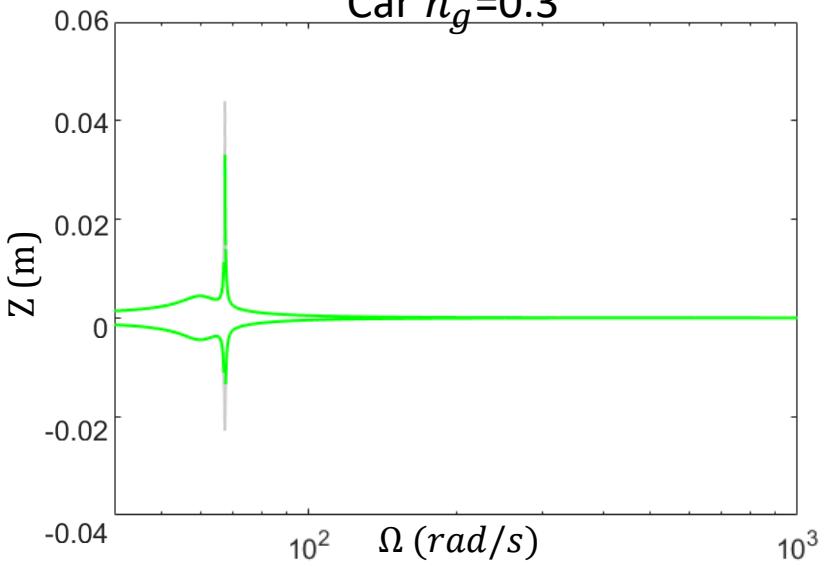
# Harmonic Forcing



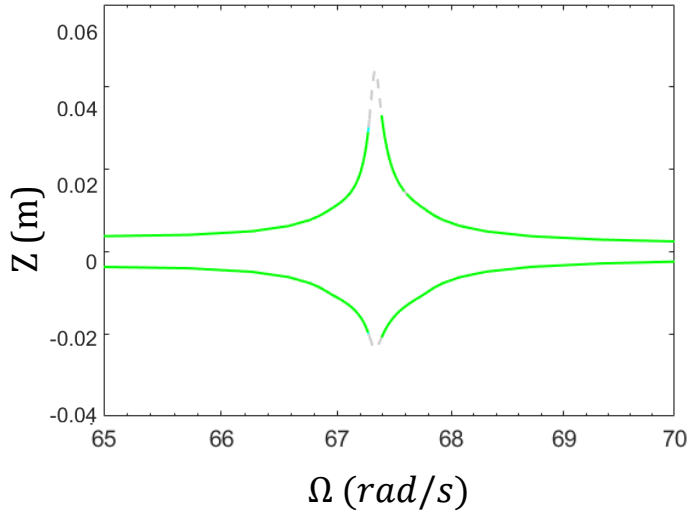
Car $n_g$=0



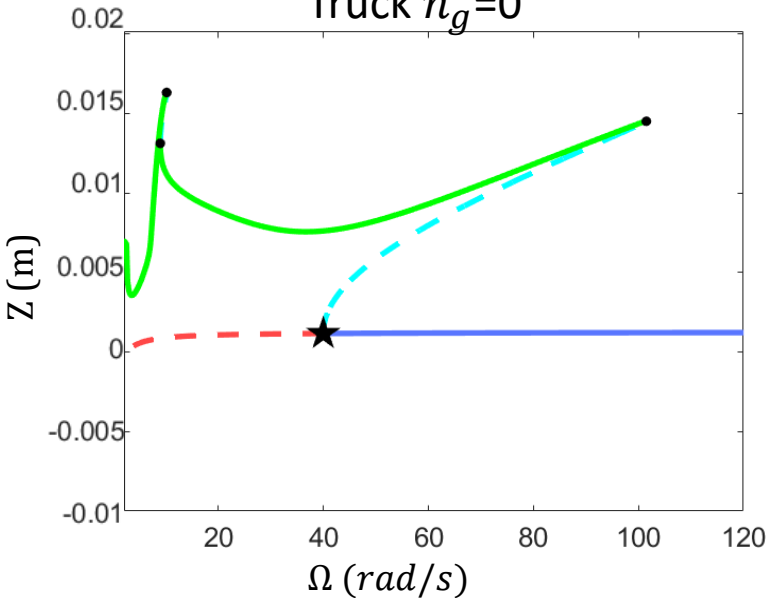

Car $n_g$=0.3



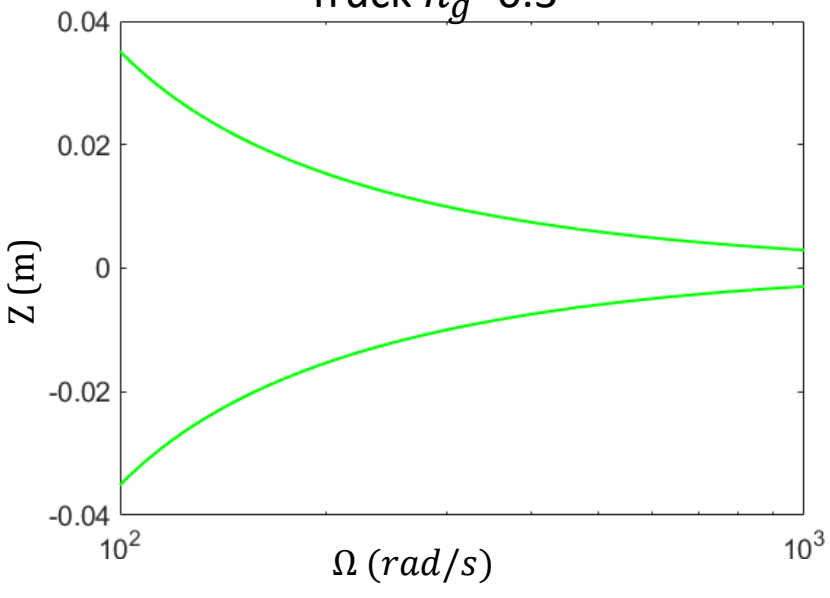

Car $n_g$=0.03 (Zoomed view)




Truck $n_g$=0




Truck $n_g$=0.3



# Conclusion

- Work presents a preliminary bifurcation study into axle tramp of an EV Truck
- Electric system appears to change the axle tramp problem
  - **System still undergoes tramp**
  - Only one limit cycle in EV parametrisation
  - Amplitudes larger in the vertical component, similar size in longitudinal displacement, smaller in angular displacement
  - **More instabilities are observed at lower speed ranges (less stable equilibria) meaning oscillations occur for a larger speed range**
- System forced with harmonic term
  - Truck does not exhibit unstable behaviour as found in the car
- Future work
  - Explore mechanisms by which bifurcations evolve

# References

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[3] S. Smith, J. Knowles, B. Mason, and S. Biggs, “International Journal of Bifurcation and Chaos A bifurcation analysis and sensitivity study of brake creep groan,” *Int. J. Bifurc. & Chaos*.

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