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3D FE-informed laboratory soil testing for the design of offshore wind turbine monopiles

by

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A dissertation submitted to the University of Bristol in accordance with the requirements of the degree of DOCTOR OF PHILOSOPHY in the Faculty of Engineering, Department of Civil Engineering

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Abstract

With the gradual increase in offshore wind turbine capacity, large monopiles having over 10 m diameter for offshore wind turbines (OWTs) are progressively employed in deeper water to exploit more steady and reliable wind power. Numerical simulations are preferred for the prediction of the behaviour of the large-diameter pile. Given the crucial importance of the cyclic long-term constitutive behaviour of the soil to the numerical simulation, appropriate laboratory testing, to determine and calibrate constitutive soil parameters, is required. The current laboratory practice typically relies on triaxial and simple shear tests. However, it is generally recognised that soil element surrounding a laterally loaded pile is subject to a complex stress path.

In this research, finite element analyses of large diameter monotonically and cyclically laterally loaded pile are presented to evaluate the actual stress paths experienced by soil element around the pile foundation. It is found that soil elements typically follow multiaxial stress paths involving rotation of principal stress axes which are more complex than assumed by current practice. Assessments of the limitation of laboratory element testing in reproducing these stress paths are provided alongside a comparison of the soil behaviour observed between these complex and the standard testing practice.

Informed by the FE analysis, granular soil element samples have been tested in the HCTA under cyclic stress paths which simulate loading conditions of soil elements around an offshore monopile foundation. The influence of the simultaneous application of high number (up to about $3 \times 10^4$) of axial and torsional stress cycles on the soil strain accumulation and stiffness evolution is presented. Results show that the accumulated stain and degradation of small strain stiffness are dependent on the direction of the principal stress axes. The results under the triaxial stress condition well fit the high cycles accumulation model (HCA) by Niemunis et al. (2005). However, a discrepancy between the experimental results under the combination of axial and torsional cycles and the HCA fitting curve is observed. A new parameter is therefore incorporated for better capturing the influence of multiaxial stress on the accumulated strain. These data will be valuable for the development and calibration of soil constitutive models to be used in numerical analyses and contribute to the pile design.
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Author’s declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

Signed: ...............................................................

Date: ...............................................................
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List of symbols

\( \varepsilon_z^{\text{amp}}, \varepsilon_r^{\text{amp}}, \varepsilon_\theta^{\text{amp}} \), Amplitudes of strain components

\( \gamma_{\theta z}^{\text{amp}} \)

\( A_0 \)

‘Intrinsic’ dilatancy parameter

\( b \)

Relative magnitude of the intermediate principal stress

\( C \)

Compression to extension strength ratio

\( c_h \)

Hardening parameter

\( D \)

Diameter of monopile (FE)

\( D_{10}, D_{30}, D_{50}, D_{60} \)

Effective particle size

\( D_r \)

Relative density

\( d_S \)

Distance between the outer membrane and the aiming point of the sensor

\( d_{S1}, d_{S2}, d_{S3}, d_{S4}, d_{S5}, d_{S6} \)

Displacement from relative non-contact sensors

\( d_z \)

Settlements of the central part of the sample (= \( d_{S2} - d_{S1} \))

\( e \)

Eccentricity (FE model)

\( e_0 \)

Initial void ratio

\( e_{\text{min}}, e_{\text{max}} \)

Minimum and maximum void ratio

\( E_{sz} \)

Secant axial stiffness

\( f(e) \)

Void ratio function

\( f_0 \)

Natural frequency of offshore wind turbine structures

\( f_{\text{coup}} \)

Coupling ratio

\( f_N \)

Function of the number of cycles

\( f_P \)

Function of the mean effective stress

\( f_Y \)

Function of the average stress ratio

\( G_0 \)

Dimensionless shear modulus

\( G_s \)

Specific gravity

\( G_{s\theta z} \)

Secant shear stiffness

\( G_{\theta z} \)

Shear modulus

\( G_{\theta z,i} \)

Shear modulus at the beginning of cycle batches
$H$  Horizontal load applied on the monopile foundation (FE)

$h_0$  Hardening parameters

$H_c$  Height of the central part of the hollow cylindrical sample

$H_{s}, \Delta H_s$  Height of the hollow cylindrical sample and its variation

$H_{ult}$  Ultimate horizontal load

$J$  Second stress invariant

$K$  Coefficient of lateral earth pressure

$K_0$  Coefficient of lateral earth pressure ‘at-rest’

$L$  Embedded length of monopile (FE)

$L_{Dcyc}$  Length of the cyclic stress path

$L_{Df}$  Linear length between the initial point of the cycle stress path

$L_e$  Length of the embedded pile (FE model)

$L_s$  Height of soil domain (FE model)

$m$  Yield locus opening parameter

$M$  Overturning moment applied on the foundation

$M_{ult}$  Ultimate pile capacity

$N$  Number of cycles

$N_{eq}$  Equivalent number of load cycles

$n^b$  Void ratio dependence parameter (Plastic modulus)

$N^d$  Void ratio dependence parameter (Dilatancy)

$p$  Mean pressure

$p^{ave}$  Average mean stress

$P_i, P_o$  Internal and external pressure of HCTA

$q$  Deviatoric stress

$q^{amp}, q^{ave}$  Deviatoric stresses cyclic amplitude and prestress level

$R$  Radius of monopile (FE model)

$R_s$  Ratio between the minimum and maximum principal effective stress
\( r_{ave,c} \) Average between outer and inner radius of the central part of the sample

\( R_{s,max}, R_{s,min}, \bar{R} \) Maximum, minimum and average value of stress ratio

\( r_o, r_i, \Delta r_o, \Delta r_i \) Outer and inner radius of the hollow cylinder sample and their variations

\( S \) Third stress invariant

\( S1, S2, S3, S4, S5, S6 \) Non-contact transducers of the small strain measurement system

\( t \) Wall thickness of monopile (FE)

\( T, \Delta T \) Torque load on the soil element and its variation

\( t_m \) Thickness of the inner \( (t_{m,i}) \) and outer \( (t_{m,o}) \) membrane

\( V \) Vertical load applied on the foundation

\( V_0 \) Initial volume of the specimen

\( V_m \) Volume due to membrane penetration

\( V_s \) Volume of the specimen

\( W \) Axial load on the soil element

\( \alpha \) Inclination of the major principal stress

\( \beta \) Dilatancy memory parameter

\( \beta_1, \beta_3, \beta_r \) Coefficients to quantify the level of non-uniformity

\( \beta_{xy} \) Inclination of stress path in \( \left( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \right) \) stress plane

\( \beta_{xz} \) Inclination of stress path in \( \left( \frac{\tau_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'} \right) \) stress plane

\( \gamma' \) Effective soil unit weight

\( \gamma_s \) Unit weight of steel

\( \Delta r_{o,c}, \Delta r_{i,c} \) Variation of the outer and inner radius of the central part of hollow cylinder sample

\( \Delta V_i, \Delta V_s \) Inner chamber volume and sample volume variations

\( E \) Deformation matrix

\( \varepsilon^{acc} \) Accumulated strain

\( \varepsilon^{ampl} \) Strain amplitude

\( \varepsilon_q \) Deviatoric strain component

\( \varepsilon_v \) Volumetric strain

\( \varepsilon_z, \varepsilon_r, \varepsilon_\theta \) Axial, radial and circumferential deformation
\( \varepsilon_{x,c}, \varepsilon_{r,c}, \varepsilon_{\theta,c} \) - Axial, radial and circumferential deformation measured by the local strain measurement system

\( \zeta \) - Memory surface shrinkage parameter

\( \zeta_b \) - Cyclic load amplitude

\( \zeta_c \) - Cyclic load asymmetry parameter

\( \zeta_{xy} \) - Inclination of stress path in \( \frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_x - p'}{p'} \) stress plane

\( \zeta_{xz} \) - Inclination of stress path in \( \frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_y - p'}{p'} \) stress plane

\( \eta \) - Stress ratio

\( \eta_{ave} \) - Average stress ratio \( (q_{ave}/p_{ave}) \)

\( \theta \) - Angle from the lateral pile loading direction to the element’s location

\( \theta_c \) - Angle of torque (relative to the central part of the sample)

\( \theta_L \) - Lode’s angle

\( \theta_{ref} \) - Reference rotation at ultimate state

\( \theta_s, \Delta \theta_s \) - Angle of torque (relative to the top of the specimen) and its variation

\( \lambda_c \) - Critical state line shape parameter

\( \nu \) - Poisson’s ratio

\( \sigma \) - Stress state matrix

\( \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz} \) - Stress components of FE soil elements

\( \sigma_z, \sigma_r, \sigma_{\theta} \) - Axial, radial and circumferential stress

\( \tau_{z\theta} \) - Shear stress

\( \varphi' \) - Friction angle

\( x \) - Horizontal distance to the pile

\( \xi \) - Critical state line shape parameter

\( z \) - Depth below ground level
Chapter 1  Introduction

1.1  Background

1.1.1  Global energy demand and the role of offshore wind

The global energy demand has been constantly rising in the last decades due to the increase in industrialization and improvement in living standards. Over 50% growth in the energy consumption by 2030 has been estimated by International Energy Agency (IEA) posting a challenge to energy security in the near future (Razi and Dincer, 2022). Although over 80% of energy is still produced by traditional fossil fuels, a reduction in fossil fuel utilization is required due to the excessive emission of greenhouse gasses, which is considered the main cause of climate change. Therefore, the production of renewable energy has attracted remarkable interest. Over 130 countries and regions have targeted lower carbon emissions by the middle of the 21st century (Dong et al., 2022). The offshore wind energy sector is leading the UK green industrial revolution to meet the net zero target by 2050 (HM Government, 2020), with a plan to quadruple the UK capacity to 40GW by 2030, including 1GW of innovative floating plants, and reach 125 GW by 2050 (Spyroudi et al. 2020; Climate Change Committee, 2020).

Compared to onshore wind, the offshore wind takes advantage of higher and more consistent wind speeds. According to the cumulative capacity recorded in the last decade, the offshore wind capacity has been significantly increasing from 0.5 GW
in 2011 to 3.3 GW in 2021 (Fig. 1.1). A record 3.6 GW of offshore wind capacity has been installed in 2019, mostly located in the North Sea waters.

**Figure 1.1** Annual offshore wind installations in Europe, 2012-2021 (Wind Europe, 2022).

The increase in installed offshore capacity is mirrored by an increase in the wind turbine in the last decades as reported in Fig. 1.2, which is expected to continue in the future. For example, in 1990, typical wind turbines had a rated capacity of about 0.5 MW with rotor diameters of about 15 m, while current design, such as the EU Horizon 2020 project COREWIND (COst REduction and increase the performance of floating WIND technology), is considering wind turbines of 15 MW power, reaching rotor diameters of 240 m (Mahfouz et al., 2021). These latter turbines are typically installed at a great distance from the shore with deep waters to take advantage of steadier and stronger wind zones.

**Figure 1.2** Evolution of offshore wind turbine dimension and capacity (from Bloomberg New Energy Finance).
1.1.2 Offshore wind turbine foundations

The appropriate selection of the foundation type is essential to ensure the stability and cost effectiveness of Offshore Wind Turbines (OWTs). Typical foundation choices for offshore wind turbines are summarised in Fig. 1.3, including gravity base, suction caisson, monopile, tripod foundation, jacket foundation, and also anchors for floating wind systems. Design and installation of the foundation system typically account for around 16% to 35% of the overall costs (Bhattacharya et al., 2021), depending on the location (water depth, seabed conditions) and size of the wind farm. To date, with offshore wind turbines being installed mostly in shallow coastal waters, monopile foundation has been typically the preferred foundation option of developers with over two-thirds of all installations in 2020 (81.2%) (Wind Europe, 2022) due to its ease of installation, economy and logistics (Fig. 1.4). The gravity base foundations accounting for 6% of installed foundations, were developed as alternatives to the monopile at shallow water depth, which consists of heavy prefabricated concrete with flat base (Fig. 1.3). Jacket foundations representing about 9.8% of installed structures, are developed as a cost-effective solution to support larger turbines with improved overturning capacity. Despite being in development, floating structures have the potential to exploit deeper water.

![Figure 1.3 Typical offshore wind turbine foundations (Bhattacharya et al., 2021).](image-url)
Chapter 1. Introduction

Despite the constant increase in turbine capacities, monopiles are expected to continue their market dominance. These open-ended steel monopiles with length ($L$) to diameter ($D$) ratios ($L/D$) of 3 - 6 (Sørensen et al., 2017), are normally employed in water depths up to about 30 – 35 m. However, with the progressive increase of wind turbine capacity and the progressive research for additional and stronger wind resources in deeper water, the size of the monopile foundation has been progressively increasing in the last decade. It is currently well conceivable to install monopiles with a diameter in excess of 10 meters for wind turbines with a capacity of 13 – 15 MW, as predicted in 2025 in Fig. 1.2.

1.1.3 Requirement and recent improvements in monopile design

The increase in monopile foundation size is associated with an increased overall cost of the material as well as with the requirement of specialised vessels and installation equipment able to deal with the larger monopile sizes and weights. However, cost savings are necessary to ensure the long-term sustainability of wind farm developments if compared to other onshore energy resources. Therefore, engineers are not only facing the challenges of designing larger foundations under stricter and harsher environmental conditions, but they are also continuously asked to increase the cost-effectiveness of their design.

Optimisation of the design of the foundation has been sustainably developed such as the very recent Pile Soil Analysis (PISA) project (Burd et al., 2020; Byrne et al., 2020), in which a new cost-effective 1D design methodology considering the
interaction between the soil and monopile derived from complex 3D analyses were established. Extensive research accounting for installation procedure (Staubach et al., 2020, 2021a and 2021b), cyclic degradation (Yang et al. 2022; Achmus et al., 2009), and derivation of soil parameters (Andersen, 2015) also has been carried out for improvement of foundation design. It should be noted, these design methodologies have highlighted the importance of the appropriate selection of the constitutive model and calibration of its parameters in any numerical analysis, and the development of new soil reaction curves, which invariably rely on correlations from in-situ testing and/or high quality, representative laboratory element testing on undisturbed or reconstituted samples.

However, most of these design procedures, considering the influence of material, relative density, stress amplitude, stress level, cyclic stress ratio, and drainage conditions on the cyclic response (e.g. Wichtmann, 2005; Andersen, 2015), follow a simplified loading conditions (either triaxial or direct simple shear condition. In real situation the stress conditions are more complex involving the simultaneous variation of normal stress and shear stress (Arthur et al., 1980). The monopiles are designed to sustain $10^8$ to $10^9$ environmental cycles for a lifetime of 25+ years. The pile capacity governed by the ultimate limit state (ULS), the accumulated rotation in the serviceability limit state (SLS) and the pile stiffness in the fatigue limit state (FLS) assessment in its long-term lifetime (Jostad et al., 2020) are inevitably affected by the changes in surrounding soil behaviour under such complex stress conditions. In light of the above considerations, the identification of accurate stress conditions in soil elements surrounding the monopile foundation will be beneficial to the improvement of the design.

### 1.2 Research questions

Appropriate laboratory element testing has been demonstrated to be essential for the pile design by offering accurate and detailed assessments of the nonlinearity, aging and anisotropy of geomaterials, which is highly dependent on the general stress paths. However, it seems that typical simplified loading conditions involving cyclic triaxial and/or simple shear tests cannot fully or reliably represent the soil stress path conditions developed around the laterally loaded pile. Therefore, this leads to the following research questions for this doctoral thesis:
(a) what are the real stress conditions followed by the soil elements surrounding the laterally loaded monopile foundation?
(b) what is the mechanical soil response under cyclic stress paths truly representative of those experienced by soil elements, and how does it differ from standard laboratory procedures?
(c) how will the exploration of the stress conditions and soil response affect the OWT monopile foundation design approaches?

1.3 Main aim and research objectives

In order to provide answers to the above research questions, the research work described in this thesis can be divided into two parts.

The first part of the thesis will investigate, using a 3-D finite element tool, the actual stress paths experienced by soil elements in the vicinity of a monopile foundation for offshore wind turbines. Informed by the obtained stress paths, the research will assess (i) whether the current laboratory testing procedures for offshore wind turbine design closely align with field situations and (ii) how laboratory procedures could be enhanced to better (and feasibly) simulate the stress paths experienced by the soil.

Building upon the results from part one, the second part of the research will aim to experimentally investigate what would be the effect of applying the enhanced testing procedures on experimental element testing results. It can be anticipated that, differently from standard practice, an advanced multiaxial loading device (i.e. Hollow Cylinder Torsional Apparatus (HCTA)) for soil testing, is expected to be used at this stage.

The specific objectives of the two parts of this thesis are summarised below:

Part 1: investigation and definition of real soil stress paths

- To identify the main features of the stress paths experienced by soil elements in different positions around the pile subjected to monotonic and cyclic loading.
- To assess which laboratory testing procedures can reproduce these stress paths.
Chapter 1. Introduction

- To provide suggestions for replicating these stress paths through laboratory element testing.

Part 2: Laboratory assessment of soil response under some prescribed stress paths

- To evaluate the strain accumulation of granular soils subjected to a large number of multiaxial cyclic loading, which resembles those in soil elements around a monopile foundation.
- To explore the stiffness evolution of granular soils under the same high number of multiaxial.

1.4 Thesis outline

This thesis consists of seven chapters, including this introduction, a review of the relevant research, the core contents, the final concluding remarks and future recommendations. The key novel findings are presented in Chapters 3 -- 6, where Chapters 3 and 4 relate to part 1 and Chapters 5 and 6 relate to part 2 of the research objectives. These four chapters are based on three journal papers, either published or in progress (see relevant references as follows).

Chapter 2 provides an overview of the adopted methodologies for designing monopile foundations and explains how laboratory element testing is blended within and affects the overall design process. The chapter also summarises past numerical and experimental evidence which investigated the effect of the complex loading conditions on the cyclic performance of monopile foundations and surrounding soils, to identify knowledge gaps and needs for further research.

Chapter 3 reports the 3D FE results for the monopile foundation under lateral monotonic loading. The aim of the chapter is to numerically investigate relevant soil pile interaction mechanisms, to determine typical stress paths for soil elements at different locations around the pile and assess whether laboratory element testing procedures can mimic those stress paths and identify the main shortcoming. This chapter is based on a published journal paper (see list of publications below): ‘Large diameter laterally loaded piles in sand: Numerical evaluation of soil stress paths and relevance of laboratory soil element testing’.
Chapter 1. Introduction

Following the work in Chapter 3, Chapter 4 extends the numerical study of the stress paths to cyclic loading. The influence of loading amplitude and relative density on soil element’s stress paths induced by horizontal pile cyclic loading is investigated. This chapter represents the base for a journal paper in preparation for submission: ‘Numerical evaluation of stress paths around large diameter cyclically laterally loaded piles and relevance of laboratory soil element testing’.

Chapter 5 details the HCTA including the loading system, measurement system and acquisition and control system. An additional small strain measurement system is particularly introduced. Test material and testing procedures including saturation, consolidation and shearing are also provided in this chapter.

Chapter 6 summarises the results of the experimental tests. Complementary laboratory testing using the HCTA is carried out on granular soil samples under cyclic stress loading (with up to about $3 \times 10^4$ cycles) which resemble those determined by 3D finite element analysis. The small strain stiffness evolution and accumulated strain throughout the experimental tests are analysed. An advanced strain accumulation model from the literature has been revisited to consider the multiaxial nature of the stress paths. Similarly, this chapter includes the work of a journal paper ‘Evolution of stiffness and accumulated strain of sand under large cyclic loading with rotation of principal stress axes which is in progress to be submitted.

Chapter 7 summarises the research outcomes and draws conclusions. Limitations and suggestions for future work are proposed.

1.5 List of supporting publications

This doctoral thesis includes the work published or prepared for publication in the following articles. Note in order to extend the application of the numerical model used in this work to the residential soil, the author has contributed to a publication by Consoli et al. (2022), which is not included in this work.

Journal publications

paths and relevance of laboratory soil element testing. Computers and Geotechnics, 154, p.105139.


Conference publications


Papers in progress

1. Numerical evaluation of stress paths around large diameter cyclically laterally loaded piles and relevance of laboratory soil element testing.

2. Evolution of stiffness and accumulated strain of sand under large cyclic loading with rotation of principal stress axes.
Chapter 2  Literature Review

2.1 Introduction

This chapter provides an overview of the main procedure and aspects of the design of large monopiles for offshore wind turbines. Initially, the discussion touches upon the key monopile design aspects including design loads, criteria and methodologies. Particular focus is devoted to the 3D finite element method, with emphasis on recent progress in numerical modelling of the laterally loaded pile, under both monotonic and cyclic conditions. It is shown that design procedures are typically underpinned by cyclic experimental testing at the soil element scale, essential to determine and calibrate constitutive soil parameters for the design of offshore wind monopile foundations or to apply cyclic degradation or deformation accumulation processes. Therefore, the latter part of the chapter provides evidence of the typical cyclic behaviour of soil obtained in laboratory element tests and discusses the governing variables and behavioural features. The final part of the chapter highlights the importance of considering the multiaxial stress conditions for reliable predictions of pile performance and pile-soil interaction and identifies key research gaps that this research will try to fill.
2.2 Design of monopile foundation

2.2.1 Design loads

Monopile foundations of offshore wind turbine structures are systematically designed to withstand: (a) static load due to the self-weight of the structure; (b) the cyclic loading characterised by varied frequency, amplitude and direction induced by the complex environmental actions (combination of wave, water currents and wind); and (c) operational turbine’s loadings (rotor frequency, 1P, and tower shielding blade passing, known as 3P) over an operative life period of 20 to 25 years, as shown in Fig. 2.1. According to Kallehave et al. (2015), a typical 3.5 - 5 MW offshore wind turbine possesses the self-weight in the range of 6 -10 MN. The contributions of the wave and wind loads acting on the shaft of OWT to the lateral force and overturning moment are dependent on the heights of the application of the loads. Within this context, the lateral force representing around 25% and the overturning moment representing 75% arising from wind are considered for pile design (Byrne and Houlsby, 2003).

Figure 2.1 Typical loads acting on offshore wind turbines 5 MW offshore wind turbine in water depth from 20 to 50 m (modified after Nikitas et al. (2016) with data from Byrne and Houlsby (2015)).
Due to the complexity, error accumulation and high cost, it is unrealistic to apply these irregular load histories to the design procedures and to the experimental tests (Andersen, 2015). In current design practice, the irregular loading histories are usually translated to idealised packages of cycles with constant amplitude, average load and load frequency in methods such as rain flow counting (Matsuishi and Endo, 1968; Kaggwa et al., 1991) (steps from a to b in Fig. 2.2). Based on the comparable fatigue damage, further simplification can be conducted by converting the individual load packages with corresponding numbers of load cycles into a single load package (typically for the largest load level) with an equivalent number of load cycles, \( N_{eq} \) (step c in Fig. 2.2). There are different ways to determine the equivalent load packages (e.g. LeBlanc et al., 2010a; Noren-Cosgriff et al., 2015; Jalbi et al., 2020), and different methods may result in different outcomes (Andersen, 2015). In addition, the multiaxial nature of the loading condition is typically neglected and the literature investigations regarding the multi-directional loading of monopile foundations are limited (Sheil and McCabe, 2017). The same applies to industrial practice, which typically carries out the monopile design for an equivalent and most demanding loading direction.

![Figure 2.2 Translation of irregular environmental load to regular load history.](image)

### 2.1.1 Design variables

Monopiles are simple steel hollow cylindrical tubes as shown in Fig. 2.3. The design of the monopile for OWTs considers the following three main geometrical degrees of freedom:

**The length of the monopile.** It is generally decided by the ultimate capacity of pile or the acceptable accumulated rotation induced by the environmental and operational loads.
The diameter of the monopile. It is confirmed by the natural frequency of the offshore wind turbine, which depends on the surrounding soil stiffness.

The wall thickness. It should be designed to withstand shell buckling at pile penetration stage and fatigue loads. The wall thickness of the monopile is typically variable over its length to minimise the amount of steel and optimise costs.

Figure 2.3 A 7.5 m diameter monopile for Gode Wind Offshore Wind Farm. DONG Energy.

2.2.2 Design criteria

Section 2.2.1 has clarified that the length, diameter and wall thickness of the monopile foundation must be designed to ensure the safety of OWT under environmental loads. The typical design criteria for monopiles are identified in the offshore design code - DNVGL-ST-0126 (DNV, 2018), with a particular focus on 7.6.2 ‘Design criteria for laterally loaded piles’. Current practice for the design of OWT monopile includes the Ultimate Limit State (ULS), Accidental Limit State (ALS), Serviceability Limit State (SLS) and Fatigue Limit State (FLS) checks (DNV, 2018):

(a) ULS design ensures the monopile foundation possesses the necessary maximum load-carrying resistance.
(b) ALS design ensures the monopile foundation resists accidental loads or post-accidental integrity for damaged structures.
(c) SLS design requires lifetime performance under repeated routine loading.
(d) FLS design predicts the fatigue life of the monopile under dynamic loading. Fig. 2.4 schematically shows examples of the SLS and ULS modes of failure.

![Diagram of ULS and SLS failure modes](image)

**Figure 2.4 Examples of ULS and SLS failure (Arany et al. 2017).**

Two particular criteria seem to be demanded for the cyclic design of a monopile foundation:

*R Rotation criteria.* A strict limit on the foundation tilt over its lifetime is $0.5^\circ$ according to the SLS check (Achmus et al., 2009; DNV, 2018), including maximum installation verticality of $0.25^\circ$ (DNV, 2018).

*D Dynamic criteria.* The dynamic criteria are particularly for monopile design which is related to the FLS check. Due to complex environmental loads acting on the OWT, it is essential to avoid the mutual effect of these loads. Fig. 2.5 illustrates the typical frequency spectrum of the wind and wave loads and rotor frequencies $1P$ and blade-passing frequencies $3P$ of the turbine. The natural frequency of the overall offshore wind turbine system is determined by the support conditions such as foundation dimensions and soil stiffness. As dynamically sensitive structures, the OWTs are usually designed with the first natural frequency of the structure lying outside the excitation frequencies triggered by the operational and environmental loading (waves and winds), to minimise the development of fatigue damage and avoid resonance. Typically, the so-called soft-stiff design is selected as the most economical option, in which the natural frequency of the OWTs lies between $f_{1P}$
and $f_{3P}$ (see Fig. 2.5). The narrow band of the target design frequency ($f_0$) necessitates an accurate prediction of the foundation stiffness. In contrast, the soft-soft design ($f_0$ falls below the 1P range) for flexible structures is hard to bear extreme aerodynamic and hydrodynamic loads since the soft-soft range approaches the wave and wind loading frequencies. While the stiff-stiff design ($f_0$ higher than the 3P) for extremely stiff structures appears to be less used due to the prohibitive cost.

![Figure 2.5 Typical excitation range for offshore wind turbines (Kallehave et al., 2015).](image)

The monopile normally works at the serviceability state under long-term cyclic loading, while the millions of loading cycles can cause the progressive rotation accumulation of the foundation and affect the strength and stiffness of the pile, as shown in Fig. 2.6. Fig. 2.6 clearly depicts the typical cyclic force–displacement response of monopile, in which the displacement accumulation rate tends to decrease (ratcheting), suggesting that the reloading secant stiffness ($E_{c}$) is higher than the initial secant stiffness, and also grows with every cycle, while the overall secant stiffness ($E_s$) decreases with increasing the number of load cycles. These features should be considered in monopile foundation design, details will be discussed in the following sections.
In conclusion, the key factors driving the design for offshore wind turbine foundations are:

(a) prediction of accumulated permanent rotation/displacement of monopile, governed by plastic soil deformation around the pile.

(b) accurate determination of foundation stiffness, governed by the soil stiffness. The strain accumulation of soil surrounding the monopile can lead to changes of small strain soil properties, which will potentially result in system resonance and consequent fatigue problems.

### 2.2.3 Design methods

The growth of offshore renewable energy in the last decade has considerably driven offshore wind farms to grow in size. This tendency naturally poses significant challenges to foundations (Pisanò & Gavin, 2017; Versteijlen, 2018; Kementzetzidis et al., 2021). Optimisation of foundation design is necessary to help reduce the uncertainty and minimise the cost.

The procedures for monopile foundation design require careful consideration and robust modelling of monopile-soil interaction mechanisms under the cyclic lateral loading induced by the operational and storm loading conditions. There are typically three possible design approaches to model the monopile soil interaction under lateral loading (Fig. 2.7): (a) macro-element approach, (b) Winkler type (or p-y) approach and (c) 3-D FE analysis.
2.2.3.1 Macro-element method

As the most simplistic approach, the macro-element model is represented by a surface spring, which is characterised by the force and displacement response at the seabed (Page et al., 2018), as shown in Fig. 2.7a. Despite the significant increase in the time efficiency of the numerical analysis due to its simplification, however, at the expense of diminished accuracy (Skau et al., 2018), this model holds no information on the deformations and forces along the entire pile embedment. Therefore, following the suggestions by Skau et al. (2018) and Muir Wood (2004), the implementation of the macro-element into 3D FE modelling or physical modelling is necessary.

The recently developed macro-element project, the REDucing cost in offshore WINd (REDWIN) by integrated structural and geotechnical design project led by NGI (Page et al., 2018), aims to reduce the costs in the design of OWTs by developing soil-foundation models. Apart from the improvement in computational effectiveness, the proposed model can also guarantee the accuracy of the analysis compared to the traditional modelling methodologies for soil-structure interaction by implementing the kinematic hardening function. Two models have been developed for monopile foundations (Fig. 2.8): (a) one model was developed on the basis of the standard p-y curve method; (b) one model acted as a structural
boundary at seabed through the macro-element approach. The REDWIN model, with the capability of capturing the hysteretic response, has been validated against the results from PISA field tests (Byrne et al., 2017). However, the influence of cyclic loading on soil strength and stiffness is not included.

<table>
<thead>
<tr>
<th>Foundation and substructure</th>
<th>Model applicable</th>
<th>Loading regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redwin model 1</td>
<td>Distributed 1D model to be applied to any DOF.</td>
<td></td>
</tr>
<tr>
<td>Redwin model 2</td>
<td>Horizontal and Moment loading</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.8 Available soil models for monopile foundation (Skau. et al., 2018).

Houlsby et al. (2017) developed the robust Hyperplastic Accelerated Ratcheting Model (HARM), which can accommodate the ratcheting, hysteretic response and evolution of loop shape by including a ratcheting element and the selection of empirical evolution functions. The HARM framework has been successfully implemented as a single 0-D macro-element model by Abadie et al. (2019) to simulate the response of monopile foundation, in which the coupled moment-rotation \((M - \theta)\) response (Fig. 2.9a) and lateral force-displacement \((H - v_G)\) response (Fig. 2.10b) induced by the wind and wave loading is replaced by a single lateral reaction \(H - v_T\) (Fig. 2.9c).
2.2.3.2 The Winkler-type approach or p-y curve method

As recommended by current design codes, such as API (2014) and DNV (2018), the soil-pile interactions are generally predicted by the Winkler-type approach (or p-y curve method), which was originally proposed by Reese & Matlock (1956) and McClelland & Focht (1958) based on the Winkler approach (Winkler, 1867). This approach follows the assumption that the pile acts as an elastic beam and the embedding soil is represented by a series of independent nonlinear springs along the length of the monopile (Fig. 2.7b).

The p-y curve is derived mainly from field tests and has been validated for flexible piles with small diameter and long length (Achums et al., 2009). However, as indicated in Chapter 1, recently installed monopiles for OWTs tend to have a large diameter and relatively small length to diameter ratios, thus it is necessary to justify the suitability of the p-y method for the current monopile. Doherty et al. (2012) compared the pile lateral force-displacement curves obtained from field tests, 3D FE analysis and API method (Fig. 2.10). The result indicated a significant underestimation of the ultimate pile capacity with the p-y method recommended.
by API guidance. Although great progress for the modified static $p$-$y$ method has been achieved by considering the stiffness degradation and strength of monopile, detailed progress such as the effect of the number of loading cycles or the amplitude of cyclic loading is not taken into account (Byrne et al., 2017).

![Figure 2.10 Load-displacement comparison predicted by the API guideline, finite element data and field test results from Doherty et al. (2012).](image)

**2.2.3.3 Finite Element (FE) method**

The finite element method appears to be an appropriate way to predict the detailed and realistic response of the monopile foundation for simulating complex soil stratum (Fig. 2.7c). However, difficulties would arise regarding the challenging and time-consuming calculation for the pile design.

Remarkable improvements in the design methodologies involving the FE analysis have been achieved by the industry in the past few years, such as the updated distributed soil-spring models - Pile Soil Analysis (PISA) project (Burd et al., 2020). A major option of the PISA design procedure requires the extraction of the $p$-$y$ curve from 3D FE analysis (Fig. 2.11a), which then was implemented into a simplified 1D model of monopile, as shown in Fig. 2.11b. The PISA project has made a fundamental contribution by the beneficial effect of recognising all pile-soil structure interaction mechanisms including distributed moment ($m$) and horizontal ($H_b$) and the distributed lateral response ($p$), and base reactions ($M_b$) and introducing them in the design (Fig. 2.11b). Appropriate choice of constitutive model and accurate calibration for the constitutive model play an important role in this approach (Burd et al., 2020; Byrne et al., 2020). The advanced approach addresses the limitation of the standard $p$-$y$ curve method as discussed in Section
2.2.3.2. In addition, an accurate prediction is ensured by the 3D FE analysis with a relatively efficient computational speed. However, at the current stage, the PISA methodology was only validated against monotonic loading and the extension to cyclic loading is still in progress.

Figure 2.11 PISA design model: (a) idealisation of the soil reaction components acting on the pile; (b) 1D finite-element implementation of the model showing the soil reactions acting on the pile (Burd et al., 2020).

Following the assumption that the failure mechanism can be characterised by laboratory testing shown in Fig. 2.12, Andersen (2015) proposed the cyclic contour diagram (Fig. 2.13) obtained from a series of undrained triaxial (CUcyc) or direct simple shear (DSScyc) tests for Drammen clay to estimate the bearing capacity of offshore shallow foundations. The contour diagrams are plotted in terms of the normalised cyclic shear stress ($\tau_{cy}$) against the average shear stress ($\tau_a$) by the undrained shear strength ($s_u$). The numbers of cycles to failure ($N_t$) are taken from the moment that either average ($\gamma_a$) or cyclic ($\gamma_{cyc}$) strain exceeds 15%. This methodology has been implemented in PLAXIS software by calculating the equivalent number of cycles using the cyclic strain accumulation procedure at the integration (Gauss) points developed by NGI, such as the UnDrained Cyclic Accumulation Model (UDCAM, see Jostad et al. (2014)). As an alternative to the
PISA approach, the UDCAM model accounts for the effect of cyclic loading in undrained conditions. However, the $p-y$ curve is not derived in this model, instead the response is ‘degraded’ by tracking contour diagrams at each integration point.

![Figure 2.12 Simplified stress conditions for typical elements along a potential failure surface beneath a shallow foundation (Andersen, 2015).](image)

Figure 2.13 Cyclic Contour diagram: (a) $DSS_{cyc}$; (b) $CU_{cyc}$ (Balaam, 2020).

In summary, the 3D finite element (FE) analyses are widely employed in the design of monopiles either for the direct prediction of pile behaviour or for extracting soil reaction curves which can be incorporated into simplified one-dimensional analyses (e.g. PISA design approach), if a large number of foundations and load cases must be scrutinised. The following section will focus on the numerical analysis on the response of monopile under lateral cyclic loading.
2.3 Numerical analysis of laterally loaded pile

2.3.1 Introduction

Beyond the current design procedure, the numerical analysis provides a valuable means to predict the behaviour of the monopile (e.g. Byrne et al., 2015; Zdravković et al., 2015; Kementzetidis et al., 2019), which can be validated by the real behaviour of monopile obtained from pile testing. For example, the previously mentioned industry-led PISA project (Fig. 2.11) involved full-scale field testing, in which a series of piles with diameters up to 2 m were tested for the validation of the 3D FE model (Burd et al., 2020; Byrne et al., 2020; McAdam et al., 2020).

Therefore, relevant research in terms of pile testing will be provided in Section 2.3.2, and numerical analysis will be reviewed in Section 2.3.3.

2.3.2 Pile testing

Full-scale, centrifuge and physical modelling tests are typically used for the investigation of the response of piles subject to cyclic lateral load.

2.3.2.1 Full-scale tests

Field tests on five full-scale piles with dimensions of outer diameters equal to 762 mm and embedded lengths varying from 4 m to 10 m in low to medium density chalk were performed by Ciavaglia et al. (2017) to investigate the response of short and long piles under cyclic loading packages up to 100 cycles. The experimental results revealed that the tangent stiffness decreases as the cyclic loading proceeds, which agrees well with the findings indicated in Fig. 2.4. A near-linear increase of the accumulated lateral pile head displacement with the logarithm of the number of cycles \((N)\) was observed for lower load amplitude, while the increasing rate of the accumulated displacement keeps increasing with the number of cycles for larger load amplitude. The author suggested that the peak loads should remain lower than 11% of the lateral capacity for the Chalk to ensure the serviceability of offshore wind turbines.

Generally, data monitoring of full-scale monopile foundations relevant to offshore loading conditions is scarce and, for practical reasons, limited to a low number of
uni-directional and regular excitation cycles (PISA project by Burd et al., 2020; Ciavaglia et al., 2017). Conversely, the investigation on a much larger number of loading cycles was possible through model-scale lateral loading pile tests (LeBlanc et al. 2010a, b; Cuéllar et al., 2012).

2.3.2.2 Physical modelling tests

Remarkable work on pile behaviour to long-term cyclic loading has been achieved through physical modelling tests. LeBlanc et al. (2010a), for example, applied up to 60000 combined lateral load and moment cycles on the 1-g scale copper-driven monopile with geometries of 80 mm for diameter and 360 mm for penetration depth in drained sand. The author evaluated the variation of the accumulated rotation and secant stiffness of monopile with the number of loading cycles, which supported the following conclusions: (a) the accumulated rotation highly relied on the cyclic modes, particularly pronounced accumulation magnitude was observed for the loading lying between one- and two-way cyclic loads; (b) An increasing trend of secant pile stiffness was induced with increasing loading cycles. Further study on the effect of random two-way cyclic loading with the same testing set-up was also performed by Leblanc et al. (2010b). A model therefore was proposed by superposing the strain increments to Miner’s rule. The framework by LeBlanc et al. (2010a, b) has the potential for predicting the performance of full-scale monopile.

An interesting physical test in 1-g conditions was performed by Cuéllar et al. (2012), who used coloured particles to track the surrounding sand movement during lateral pile loading cycles (Fig. 2.14). They identified the convective domain, which may be helpful for better understanding the ratcheting behaviour of the pile.
Figure 2.14 Particle migration at soil surface and evidence of convective region following (Cuéllar et al., 2012).

These physical modelling tests at 1-g scale appear faster and more cost-effective approaches to predict the behaviour of full-scale prototypes. However, the stress-state in surrounding soil substantially different from that around a full-size monopile foundation. In this sense, further validation against the field stress condition around the prototype is required, which can be achieved through centrifuge tests. The following section will show the experimental evidence in terms of centrifuge modelling tests.

2.3.2.3 Centrifuge modelling tests

Verdure et al. (2003) employed a number of centrifuge tests to evaluate the behaviour of monopile under lateral cyclic loading up to 50 cycles. The authors observed that the secant stiffness of each cycle increases slightly as the cycles proceed. This result is consistent with the evolution of $E_c$ in Fig. 2.4. Remarkable accumulated permanent pile head displacements were found to increase with loading cycles, in which the relationship between them appeared in the logarithmic form.

The rigorous PISA design methodology has emphasised the importance of considering the lateral and base moment resistance of the monopile foundation (Taborda et al., 2020; McAdam et al., 2020;). In order to investigate the base resistance as well as the degradation of the soil resistance under short-term symmetrical cyclic lateral loading, Takahashi et al. (2022) recently conducted
centrifuge model tests on rigid monopiles with slenderness ratio (embedded pile length to diameter) varying from 3.75 to 8 in partially drained sand. This study revealed an important result that the slenderness ratio has a significant influence on the moment resistance at the base. It was also found that the same loading and natural frequencies can lead to the degradation of the soil resistance even for the foundation surrounded by dense sand.

The stress levels vary significantly among the aforementioned pile testing. In order to assess the influence of stress levels on the pile behaviour in dry dense sand, Richards et al. (2021) performed centrifuge model tests at three different g-levels (1-g, 9-g and 80-g). The results showed that the rate of ratcheting and change of secant stiffness decrease logarithmically with the increasing stress level, which highlighted the need to perform full-scale stress level to gain an accurate understanding of pile behaviour.

Both 1-g physical modelling tests and centrifuge tests have shown an exponential or logarithmic relationship between the lateral pile displacement accumulation and the number of cycles under one-directional loading (Leblanc et al., 2010a, b; Verdure et al., 2003). However, the loading from waves and wind is multidirectional in nature. A centrifuge test at 200g was conducted by Rudolph et al. (2014), representing a prototype pile with diameter of 5 m and embedded length of 25m under one-way cyclic lateral loading of 2 MN with varying direction. The results indicated that the displacement accumulation increases significantly when the direction of cyclic loading varies during the experiments. It should be emphasised that further investigation on the multiaxial response of pile is required in order to gain confidence in predicting the effects of stress close to field conditions.

In summary, experimental modelling testing provides indication of the overall soil-pile response including the account of local phenomena (Cuéllar et al., 2012), which can inform or used to validate numerical approaches.

### 2.3.3 Numerical analysis of the cyclic response of monopile

Robust constitutive modelling approaches and advanced modelling techniques are essential for numerical analysis. Predicting the cyclic pile-soil response over a
number of loading cycles \((N)\) typically relies on two computational strategies for obtaining approximations to the numerical solutions, known as ‘explicit’ and ‘implicit’ approaches (Niemunis et al., 2005). This section is not intended to cover all literature related to numerical analysis on the cyclic response of monopile but focuses on the latest state of the art in laterally loaded pile modelling: Sections 2.3.3.1 and 2.3.3.2 respectively discuss the performance of some advanced constitutive models with implicit and explicit implementation methods for prediction of soil behaviour under high-cyclic loading, while Section 2.3.3.3 reviews the cyclic pile behaviour accounting for the pile installation effect.

2.3.3.1 Implicit method

The implicit method calculates the accumulated stress/strain increments step-by-step, which involves both the current state of the system and the later one to capture the overall response of the considered system, as expressed in Equation 2.1.

\[
(Y(t), Y(t + \Delta t)) = 0
\]  

Where \(Y(t)\) denotes the current system state, while \(Y(t+\Delta t)\) is the state after time step \(\Delta t\).

However, the advantage of such accurate time integration over numerous loading cycles comes at the expense of high computational costs and systematic error accumulation, which may lead to inaccuracy in the prediction of strain accumulation (Niemunis et al., 2005).

One of the latest developments in cyclic soil constitutive modelling based on the implicit method was proposed by Liu et al. (2019), named SANISAND-MS. This model was developed by introducing the concept of hardening memory surface (Corti et al., 2016) into the previous SANISAND model by Dafalias and Manzari (2004) to accurately predict the ratcheting soil response under a large number of loading cycles. The SANISAND-MS model consists of yield surface, dilatancy surface, critical surface, bounding surface and memory surface, as shown in Fig. 2.15. The implementation of the memory surface enables the model to capture the stiffening behaviour of soil under the high number of cycles.
Figure 2.15 SANISAND-MS model loci in the deviatoric stress ratio plane (Liu et al., 2019).

The SANISAND-MS constitutive model has been successfully employed to 3D FE problems concerning the cyclic drained response of monopile (Liu et al., 2021b; Liu et al., 2022), as shown in Fig. 2.16. The 3D FE modelling (Fig. 2.16a) combined with the SANISAND-MS model is capable of simulating the global ratcheting behaviour of the pile (Fig. 2.16b). The author also demonstrated that the 3D FE SANISAND-MS framework has the potential to provide a reliable prediction of pile-soil interaction by showing the good agreement between the numerical simulation and the experimental tests by LeBlanc et al. (2010b) and Richards et al. (2020) in terms of $T_c - \zeta_c$ relationship in Fig. 2.16c. Such a relationship was defined by LeBlanc et al. (2010b), where $\zeta_c$ is the ratio between the minimum and maximum in a load cycle and $T_c$ is the dimensionless function to described the accumulated rotation.
2.3.3.2 Explicit method

In contrast to the implicit method, the explicit method calculates the later state of a system based on the state of current time, which can be expressed as:

\[ Y(t + \Delta t) = F(Y(t)) \]  \hspace{1cm} (2.2)

Such a method possesses the advantage of computational efficiency, which only accounts for the accumulated stress/strain at a certain point of each loading and unloading cycle. However, this therefore highly relies on extensive laboratory testing.

Niemunis et al. (2005) and Wichtmann et al. (2007a) have proposed and developed a high-cycle explicit model (HCA) to predict the accumulated strain induced by small-amplitude cycles for sand. Based on numerous cyclic drained triaxial tests, the empirical expression is described as follows:

\[ \varepsilon^{acc}(N) = f_{amp}f_{N}f_{e}f_{p}f_{Y} \]  \hspace{1cm} (2.3)

The functions \( f_{amp} \), \( f_{e} \), \( f_{p} \), \( f_{Y} \) and \( f_{N} \) describe the effect of the strain amplitude \( \varepsilon^{acc} \), the initial void ratio \( e \), the average mean pressure \( p \), the average stress ratio,
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$Y$ and the number of cycles $N$, respectively. The details are summarised in Table 2.1.

The hypoplastic model is used to implicitly calculate the first several cycles in the proposed HCA model (Fig. 2.17). Then the accumulated strain under a large number of cycles is predicted explicitly. The involvement of the so-called ‘control cycles’ for updated hysteretic response and extension allows for the drastic reduction of computation time.

![Figure 2.17 Schematic of the High Cyclic Accumulation model explicit formulation (Niemunis et al., 2005).](image)

Table 2.1 HCA model functions implemented and reference values (Niemunis et al., 2005).

<table>
<thead>
<tr>
<th>Function</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ampl} = \left( \frac{\varepsilon_{ampl}}{\varepsilon_{ref}} \right)^{C_{ampl}}$</td>
<td>$\varepsilon_{ref} = 10^{-4}$</td>
</tr>
<tr>
<td>$f_N = C_{N1} \ln(1 + C_{N2}N + C_{N3}N)$</td>
<td>$C_{N1}, C_{N2}, C_{N3}$</td>
</tr>
<tr>
<td>$f_p = \exp \left( -C_p \left( \frac{p_{av}}{p_{ref}} - 1 \right) \right)$</td>
<td>$p_{ref} = 100kPa$</td>
</tr>
<tr>
<td>$f_Y = \exp \left( C_Y \frac{Y_{av}}{Y_{ref}} \right)$</td>
<td>$C_Y$</td>
</tr>
<tr>
<td>$f_e = \frac{(C_e - e)^2}{1 + e_{ref}}$</td>
<td>$e_{ref}$</td>
</tr>
</tbody>
</table>

The successful application of this method was conducted by Staubach & Wichtmann, 2020, who performed a finite element analysis of monopile in fine sand based on the aforementioend high-cycle accumulation (HCA) model. The
HCA model has been experimentally proved to be a powerful cyclic constitutive model for the prediction of accumulated strain after a large number of cycles. Up to $10^6$ drained loading cycles were applied in the FE analysis to investigate the influence of several governing factors including pile dimensions, soil densities and magnitude of cyclic loading on the permanent deformation of monopile foundation. The permanent horizontal displacements were found to increase with the increase of the mean value of cyclic bending moment $M$ and its amplitude and with the decrease of soil density. The effect of multidimensional cyclic loading characterised by a circular shape in the $M_x - M_y$ plane ($M_x$ and $M_y$ are the bending moments along $x$ and $y$ direction) was also simply assessed, as shown in Fig. 2.18. It was interesting to be noticed in Fig. 2.18 that a significant deformation occurred at the $x$ direction in the loading plane, while a small deflection was observed in the $y$ direction.

Figure 2.18 Horizontal pile deformations in the $x$ and $y$ direction in a simulation on the monopile model with $10^6$ circular cycles (Staubach and Wichmann, 2020).

Another application of the explicit method is the stiffness degradation model by Achmus et al. (2009), which is capable of capturing the stiffness degradation ($E_{sn}$ shown in Fig. 2.4) to simulate the realistic lateral deflection of piles subjected to long-term cyclic lateral load. This model linked the laboratory element tests and the
FE model, in which the soil behaviour of secant stiffness ($E_{sn}$) associated with the strain accumulation obtained from cyclic triaxial tests (Fig. 2.19b) was implemented to the FE model (Fig. 2.19a). This method has demonstrated the ability to evaluate the effect of pile dimensions, loading conditions and void ratio of soil on the cyclic pile behaviour. However, further implementation of soil response under multi-amplitude loads is still unclear.

![Figure 2.19 Degradation of secant modulus under cyclic loading in the pile–soil model (Achmus et al., 2009)](image)

2.3.3.3 Modelling of pile installation

As a standard in the current practice of FE modelling of monopiles, including the very latest design methodology proposed by the PISA project (Burd et al., 2020 and Byrne et al., 2020), the pile is typically generated in a manner of ‘wished in place’ (WIP). However, the experimental and numerical assessments of the effect of installation on the monopile response (e.g. Fan et al., 2021a, b and c; Heins et al., 2020; Staubach et al., 2020), have shown that the change of initial stress state, such as densification and soil plug, has a profound influence on the subsequent axial and lateral response of pile. Staubach et al. (2020) conducted a numerical analysis on the lateral displacement to a large number of cyclic loading when accounting for the jacking and the impact driving installation process. The authors found that the simulation considering the pile installation effect shows a relatively conservative result regarding the pile deflection compared to the pile generated
with ‘wished in place’ for the initial medium dense sand ($D_{r0} = 40\%$), while the results for dense sand ($D_{r0} = 80\%$) show an opposite trend (Fig. 2.20). Therefore, accounting for the pile installation effect in numerical modelling inevitably attracts more interest in recent years.

**Figure 2.20 Pile deflection curves after the second cycle (dashed) and after five million loading cycles for different initial conditions (Staubach et al., 2020).**

Similar research was conducted by Le et al. (2021), who explored the effect of soil densities and loading amplitude on the long-term deformation of monopile with the impact and vibratory driving installation processes. Given the involvement of advanced calculation techniques for dealing with large deformations caused by pile penetration and the special numerical method for simulating long-term cyclic soil behaviour (i.e. HCA model), it is difficult to conduct the 3D FE analysis on offshore monopile foundations under the application of the combined the installation process and the high number of loading cycles. For this reason, two numerical models separately for pile installation with MMALE formulation (Multi-Materials Arbitrary Lagrangian-Eulerian, see Bakroon et al. (2020)) and for the horizontal cyclic loading scenario were created in this research, where the post-installation state conditions (void ratio and stress condition) obtained from the former are considered as the initial soil conditions for the cyclic loading phase, as shown in Fig. 2.21.
Figure 2.21 Schematic view of calculation concept with models for the installation process (left) and horizontal loading (right) (Le et al., 2021).

Fig. 2.22 presents the horizontal displacement at ground level obtained from the model previously described. The consistency between the model test results and FE calculation in Fig. 2.22 again suggested that the numerical simulation considering the pile installation effect can well reflect the actual pile-soil interaction, while an overestimation of the accumulated horizontal displacement was clearly observed for the simplified wished-in-place pile. It was also found that the displacements for vibratory-driven piles were smaller than that for impact-driven piles (Fig. 2.22). Despite the applicability of such FE framework, a larger scale or prototype is still required for the validation of this proposed simple approach for considering the installation process.
2.4 Soil element testing: cyclic behaviour

Numerous studies on the cyclic stress-strain response of sand have been conducted under various conditions through soil element testing controlled with cycles of stress or strain, such as cyclic triaxial (TX), direct simple shear (DSS), cubical triaxial cell or hollow cylinder torsional apparatus (HCTA) tests. This section will present a summary of the main studies concerning the response of soil subjected to cyclic loading.

2.4.1 Current assumption of stress path imposed on laboratory element testing

Soil stress-strain response is dependent on the specific stress or strain path imposed by the nearby geotechnical structure. Laboratory tests used in monopile design procedures are usually conducted under simplified assumptions of the stress states around the laterally loaded pile. Typically, the soil elements in front of
a laterally loaded pile are considered to be subjected to horizontal triaxial compression loading, while pure tangential shearing is experienced by elements on the side of the pile (Randolph and Houlsby, 1984, Fan and Long, 2005, Won et al., 2015 and Ahmed and Hawlader, 2016), as shown in Fig. 2.23.

![Figure 2.23 Typical stress paths experienced by soil elements around laterally loaded pile (after Ahmed and Hawlader, 2016).](image)

Fan & Long (2005) explored the stress paths in $p-q$ plane followed by soil elements around laterally loaded pile at five representative positions, as shown in Fig. 2.24. Conventional triaxial compression (CTC), conventional triaxial extension (CTE), triaxial compression with an increase in radial stresses (TCR), and direct simple shear (DSS) are also provided as references. It was shown that the soil element in line with the loading direction (Point 1) can be simulated by TCR. The stress path for soil element 2 can be simulated by conventional triaxial compression tests. The soil element at the orthogonal direction of the lateral loading can be simulated by direct simple shear tests. While soil element 5 located at the back of the pile can be linked to the conventional triaxial extension conditions.
2.4.2 Triaxial and direct simple shear tests

2.4.2.1 General cyclic response of soil

The results reported in Fig. 2.25 by Escribano (2014) systematically illustrate the typical response of sand subjected to drained cyclic loading. From Fig. 2.25b, it can be noticed that under stress path shown in Fig. 2.25a, much larger accumulated displacement in the first cycle is observed compared to those in the successive cycles. As the cyclic loading progresses, a stable state is reached where the rate of strain accumulation tends to decrease (Figs. 2.25b and d), which is related to a progressive stiffening behaviour associated with reduced plastic dissipation. Such a stable state with a volumetric state of medium-loose soils subjected to drained triaxial cyclic loading conditions can “cross” the critical state line in the specific volume (ν) – mean pressure plane (p) (López-Querol and Coop, 2011), is considered to be an asymptotical state (Wichtmann et al., 2005; Wichtmann et al., 2007; Wichtmann and Triantafyllidis, 2012; Wichtmann et al., 2010), as shown in Fig. 2.25c.
Figure 2.25 Typical response of a sand sample subjected to drained cyclic loading: (a) stress path; (b) stress-strain response; (c) volumetric response; (d) accumulated strain with the number of cycles. (Escribano, 2014).

As stated in Section 2.2.3, two main aspects, accumulated rotation of pile and change in stiffness of surrounding soil, should be considered for the cyclic design of monopile. Wichtmann et al. (2004a, 2004b) have demonstrated that the accumulation of soil deformation not only depends on the void ratio, average stress and strain amplitude during cycles. The changes in the fabric caused by the re-orientation of soil particles should also be considered for the prediction of accumulated strain. Such changes further lead to the changes of small strain soil properties, which in turn may affect the serviceability of OWTs.

Generally, the shear stiffness for a wide range of shear strains is illustrated in the stiffness degradation curve, as shown in Fig. 2.26. Atkinson and Sallfors (1991) suggested that the soil strain levels can be typically classified into: (a) the very small strain level, in which the soil show elastic behaviours; (b) the small strain level, where the non-linear stress and strain response is observed; and the large strain level, where the critical stress state associated with a relatively small soil stiffness is reached.
The cyclic soil behaviour was found to strongly depend on the initial void ratio \( (e) \), average mean effective stress \( (p') \), average stress ratio \( (\eta_{ave} = q_{ave}/p_{ave}) \), stress amplitude \( (q_{ampl}) \), loading history and loading direction, while the loading frequency has the negligible (or unclear) impact of such factors on the strain accumulation based on Youd (1972) and Shenton (1978). The following sections will focus on the effect of cyclic loading on soil responses regarding either strain accumulation or evolution of stiffness.

### 2.4.2.2 Effect of initial void ratio/relative density

The initial void ratio plays an important role in soil strain accumulation. The results obtained from the drained cyclic simple shear tests conducted by Silver & Seed (1971) revealed that the accumulated axial strain \( (\varepsilon_{acc}^1) \) with low initial relative density is larger than that with high initial relative density (Fig. 2.27a). Wichtmann (2005) confirmed these trends by performing a series of drained triaxial tests with the initial void ratios \( (e_0) \) varying from 0.58 to 0.80 (corresponding the \( e_{0,ldo} \) between 0.24 and 0.99). Other stress conditions \( (q_{ampl} = 60 \text{ kPa}, p_{av} = 200 \text{ kPa}, \eta_{av} = 0.75) \) remain unchanged throughout the tests. As shown in Fig. 2.27b, the accumulation rate increases with an increasing void ratio.
2.4.2.3 Effect of stress amplitude

The stress amplitude can significantly affect the pattern of accumulated strain. For example, the accumulated strain with stress amplitude less than a threshold shear strain amplitude $\gamma_{\text{ampl}} = 10^{-4}$ is neglectable, while an approximately quadratic relationship between the accumulation rate and $\gamma_{\text{ampl}}$ can be observed for larger stress amplitude (Wichtmann, 2005).

Several experimental studies on the effect of the cyclic amplitude on soil response were conducted in both the triaxial cell (Allen Marr and Christian, 1981; Martin et al., 1975; Wichtmann, 2005; Wichtmann et al., 2005; Wichtmann et al., 2007; Wichtmann and Triantafyllidis, 2012) and in the ring torsional apparatus (Oh-oaka, 1976). A series of tests in the dry condition with various stress amplitudes but with the same mean average stress, cyclic stress ratio and initial void ratio was conducted by Wichtmann et al. (2005). The strain amplitude remained constant with low-stress amplitude during cycles, while a decrease of the strain amplitude with the number of cycles was noticed, especially for larger amplitudes within the first 100 cycles (Fig. 2.28a). The experimental results indicated that the cyclic amplitude affects marginally the strain accumulation and such effects become more obvious as the increasing number of cycles in the semilogarithmic plot in Fig. 2.28b. Wichtmann (2005) also argued that the relationship between the strain amplitude and accumulated strain appeared to be quadratic ($\varepsilon_{\text{acc}} \sim (\varepsilon_{\text{ampl}})^2$) and such a relationship will lose its validity for large strain amplitude when the strain
amplitude exceeds $10^{-3}$. A similar conclusion was drawn through the cyclic simple shear tests on fine sand with different amplitudes $\gamma_{\text{ampl}}$ by Sawicki & Swidzinski (1987).

Despite the profound influence of stress amplitude on strain accumulation, large increases in small strain stiffness (e.g. shear modulus) could not be observed for the specimens under cycles with large amplitudes in the resonant column device (Wichtmann and Triantafyllidis, 2004a). Neither the small strain stiffnesses measured by the bender element in cyclic triaxial tests show significant changes during 100000 load cycles (Wichtmann and Triantafyllidis, 2004b). Lo Presti et al. (1993) confirmed this conclusion by performing resonant column tests (RC) equipped with high-resolution sensors. The results suggested that the small strain shear modulus $G_0$ was independent on the application of torsional cycles with low amplitude.

2.4.2.4 Effect of mean effective stress

Six cyclic triaxial compression tests with mean effective stress varying from 50 kPa to 300 kPa were performed by Wichtmann (2005) to investigate the influence of the average mean pressure on stain accumulation. The average stress ratio $\eta_{\text{av}}$ of 0.75 and amplitude ratio of 0.3 ($\zeta = q_{\text{ampl}}/p_{\text{av}}$) remained constant ($p-q$ stress path in Fig. 2.29a). The author concluded, as illustrated in Fig. 2.29b, there is no clear dependence of the accumulated strain on $p_{\text{av}}$ for the number of cycles less than $10^4$. However, the increasing rate of the accumulated strain becomes predominant.
for smaller mean effective pressure when the number of cycles exceeds $10^4$. Similar findings were obtained from the cyclic simple shear tests with a small number of cycles (Silver and Seed, 1971; Youd, 1972; Sawicki & Swidzinski, 1987; Martin et al., 1975).

The results by Timmerman and Wu (1988), who performed the cyclic triaxial tests with $\eta_{av} = q_{av}/p_{av} = \text{constant}$ but different horizontal stresses $\sigma_3 = 48 \text{ kPa}$ and $138 \text{ kPa}$, were in contradiction with the results mentioned before. It was found that the permanent soil strain under smaller stress amplitudes after $10^4$ cycles in the tests with smaller horizontal stress ($\sigma_3$) showed comparable strain accumulation with that observed in the tests with the larger $\sigma_3$, which is probably attributed to the ignorance of the pressure-dependent stiffness.

![Figure 2.29](image_url)  
*Figure 2.29 (a) Stress cycles in the tests on the influence of $p_{av}$; (b) Curves of accumulated strain $\varepsilon_{acc}$ with number of cycles for different average mean pressure $p_{av}$ (Wichtmann, 2005).*

2.4.2.5 Effect of mean cyclic stress ratio

The effect of the mean cyclic stress ratio ($\eta_{av}$) on the plastic strain accumulation again has been systematically investigated by Wichtmann (2005), who performed eleven tests with $0.375 \leq \eta_{av} \leq 1.375$ and constant $p_{av} = 200 \text{ kPa}$ and $\eta = 0.3$, as shown in Fig. 2.30a. The accumulated strain in Fig. 2.30b shows a strong dependence on the average stress ratio, in other words, the magnitude of strain accumulation increases with the increase of the average stress ratio.
2.4.2.6 Effect of high loading cycles

Generally, the investigation of strain accumulation either through experimental or numerical analysis involves limited numbers of cycles due to the high computation cost and high time consumption. In practical offshore geotechnical engineering, the foundation of offshore wind turbines is typically subjected to 10 to 100 million load cycles, which causes soil around the foundation to tend to bear with a large amount of cyclic loading, therefore the long-term performance of soil must be paid much attention to.

The load on the wind turbine is featured with low amplitude and high cycles. Low amplitude refers to the cyclic loading part relative to the average load applied on the soil is small (cyclic stress ratio, $\eta = q_{\text{amp}}/p_{\text{ave}}$). For the cyclic load itself, it can be completely analysed using the elastic model. High cycles refer to the long-term action of cyclic load (cycle number $N > 10000$) (Wichtmann, 2005). It is verified from both laboratory tests and field tests that even under such a cyclic load of low amplitude, large cumulative plastic deformation will occur in soil mass with the continuous increase of cycle numbers.

2.4.2.7 Effects of loading sequences

The abovementioned studies mainly focused on cases with regular and single amplitude cyclic loading histories. It should be reminded that the environmental...
loading acting on the offshore wind turbines is irregular and multi-directional in nature. Fig. 2.3 in Section 2.2.1 depicts the translation from such irregular loading history to idealised loading history for current design practice. However, the effect of such irregular loading history is probably quite complex and shouldn’t be ignored.

Some insightful investigations on the influence of irregular loading on monopile response just by considering simple cases were performed. Significant work has been done by Wichtmann et al. (2010), who performed six cyclic triaxial tests with the same average stress. Four packages of 25000 cycles with different stress amplitudes $q_{ampl} = 20, 40, 60$ and $80$ kPa in different sequences were applied on each sample (Fig. 2.31a). It was found that the residual strain recorded at the end of the cyclic stage was similar and independent on the cyclic sequence (Fig. 2.31b). Similar tests on Hostun sand performed by Escribano (2014) again demonstrated the dominant effect of the largest cyclic amplitude on the strain accumulation.

![Figure 2.31 (a) Tested sequences of the amplitudes $q_{ampl}$ = 20, 40, 60 and 80 kPa; (b) Strain accumulation curves (Wichtmann et al., 2010).](image)

2.4.3 Multiaxial stress paths imposed on laboratory element testing

In practice, the stress state of soil surrounding a laterally loaded pile is expected to be more complex with the variation of six independent stress variables (see Fig. 
2.32), which inevitably induces the rotation of principal stress axes within the soil around the pile. Such a phenomenon has been noticed by many researchers. The previously mentioned stiffness degradation model by Achmus et al. (2009), overcame the problem that the minor principal stress ($\sigma_3$) as well as the orientation of the principal stress axes continuously vary during cyclic loading, as schematically shown in Fig. 2.33.

![Diagram of soil element under general stress state](image1)

**Figure 2.32** Soil element under general stress state.

![Diagram of initial stress state and stress state under cyclic load](image2)

**Figure 2.33** Initial stress state and stress state under cyclic load for the pile-soil system (Achmus et al., 2009).
Mandolini et al. (2019a) numerically analysed the evolution of the direction of the principal stress axes for the soil elements around the laterally loaded pile, in which the x-axis is the radial direction from the pile centre, the z-axis is the vertical axis and the y-axis is the circumferential direction, as shown in Fig. 2.34. The unit vectors were coloured for better visualising and understanding the variation of the principal stress axes. Specifically, the red vector refers to the direction of the principal stress which is in the same direction of x at the geostatic stage. The green vector is the direction of the principal stress which is originally vertical. While the blue colour denotes the circumferential principal stress direction. It was found that the rotations of two principal stress axes occurred for soil elements in front of the loading direction (\(\alpha = 0^\circ\) in Fig. 2.34a), while for the soil elements located out of the loading direction (e.g. \(\alpha = 45^\circ\) in Fig. 2.34b and \(90^\circ\) in Fig. 2.34c), all three principal stress axes re-orientate during loading.

![Figure 2.34 Rotation of principal stress axes for typical soil elements during lateral loading of a pile: (a) in front of the pile (\(\alpha = 0^\circ\)); (b) at 45° from load direction (\(\alpha = 45^\circ\)); (c) perpendicular to load direction (\(\alpha = 90^\circ\)) (Mandolini et al. 2019a).](image)

### 2.4.4 Testing devices for multiaxial stress path

Traditional testing equipment such as triaxial device and direct simple shear device have two degrees of freedom, which are unable to reproduce multiaxial stress space. In order to better mimic the stress conditions as close as possible to those existing in the field, advanced testing apparatuses with more sophisticated capabilities have been developed.

#### 2.4.4.1 The true triaxial apparatus

Compared to the conventional triaxial device with equal horizontal stresses, the true triaxial apparatus is designed to independently control the stresses or strain of a cubical sample in three orthogonal directions (Fig. 2.35), which allows for the evaluation of the effects of the intermediate principal stress on soil response.
However, the continuous rotation of principal stress axes can’t be achieved as no shear stress is applied to the sample, which therefore limits the orientation of the principal stress axes at 0° and 90°.

![Diagram of principal stress axes](image)

**Figure 2.35** True triaxial apparatus: (a) photograph of the frame (University of Bristol, Cubical Cell Apparatus); (b) stress condition in an element tested in the true triaxial apparatus (Sadek, 2006; Hamlin, 2014).

### 2.4.4.2 The hollow cylinder apparatus

The hollow cylinder apparatus (HCA) allows for controlling four degrees of freedom. With the independent control of axial load ($W$), torque ($M_T$), inner ($p_i$) and outer ($p_o$) pressures (Fig. 2.36), the HCA is capable of controlling the orientation and magnitude of the principal stresses. As a result, the triaxial and direct simple shear stress paths along with slightly more complex field stress paths can be reproduced by the equipment. One disadvantage of the HCA lies in the inevitable non-uniformity of stresses and strains across the samples, which can be minimised by selecting an appropriate geometry. Separating the inherent and induced anisotropy of the sample is difficult as the samples cannot be tilted (unless frozen sand samples or intact clay samples are used) or the direction of pluviation changed. Detailed descriptions of the HCA will be presented in Chapter 5.
2.4.5 Cyclic behaviour in multiaxial stress space

Existing studies show that the re-orientation of the principal stress axes shows a complex impact on the stress-strain behaviour of sand (Arthur et al., 1980; Ishihara and Towhata, 1983; Gutierrez et al., 1993). Even if the stress level remains unchanged, the strain of sand will gradually accumulate with the rotation of principal stress axes (Gutierrez et al., 1991). Neglecting the influence of rotation of principal stress axis can cause underestimation of accumulated strain of soil. This is also confirmed by (Wichtmann et al., 2007b; Tong et al., 2010; Mandolini et al., 2021). However, such aspect, which has been generally recognised to have a significant influence, has not been appropriately explored so far.

In the early stage, due to the limitation of test equipment and test conditions, researchers generally carried out axial shear tests with varying major principal stress directions from the samples themselves. Saada and Bianchini (1977), Kirkgard and Lade (1993) cut clay samples from different angles to conduct triaxial compression test to obtain the static mechanical characteristics of soil with different principal stress directions. The same method was adopted by Miura and Toki (1984), they found the strength obtained by cutting undisturbed sand samples in different directions was obviously anisotropic, and the strength obtained by cutting samples in the vertical direction was significantly higher than that obtained by cutting samples in the horizontal direction.
The drained tests on medium dense to dense Toyoura sand were carried out by Tong et al. (2010) through the hollow cylinder torsional apparatus to investigate the effect of continuous cyclic rotation of principal stress axes on the behaviour of soil, in which the amplitude of the cycle remains constant. The plastic deformation triggered by the rotation of the principal stress axes was found to increase with the number of cycles, while the rate of accumulation tended to decrease. The experimental results also revealed a highly dependence of the volumetric strain on the intermediate principal stress parameter \( b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3) \). The secant stiffness calculated from stress and strain cycles exhibited an increasing trend with the growth of intermediate principal stress. The hysteretic shear stress - strain loops was found open at the beginning and has a tendency to be closed as cyclic loading proceeds.

A modified cyclic multidimensional simple shear (CMDSS) device was used to study the effect of the polarization and the shape of the strain loop on accumulated strain (Wichtmann et al., 2007b), which is capable of controlling the horizontal movement of the sample’s bottom in two directions. Fig. 2.37a compares the accumulated strain \( \varepsilon^{acc} \) with and without polarization changes. It is clear that the sudden 90° change of loading axis after 1000 cycles generates the sudden acceleration on the strain accumulation ratio. The results show that strain accumulation was dependent on the loading direction. The influence of the shape of the cyclic loading is presented in Fig. 2.37b. The accumulated strain under circular cyclic shearing is approximately twice larger than that under uniaxial cyclic shearing.
Significant work on the effect of rotation of principal stress has also been done on cyclic traffic loads. Cai et al. (2015), for example, analysed the axial permanent deformation of sand under a heart shape stress path in the deviatoric stress space (Fig. 2.38a, which was generally a typical traffic loading) using hollow cylinder apparatus. The influence of confining pressures ($p$), cyclic vertical stress ratios ($\text{CVSR} = \frac{\sigma_z^{\text{ampl}}}{2p'}$, where $\sigma_z^{\text{ampl}}$ is amplitude of vertical stress), and cyclic torsional stress ratios ($\eta$) were investigated. The results reported in Fig. 2.38b reveal that a larger cyclic torsional stress ratio ($\eta$) results in a more significant axial plastic displacement ($\varepsilon_z^p$), which emphasises the importance of considering the effect of principal stress rotation on plastic displacement.

A very latest research performed by Toyota and Takada (2021) again highlighted the importance of considering the rotation of principal stress axes. The tests were
designed to replicate the actual cyclic traffic load conditions using a hollow-cylindrical torsional apparatus. The comparison of accumulated vertical strain under different combinations of stresses (Fig. 2.39a) shows that the largest accumulated volumetric strain was observed under the stress combination of $\sigma_{vd}$ and $\tau_{di}$, which includes principal stress rotation (Fig. 2.39b). However, the additional application of horizontal stress $\sigma_{hd}$ seemed to reduce the progress of accumulated vertical strain.

![Figure 2.39](image)

**Figure 2.39** Effects of combined stresses: (a) stress paths; (b) progress of vertical strain (Toyota and Takada, 2021). — Notation: $\sigma_v$, $\sigma_h$ and $\tau$ in the figure refer to the vertical stress, horizontal stress and torsional stress, respectively.

The aforementioned research mainly focused on the effect of the application of cyclic loading with rotation of principal stress axes. It should be noticed that the initial $K_0$ stress state can be greatly modified by the construction of geotechnical structures and the installation procedure of pile foundation. The influence of such initial stress states involving the rotation of the principal stress axes prior to the application of cyclic loading should not be ignored, which may significantly affect the soil response (Timmerman and Wu, 1969; Bouckovalas and Whitman, 1984; Chang and Whitman, 1988).

One recent study on this aspect comes from Xiong et al. (2017). This study was carried out with Dynamic Hollow Cylinder Apparatus (DYNHCA) on Toyoura sand. Sixteen tests with different initial stress conditions (different orientations of principal stress axes and preloading) were performed to evaluate the influence of the subsequent cyclic loading on the evolution of axial and volumetric strains with the same stress amplitudes (Fig. 2.40a). The results are summarised in Fig. 2.40b. The comparison of the tests located in the same arc with the same length of stress...
path \( (q_s) \), shows that the larger increase in the re-orientation of principal stress axes \( (\alpha^s_\Delta \sigma) \) leads to a lower growth rate of axial strain \( (\varepsilon_p) \) at the initial cyclic stages (within 100 cycles), and the rate of reduction is found in relation to the length of static shear stress path \( q_s \).

![Diagram](image1)

**Figure 2.40** (a) Scheme for drained static shear; (b) Evolution of vertical permanent strain with number of cycles in semi-logarithmic coordinates for all tests (Xiong et al., 2017).

Mandolini et al. (2021) experimentally investigated the stiffness (Young’s \( E_z \) and shear \( G_{xz} \) moduli) evolution of granular soil under long-term multiaxial cyclic loading, in which the Hostun sand samples were subjected to the combination of axial and torsional loading or either of them (Fig. 2.41). The experimental results revealed that the high number of cyclic loadings with small amplitudes can induce the degradation of small strain stiffness. A larger or maybe quicker decrease in the normalised elastic moduli was found as the cyclic amplitudes increased, as shown in Fig. 2.42. In addition, the stiffness degradation under the application of combined axial and torsional cycles with low cyclic amplitude seems to be more significant than that under pure axial or torsional cycles with larger stress amplitude.
Recently published research allowed further relations to be established between advanced laboratory soil investigation under multiaxial stress conditions and numerical modelling. Yang et al. (2022), for example, presented FE analyses to predict the accumulated displacement of large-diameter monopile, in which they adopted an improved degradation stiffness approach based on the multiaxial tests on hollow cylinder apparatus (Fig. 2.43). Despite the improvement in the soil element test calibration, actual stress paths experienced by soil elements around laterally loaded monopile is not well-understood yet, which plays a key role in controlling the soil response.
2.5 Conclusions

As shown in Chapter 1, following the recent outbreak of offshore renewable energy, attention on the behaviour of large diameter piles subjected to laterally loading has considerably increased. The offshore energy market in search of more power through larger turbines and deeper water sites has brought the foundation of monopiles to extra-large size, in excess of 10 m for the recently installed wind turbine with capacity currently up to 14-15MW. There are challenges in manufacturing, transporting and installing such large-size foundations and any design optimisation is welcome to decrease project costs and minimise project risk.

Main aspects of the monopile design are outlined. Few variables (embedded length, diameter and wall thickness) are considered for monopile design due to their simple shape. The rotation and dynamic criterion must be ensured for the cyclic design. In light of the main criterion, any improvement to the accuracy of cyclic foundation design for OWTs should consider: (i) permanent displacement or accumulated rotation of the pile head under serviceability limit state conditions; (ii) the effects of cyclic loading on the strength and stiffness. The strain accumulation of soil surrounding the monopile can lead to the changes of small strain soil properties, which will potentially result in the system resonance and consequent fatigue problems.

The methodologies currently used in the monopile design, as well as their limitations and the progress to improve design guidance are summarised in this chapter. Traditional models such as the p-y curve method have proved unreliable and non-optimal for large-diameter pile design. Numerical modelling provides an
efficient way for the prediction of the behaviour of the monopile (e.g. Byrne et al. 2015, Zdravković et al. 2015, Kementzetzidis et al. 2019).

Pile testing is essential to validate the numerical approaches. Although full-scale monitoring and model testing provides indication of the overall soil-pile response including account of local phenomena (Cuéllar et al., 2012), they are restricted to evaluate the cyclic behaviour of monopile with limited number of loading cycles. Salient numerical studies on the cyclic response of monopile foundation have been overviewed, with focus on the state of the art in the application of advanced constitutive models and pile installation effects.

Reliable numerical analysis requires knowledge of the cyclic long-term constitutive behaviour of the soil (e.g. Corti et al., 2016; Liu et al., 2019) which can be explored through appropriate laboratory testing. Soil response under high numbers of cyclic loading has been widely investigated (e.g. Wichtmann, 2005) through the variation of the cyclic amplitude, stress ratio, average stress level and soil density. The abovementioned laboratory practice, to determine and calibrate constitutive soil parameters for the design of offshore wind monopile foundations, typically relies on triaxial and simple shear tests. However, the stress state of soil surrounding a laterally loaded pile is more complex and rotation of principal stress axes within the soil around the pile invariably occurs (e.g. Andersen, 2013; Mandolini, 2018). Laboratory element studies on hollow cylinder torsional apparatus have shown that the application of multiaxial cyclic stress was found to be an important factor affecting the soil stiffness and strain accumulation.
Chapter 3  Numerical evaluation of soil stress paths under monotonic lateral pile loading

Statement

Most extracts of this chapter have been published as a journal paper in Computers and Geotechnics.


3.1 Introduction

The experimental characterisation of soils through the use of laboratory element testing is crucial for any robust geotechnical system design process. Advanced
design procedures benefit from laboratory tests which eventually mimic stress paths experienced by critical soil elements around geotechnical systems. The prediction of a geotechnical structure response invariably relies on accurate mechanical soil characterisation which must be appropriately embedded (e.g. through constitutive modelling) in analytical and/or numerical methods. Chapter 2 has indicated that soil response could be significantly changed under different stress paths, whereas current laboratory practice for laterally loaded monopile design following the simplified triaxial or direct simple shear stress condition may underestimate the stiffness degradation and accumulated strain of soil. In this respect, the identification of accurate stress paths is of vital importance to the prediction of foundation behaviour.

Numerical simulations can provide insight into the fundamental behaviour of the system that is normally not available by means of experimental investigations. The stress path followed by the soil elements surrounding the pile can be studied with a numerical model but are hardly traceable in model tests or in-situ conditions.

Therefore, this chapter seeks, for the first time, to investigate the stress paths experienced by the soil around laterally loaded piles in order to:

1) discover the characteristics of the stress paths experienced by representative soil elements;
2) assess the relevance of current laboratory testing practices to reproduce the stress paths;
3) provide suggestions for replicating these stress paths through laboratory element testing.

The research work employs 3D FE analyses, using implicit constitutive modelling, to assess and extract stress paths in the soil around laterally loaded piles. The work focuses on offshore wind monopile foundations installed in sand under drained monotonic lateral loading conditions. Assessing the monopile behaviour under monotonic conditions invariably forms the first fundamental step of monopile design to identify the monotonic backbone curve on which the ultimate limit state and cyclic design procedures build upon.
3.2 Reference system and notation

This section introduces the spatial coordinate and stress reference systems used in this investigation. Fig. 3.1a illustrates the 3D FE model of the reference pile-soil system (half-problem due to plane of symmetry), including the cylindrical coordinate system \((x, \theta, z)\) employed for defining the position of the soil elements around the pile, where \(x\) axis is the radial direction from the pile centre, the \(z\) axis is the vertical axis and \(\theta\) is the angle from the lateral pile loading direction to the element’s location. As shown in Fig. 3.1b, the stress state is defined using six stress components \((\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz})\) in the local coordinate system \((x, y, z)\).

Compressive normal stresses are positive. The stress invariants are the mean pressure \(p\), the deviatoric stress \(q\) and the Lode’s angle \(\theta_L\), defined as following:

\[
p = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \tag{3.1}
\]

\[
q = \left\{ \frac{1}{2} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 \right] + 3 \ast (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \right\}^{1/2} \tag{3.2}
\]

\[
\theta_L = \frac{1}{3} \arcsin \left( -\frac{3\sqrt{3}}{2} \frac{S}{J^{3/2}} \right) \tag{3.3}
\]

where

\[
\theta_L \text{ varies between } -30^\circ \text{ and } 30^\circ \text{ with } \theta_L = -30^\circ \text{ corresponding to triaxial compression and } \theta_L = 30^\circ \text{ to triaxial extension.}
\]

\[
S = (\sigma_x - p)(\sigma_y - p)(\sigma_z - p) - (\sigma_x - p)\tau_{yz}^2 - (\sigma_y - p)\tau_{xz}^2 - (\sigma_z - p)\tau_{xy}^2 + \tau_{xy}\tau_{yz}\tau_{xz} \tag{3.4}
\]

\[
J = \frac{\sqrt{3}}{3} q \tag{3.5}
\]
3.3 FE Modelling of soil-monopile system and overall outputs

3.3.1 3D FE modelling configuration

The 3D FE model has been built in the OpenSees software (Mckenna, 1999) which was selected following the extensive SANISAND-MS implementation and validation work for offshore monopiles performed by Kementzetzidis et al. (2019), Liu et al. (2021b) and Liu et al. (2022). Therefore, the numerical procedure follows that already adopted in these previous work and also the work by Corciulo et al. (2017), Cheng et al. (2021). A representative steel hollow monopile foundation of a typical 8 MW capacity wind turbine driven in medium-dense sand (relative density $D_r = 50\%$) has been selected as a case study, based also on data from Kementzetzidis (2019). A sole sand type and density conditions has been selected in this study, since its aim is to understand the qualitative shape of the load induced stress paths rather than performing a full quantitative analysis. The overall dimensions of the soil domain are 70 m x 35 m x 47 m to minimise the effect of the boundary conditions on the stress distribution in the soil adjacent to the monopile,
as shown in Corciulo et al. (2017) and Liu et al. (2021b). The dimensions of the monopile foundation, diameter $D = 8.0 \text{ m}$, embedded length $L = 27.0 \text{ m}$ and wall thickness $t = 0.062 \text{ m}$, are illustrated in Fig. 3.1.

### 3.3.1.1 Pile modelling

The pile is modelled as a linear-elastic 3D hollow cylinder represented by a Young’s modulus, $E = 200 \text{ GPa}$, the unit weight of steel, $\gamma_s = 77 \text{ kN/m}^3$ and Poisson’s ratio, $\nu = 0.30$. The above-ground part of the pile is modelled as an elastic beam by using 20 Timoshenko beam elements with consistent (non-diagonal) mass matrix to account for combined bending and shear deformations (De Borst et al., 2012). Constant section area $A_{\text{sec}} = 0.7776 \text{ m}^2$ and moment of inertia $I_{\text{sec}} = 2.3818 \text{ m}^4$ are assumed along the superstructure. The embedded pile is represented by eight-node hexahedral one-phase SSPbrick elements, which are effective to against shear/volumetric locking issues in FE calculations (McGann et al., 2015).

### 3.3.1.2 Soil modelling

The soil domain is also represented with SSP bricks elements. The soil behaviour has been modelled using the enhanced version of the SANISAND04 model (Dafalias and Manzari, 2004) proposed by Liu et al. (2019), named SANISAND-MS after the inclusion of an additional memory surface to capture the cyclic ratcheting behaviour of the soil under long term cyclic loading (It has been briefly described in Chapter 2). The term SANSAND (acronym for ‘Simple ANIsotropic SAND’) constitutive model family based on the theories of bounding surface plasticity and critical state soil mechanics, was first introduced by Manzari and Dafalias (1997) in 1997, then extended over the past years (Liu et al., 2019; Dafalias and Taiebat, 2016; Pisanò and Jeremić, 2014; Loukidis and Salgado, 2009; Taiebat and Dafalias, 2008; Dafalias and Manzari, 2004; Papadimitriou and Bouckovalas, 2002; Papadimitriou et al., 2001). However, as a first step, this chapter focuses on the monotonic response of monopile in dry sand, details about the cyclic characteristics of SANISAND-MS will be discussed in Chapter 4. It should be noted that the upgraded SANISAND-MS constitutive model has retained
the well-established predictive capability of the monotonic response of the SANISAND model.

The adopted monotonic values of the soil constitutive parameters are the same as those proposed by Liu et al. (2019, 2021b) for Karlsruhe sand, which have been calibrated with the experimental tests by Wichtmann (2005), as summarised in Table 3.1. Calibrations of drained monotonic triaxial tests with different initial void ratios and confining stresses are shown in Fig. 3.2. The stress-strain response \( (q-\varepsilon_a) \) and volumetric strain behaviour \( (\varepsilon_{vol}-\varepsilon_a) \) in Fig. 3.2 show a good agreement between the simulations and experimental results, which confirms the good capability of capturing the critical state for the selected constitutive model and its validation in this work. The soil is considered to respond in drained conditions and an effective soil unit weight \( \gamma' = 9.4 \text{ kN/m}^3 \) is also considered.

**Table 3.1 Values of parameters used in the memory surface constitutive model (Liu et al., 2021).**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Karlsruhe sand</th>
<th>Pile-sand interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>Dimensionless shear modulus</td>
<td>( G_0 )</td>
<td>110</td>
<td>73.3</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio</td>
<td>( \nu )</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Critical state</td>
<td>Critical stress ratio in compression</td>
<td>( M )</td>
<td>1.27</td>
<td>0.953</td>
</tr>
<tr>
<td></td>
<td>Compression to extension strength ratio</td>
<td>( c )</td>
<td>0.712</td>
<td>0.712</td>
</tr>
<tr>
<td></td>
<td>Reference critical void ratio</td>
<td>( \varepsilon_0 )</td>
<td>0.845</td>
<td>0.845</td>
</tr>
<tr>
<td></td>
<td>Critical state line shape parameter</td>
<td>( \lambda_c )</td>
<td>0.049</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>Critical state line shape parameter</td>
<td>( \xi )</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>Yield locus opening parameter</td>
<td>( m )</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Hardening parameter</td>
<td>( h_0 )</td>
<td>5.95</td>
<td>5.95</td>
</tr>
<tr>
<td></td>
<td>Hardening parameter</td>
<td>( c_h )</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>Void ratio dependence parameter</td>
<td>( n^b )</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Dilatancy</td>
<td>‘Intrinsic’ dilatancy parameter</td>
<td>( A_0 )</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>Void ratio dependence parameter</td>
<td>( n^d )</td>
<td>1.17</td>
<td>1.17</td>
</tr>
</tbody>
</table>
3.3.1.3 Modelling of pile-soil interface

The soil-pile interface is modelled according to Griffiths (1985) by inserting a thin layer of solid elements, with a thickness equal to 4% of the monopile diameter along the shaft, and 8% of the pile diameter under the pile tip. The elastic shear modulus and critical stress ratio of these thin layers of solid elements are set to, respectively, 2/3 and 3/4 times lower than those of the surrounding soil, as recommended by Kementetzidis et al. (2019) to account for the ‘friction fatigue’ promoted by initial pile driving (Randolph and Gourvenec, 2017).

3.3.1.4 Boundary conditions

The soil model is fully fixed at the bottom surface, the ground surface is free to move in all directions, and the horizontal displacement along the direction perpendicular to the lateral surface is restrained.

3.3.1.5 Mesh sensitivity

Compared to the mesh with 6000 SSP elements used by Kementetzidis et al. (2019), despite the high computational cost of the simulation, a finer mesh, generated in a total of 10764 SSP elements shown in Fig. 3.1, is adopted to provide a stable and smooth solution of the pressure field and to further minimise the
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possibility of locking phenomena (SSP brick elements have been used to against the locking).

The capability of reproducing the monopile-soil interaction of the model has been validated by Corciulo et al. (2017), Kementzetidis et al. (2019) and Liu et al. (2021) who used the same 3D FE model with slightly different geometries or soil parameters (Fig. 3.3). For example, successful application of the SANISAND-MS model to a 3D FE analysis of a monotonically loaded monopile-soil interaction system was done by Liu et al. (2021), which has been validated against the field data by Achmus et al. (2020) in Fig. 3.3.

![Figure 3.3 Comparison between monotonic monopile responses from 3D FE modelling and field testing (after Liu et al., 2021).](image)

3.3.1.6 Loading procedure

The procedure of the numerical analysis has been performed in the following steps:

1) Application of soil’s self-weight (geostatic pressure).

2) Generation of a ‘wished in place’ monopile. The effects of disturbance of the soil surrounding the monopile are not explicitly considered in this simulation (experimental and numerical assessments of the effect of installation on monopile response can be found in the very recent studies by Fan et al. (2019) and Staubach
et al. (2020) in Chapter 2). However, some considerations of the installation effects on the soil stress paths will be briefly discussed.

3) Application of the lateral monotonic load ($H/2$ due to the symmetry of the problem) with an eccentricity ($e$) of 20 m above ground level ($H$ and $e$ as defined in Fig. 3.1).

### 3.3.1.7 Inspected soil elements

The response of the soil elements around the pile is expected to be inspected in the following analysis. While all the soil elements have been inspected, the forthcoming representation of the stress paths will focus on representative soil elements belonging to the domain of passive soil resistance. These elements are pinpointed with horizontal distance ($x$) and depth ($z$) normalised by the pile radius ($R$) and length ($L$), respectively and angle from the direction of lateral loading ($\theta$), as shown in the previous Fig. 3.1. Details are provided in Table 3.2, with a schematic shown in Fig. 3.4 for better visualisation (note a indicative rotation point in the figure is included for improving the understanding). The soil elements EF1-4 located in front of the pile (EF1-3 at the same depth) and the EB element, at the back of the pile, are all in-plane with the lateral pile loading direction (Fig. 3.4). The soil elements ES1-3 are located at the same depth as EF1-3 but off-plane with respect to the lateral loading direction, at different angles $\theta$ as defined in Fig. 3.4.

**Table 3.2 Locations of inspected soil elements.**

<table>
<thead>
<tr>
<th>Element</th>
<th>$z/L$</th>
<th>$x/R$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-]</td>
<td>[-]</td>
<td>[°]</td>
<td></td>
</tr>
<tr>
<td>EF1</td>
<td>0.15</td>
<td>1.375</td>
<td>0</td>
</tr>
<tr>
<td>EF2</td>
<td>0.15</td>
<td>1.875</td>
<td>0</td>
</tr>
<tr>
<td>EF3</td>
<td>0.15</td>
<td>2.625</td>
<td>0</td>
</tr>
<tr>
<td>EF4</td>
<td>0.5</td>
<td>1.375</td>
<td>0</td>
</tr>
<tr>
<td>EB</td>
<td>0.85</td>
<td>1.375</td>
<td>180</td>
</tr>
<tr>
<td>ES1</td>
<td>0.15</td>
<td>1.375</td>
<td>30</td>
</tr>
<tr>
<td>ES2</td>
<td>0.15</td>
<td>1.375</td>
<td>60</td>
</tr>
<tr>
<td>ES3</td>
<td>0.15</td>
<td>1.375</td>
<td>90</td>
</tr>
</tbody>
</table>
3.3.2 Overall FE outputs and inspection of soil behaviour

The overall monotonic moment-pile head rotation responses of the monopile foundation in the sand with different densities are reported in Fig. 3.5. As expected, the monopile exhibits a stiffer monotonic response in dense sand than the response in loose sand. The pile head moment–rotation curves (moment = $e$ (eccentricity) $\times$ $H$ (lateral force)) in Fig. 3.5 show continued hardening to large rotations and typical well-defined failure is hard to identify. Therefore, an ultimate limit state condition (ULS) corresponding to a $1.5^\circ$ pile head rotation (an arbitrary value within the typical limits used in monopile design at ULS, which varies between $1^\circ$ and $4^\circ$, e.g. Leblanc et al., 2010b) is considered here to define the ultimate pile capacity. Accordingly, $M_{rel}$ values equal to 542.4 MN, 693.6 MN and 902.4 MN are determined for $D_r = 30\%$, $50\%$ and $70\%$, respectively. However, this research mainly focuses on the case of $D_r = 50\%$ (the following results are all for case of $D_r = 50\%$ except results in Fig. 3.13) and the stress paths in the soil elements will be analysed up to the defined arbitrary load level (rotation of $1.5^\circ$), which corresponds to an ultimate horizontal load $H_{ult} = 34.08$ MN and a relative pile head displacement of 0.51 m.

Figure 3.4 Locations of inspected soil elements.
The contours in Fig. 3.6 reveal the development of deviatoric stress ($q$) around the pile shaft after ‘wished in place’ pile generation (Fig. 3.6a) and during the application of the lateral load at two stages $0.5H_{ult}$ (Fig. 3.6b) and $H_{ult}$ (Fig. 3.6c). Please note that the pile is still modelled as a hollow cylinder and its solid shading is provided only for the sake of clarity. As expected, the most loaded soil sections appear to be arch shape generated in the upper part of the monopile and at the back toe of the pile. As the applied load increases, the extent of the pressure arch in the surrounding soil also increases. At the ultimate load conditions, the range of the influence of the upper pressure arch in front of the pile reaches to about $2R$ (where $R =$ pile radius) radial distance from the pile shaft and extends over the top three quarters of the pile length. The negligible change of deviatoric stress $q$ at the boundary validates the sufficient size of the soil domain.
Chapter 3. Numerical evaluation of soil stress paths under monotonic lateral pile loading

Figure 3.6 Contours of deviatoric stress at (a) initial stage (after pile generation), (b) half of the ultimate loading condition and (c) ultimate loading condition.

The 3D spatial distribution of the Lode’s angle ($\theta_L$) is shown in Fig. 3.7. The soil’s stress state before the application of lateral loading is largely characterized by $\theta_L = -30^\circ$, which corresponds to triaxial compression (Fig. 3.7a). Note although the pile was generated in a manner of ‘wished-in-place’, the unit weight of the pile is larger than the surrounding soil, and a certain perturbation of the stress state is inevitably introduced. Therefore, a concentric variation in Lode’s angle at the shallow layer is observed. As the lateral load is applied (Fig. 3.7b shows stress states for 50% ultimate load), the Lode’s angle in soil elements in front of the pile shows areas of triaxial compression ($\theta_L = -30^\circ$) transitioning to triaxial extension ($\theta_L = 30^\circ$) further away from the pile. The triaxial extension is also experienced by soil elements at the back of the pile. As the lateral loading reaches ultimate conditions (Fig. 3.7c), the areas subjected to triaxial compression in front of the pile and to triaxial extension at the back of the pile increase. Such observations are consistent with those reported in other FE studies of monopile foundations in sand, such as Taborda et al. (2020).
3.3.3 Analysis of stress invariants

The analysis of the stress invariants is conducted for the selected eight representative soil elements in the vicinity of the laterally loaded pile shown in Fig. 3.4.

The overall stress paths experienced by the eight representative samples in terms of the stress invariants $p'-q-\theta_L$ are reported in Fig. 3.8. The stress paths are grouped in such a way they provide the effects of the horizontal distance (Figs. 3.8a-c), depth (Figs. 3.8d-f) and orientation (Figs. 3.8g-i) of the soil elements with respect to the pile. For each group, three-dimensional views of the stress paths (subplots a, d, g), their projections on the $p'-q$ plane (subplots b, e, h) and the evolution of the Lode’s angle $\theta_L$ versus the pile displacement (subplots c, f, i) are reported. All the soil elements undergo shearing under the increase of mean effective pressure, which is due to the increase of all normal stress components as shown in Figs. 3.8b, e, h. For all the elements, the Lode’s angle $\theta_L$ shows a sharp increase from an initial stress condition close to triaxial compression ($\theta_L \sim -25^\circ$ to $30^\circ$) to reach a peak before a decrease towards an asymptotic value (Figs. 3.8c, f, i). While the asymptotic value is close to triaxial compression for all the elements at $\theta = 0^\circ$ (Figs. 3.8c and f), the rate with which the triaxial condition is reached is reduced as the elements move away from the pile (Fig. 3.8c), which agrees with the observations from Fig. 3.7. The trends of Fig. 3.8i (for $\theta \geq 0^\circ$) show that the ultimate loading conditions depart from triaxial compression with increasing $\theta$ (i.e. as the soil element moves away from the direction of lateral pile loading). Close
inspection of all the stress paths reveals that the peak point of the Lode’s angle $\theta_L$ in Figs. 3.8c, f, i is associated with the horizontal radial stress ($\sigma_x$) becoming the largest normal stress component.

However, while the Lode’s angle provides information on the overall loading conditions (i.e. compression, extension or in-between), it does not consider the loading directions with respect to the material axes. Therefore, the full six-dimensional stress state needs to be analysed, as shown in the following section.

![Figure 3.8 Stress paths in $p'-q-\theta_L$ space, projection on $p'-q$ plane and relationship between $\theta_L$ and pile head horizontal displacement for soil elements at: different distances to pile (a, b, c), different depths below ground level (d, e, f) and different orientation with respect to loading direction (g, h, i). Note that the failure line has been drawn for triaxial compression conditions.](image)

**3.4 Stress variation in soil elements around the pile**

**3.4.1 Generalised stress states**

The analysis of the six-dimensional stress variation is conducted for the eight representative soil elements (Fig. 3.4) and illustrated in Fig. 3.9. For all the elements in-plane with the direction of loading ($\theta = 0^\circ$, elements EF1-4 and EB), the three normal stresses ($\sigma_x$, $\sigma_z$, $\sigma_y$) and the shear stress $\tau_{xz}$ show significant
changes as the horizontal load increases, while the two other components of the shear stress outside the \( x \)-\( z \) plane (\( \tau_{xy} \) and \( \tau_{yz} \)) appear to be negligible (Figs. 3.9a-e).

For the soil elements outside the direction of loading (i.e. \( \theta > 0^\circ \), elements ES1-3), the magnitude of variation of all stress components decreases with the increasing of the angle \( \theta \). The soil element located orthogonally to the loading direction (\( \theta = 90^\circ \), element ES3) developed the lowest changes in all stress components (Fig. 3.9h). It can also be noticed that the shear stress \( \tau_{yz} \) remains negligible for all the samples with \( \theta \neq 0^\circ \) during the whole loading process (Figs. 3.9f-h). Careful inspection of Figs. 3.9a, f, g, h confirms the expectation that, as \( \theta \) increases, the \( \tau_{xz} \) stress component becomes progressively less influential while the shear stress component \( \tau_{xy} \) gradually becomes the largest shear stress component.
3.4.2 Rotation of principal stress axes

Given a specific stress matrix, the determination of eigenvalues and corresponding eigenvectors enables the derivation of the principal stresses and their directions with respect to the original ($x$, $y$, $z$) coordinate system. Therefore, the evolution of the principal stress orientation has been investigated for all the inspected representative soil elements (see Fig. 3.4) and all trends are reported in Fig. 3.10.
The orientations of the principal stress axes, without distinguishing between major, intermediate, and minor principal stresses, are represented by unit vectors, $r_i$, at three stages: (0) initial condition prior to pile generation ($r_0$, black vectors), (1) after pile installation ($r_1$, blue vectors) and (2) when the lateral load reaches $H_{ult}$ ($r_2$, red vectors). Three-dimensional view of the unit vectors orientation, as well as their projection on the $x$-$z$ and $x$-$y$ planes, are also given in Fig. 3.10. It can be noted that for all the elements aligned with the direction of lateral loading (i.e. $\theta = 0^\circ$) the rotation of principal stress axes takes place in the $x$-$z$ plane only. For the elements in front of the pile (EF1-4), the rotation of principal stress axes from pile generation to end of loading is between $35^\circ$ and $70^\circ$ in the $x$-$z$ plane. An opposite rotation of principal stress axes for the soil element at the back toe of the pile (element EB), could be noticed compared to the other soil elements, due to the inverse variation of shear stress $\tau_{xz}$. The rotation of the principal stress axes for element EB from pile generation to end of loading is about $45^\circ$. As the location of the soil elements moves away from the lateral load direction (increasing $\theta$), the rotation of the principal stress axes in the other two stress planes becomes more pronounced, such that all three principal stress re-orient during loading for elements ES1-3.
3.4.3 Multiaxial Stress paths

The analysis of the multiaxial stress paths is conducted for the lateral pile loading up to $H_{ult}$. The effects of the radial distance from the pile, depth and orientation with respect to the loading direction of the representative soil elements are taken into consideration.

3.4.3.1 Effect of radial distance from the pile

The analysis of the variation of the six stress components shown in Fig. 3.9 revealed that, for all the soil elements in the direction of loading ($\theta = 0^\circ$), the two stress components $\tau_{xy}$ and $\tau_{yz}$ are negligible. Therefore, the evolution of the remaining four stress components is analysed in the three dimensional stress space $\left( \frac{\tau_{xz}}{p_{r}}, \frac{\sigma_x-\sigma_z}{2p_{r}}, \frac{\sigma_y-p_{r}}{p_{r}} \right)$, which combines the deviatoric stress plot in the plane $x\cdot z \left( \frac{\tau_{xz}}{p_{r}} \right)$ stress plane with the addition of the deviatoric stress axis $\left( \frac{\sigma_y-p_{r}}{p_{r}} \right)$ to consider the influence of the out of plane (intermediate) stress (Muir Wood, 2017),
as shown in Fig. 3.11. The Matsuoka-Nakai (1974) peak failure envelope for the sand material for an assumed friction angle $\varphi'$ of 37° is provided for reference in the $\left(\frac{t_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'}\right)$ plane (Figs. 3.11b, e and h).

The three-dimensional stress paths, alongside two planar projections, are presented in Figs. 3.11a-c for all the elements at $\theta = 0^\circ$ and depth $z/L = 0.15$ at different distance in front of the pile (elements EF1-3). The stress paths in $\left(\frac{t_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'}\right)$ plane (Fig. 3.11b) follow an almost linear trajectory, whose slope is defined here as $\beta_{xz} = \frac{\Delta t_{xz}}{p'} / \Delta \frac{\sigma_x - \sigma_z}{2p'}$. The inclination $\beta_{xz}$ is rather similar for all considered elements and its numerical values will be analysed in the next section. However, it can be noticed from Fig. 3.11b that the closer the element to the pile, the larger the shear component $t_{xz}$ becomes.

The projections of the stress paths in the stress plane $\left(\frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_y - p'}{p'}\right)$ superimpose (see Fig. 3.11c). They also appear rather linear and could be characterised by a sole inclination ($\zeta_{xz} = \frac{\Delta \sigma_y - p'}{p'} / \Delta \frac{\sigma_x - \sigma_z}{2p'}$) of about -15° (adopted sign convention for angles is positive anticlockwise).

### 3.4.3.2 Effect of depth along the pile

The three-dimensional stress paths and their projections for the soil elements located in line with the loading direction but at different depths are reported in Figs. 3.11d-f. Fig. 3.11d shows that, while the two elements in front of the pile follow a qualitatively similar stress path, the element EB is subjected to a rather different evolution of the stress state. Fig. 3.11e demonstrates that the difference is mostly related to the stress components of the vertical deviatoric projection plane $\left(\frac{t_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'}\right)$ and particularly to the reversed value of the shear stress $t_{xz}$ as well as to the $\sigma_x$ stress components not exceeding the magnitude of the $\sigma_z$ components. The slope $\beta_{xz}$ for EB soil element is about -60°.

The values of the slope $\beta_{xz}$ for a larger set of soil elements located at varying distance from the front face of the pile and with soil depth is reported in Fig. 3.12. The $\beta_{xz}$ values show limited changes for element locations up to 4.5$R$ distance from the pile shaft, then decreases at a further distance from the piles. For soil
elements within the top third of the pile, expected to be the most loaded and contribute to the lateral loading soil resistance, $\beta_{xz}$ values lie in the range of $10^\circ$ to $25^\circ$.

The stress paths projection in Fig. 3.11f are rather coincident and independent whether the element is in front or at the back of the pile. The stress paths are characterised by the same inclination $\zeta_{xz}$ of about $-15^\circ$.

Figure 3.11 Stress paths in $\left(\begin{array}{c} \tau_{xz} \\ \sigma_{x}' - \sigma_{z}' \end{array} \right)$ space, projection on plane $\left(\begin{array}{c} \tau_{xz} \\ \sigma_{x}' \end{array} \right)$ and on plane $\left(\begin{array}{c} \sigma_{x}' - \sigma_{z}' \\ \sigma_{y}' - \sigma_{z}' \end{array} \right)$ for soil elements at: different distances to pile (a, b, c), different depths below ground level (d, e, f) and different orientations with respect to loading direction (g, h, i). Stress paths in $\left(\begin{array}{c} \tau_{xy} \\ \sigma_{x}' - \sigma_{y}' \end{array} \right)$ space, projection on plane $\left(\begin{array}{c} \tau_{xy} \\ \sigma_{x}' \end{array} \right)$, and on plane $\left(\begin{array}{c} \sigma_{x}' - \sigma_{y}' \\ \sigma_{z}' - \sigma_{y}' \end{array} \right)$ for soil elements at different orientations with respect to loading direction (j, k, l).
3.4.3.3 Effect of orientation with respect to the loading direction (same distance from the pile shaft)

It has been shown in Fig. 3.9 that for the soil elements outside the direction of loading (i.e. $\theta > 0^\circ$, elements ES1-3), the stress state is defined by five stress components, with the component $\tau_{yz}$ being negligible. Therefore the representation of the stress paths for these elements requires two 3D stress spaces: 

$$\left( \frac{\tau_{xz}}{p_t} \sim \frac{\sigma_x - \sigma_z}{2p_t} \sim \frac{\sigma_y - p_t}{p_t} \right) \quad \text{and} \quad \left( \frac{\tau_{xy}}{p_t} \sim \frac{\sigma_x - \sigma_y}{2p_t} \sim \frac{\sigma_z - p_t}{p_t} \right)$$

as shown in Figs. 3.11g and 3.11j, respectively.

The stress paths in the $(\frac{\tau_{xz}}{p_t} \sim \frac{\sigma_x - \sigma_z}{2p_t} \sim \frac{\sigma_y - p_t}{p_t})$ space show larger increase of the normalised shear stress component in the x-z plane for low values of $\theta$, while becoming constant for $\theta = 60^\circ$ to then showing a decreasing trend for the element positioned on the side of the pile ($\theta = 90^\circ$). As a result, $\beta_{xz}$ decreases as $\theta$ increases. The projections of the stress paths on the stress plane $(\frac{\sigma_x - \sigma_z}{2p_t} \sim \frac{\sigma_y - p_t}{p_t})$, Fig. 3.11i, superimpose and show the same $\zeta_{xz}$ inclination as for the other elements in front of the pile except for $\theta = 90^\circ$ which shows $\zeta_{xz} = 0^\circ$.

Conversely, the analysis of the vertical deviatoric projection plane $(\frac{\tau_{xy}}{p_t} \sim \frac{\sigma_x - \sigma_y}{2p_t})$ stress plane in Fig. 3.11k) reveals that all the stress paths start close to the origin and follow a rather linear trajectory with an inclination 

$$\beta_{xy} = \frac{\Delta \frac{\tau_{xy}}{p_t}}{\Delta \frac{\sigma_x - \sigma_y}{2p_t}}$$
dependent on the element location ($\theta$). Fig. 3.13a shows the variation of $\beta_{xy}$ for several soil elements around the pile at distance $x/R = 1.375$ but with varying $z/L$ and $\theta$. Evidently, the $\beta_{xy}$ in Fig. 3.13a shows only little variations from the initial values with the increase of depth. The relationship between $\beta_{xy}$ and $\theta$ is summarised in Fig. 3.13b and it appears that can be reasonably approximated by $\beta_{xy} \approx \theta$.

The projection of the stress paths in the $\left(\frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_z - p'}{p'}\right)$ stress plane (Fig. 3.11l) appears also rather linear and could be ideally characterised by an inclination ($\zeta_{xy} = \Delta \frac{\sigma_z - p'}{p'} / \Delta \frac{\sigma_x - \sigma_y}{2p'}$). The projection is rather similar for all the soil elements with a limited variation for ES3. The value of the inclination $\zeta_{xy}$ for all the elements is between -55° and -70°.

![Figure 3.13 Variation of $\beta_{xy}$ for elements at distance $x/R = 1.375$: (a) $\beta_{xy}$ versus the depth $z/L$; (b) relationship between $\beta_{xy}$ and orientation $\theta$.](image-url)
Chapter 3. Numerical evaluation of soil stress paths under monotonic lateral pile loading

3.4.3.4 Effect of initial relative density

As mentioned in Section 3.3.2, in addition to the analysis on case of an initial void ratio $D_r = 50\%$, simulation undertaken with both looser ($D_r = 30\%$) and denser ($D_r = 70\%$) cases. Fig. 3.14 presents influence of the initial relative density on the stress paths in the $(\frac{\tau_{xz}}{p'} \sim \frac{\sigma_x-\sigma_z}{2p'} \sim \frac{\sigma_y-p'}{p'})$ stress plane for soil elements EF1 and the stress paths in the $(\frac{\tau_{xy}}{p'} \sim \frac{\sigma_x-\sigma_y}{2p'} \sim \frac{\sigma_z-p'}{p'})$ stress plane for soil elements ES2 and ES3.

Although the effect of the initial relative density is prominent through the differences in moment–rotation response of pile from $D_r = 30\%$ to $D_r = 70\%$ (Fig. 3.5), negligible variation in the stress paths experienced by the selected soil elements for different $D_r$ is observed, suggesting the monotonic stress path is independent of the relative density.

Figure 3.14 Influence of relative density ($D_r$) on: stress paths in $(\frac{\tau_{xz}}{p'} \sim \frac{\sigma_x-\sigma_z}{2p'} \sim \frac{\sigma_y-p'}{p'})$ space, projection on plane $(\frac{\tau_{xz}}{p'} \sim \frac{\sigma_x-\sigma_z}{2p'})$, and on plane $(\frac{\sigma_x-\sigma_y}{2p'} \sim \frac{\sigma_y-p'}{p'})$ for soil elements EF1 (a, b, c); on stress paths in $(\frac{\tau_{xy}}{p'} \sim \frac{\sigma_x-\sigma_y}{2p'} \sim \frac{\sigma_z-p'}{p'})$ space, projection on plane $(\frac{\tau_{xy}}{p'} \sim \frac{\sigma_x-\sigma_y}{2p'})$, and on plane $(\frac{\sigma_x-\sigma_y}{2p'} \sim \frac{\sigma_z-p'}{p'})$ for soil elements ES2 (d, e, f) and soil elements ES3 (g, h, i).
3.5 Relevance of laboratory testing to reproduce the stress paths

This section aims to assess whether available laboratory testing procedures can reproduce field situations (as simulated through 3D FE analysis).

The 3D FE analysis revealed that four stress components vary in magnitude for all the elements in front and back of the pile, whereas a change of five stress components is involved for those soil elements out-of-plane with the lateral loading direction. Given these observations, it is clear that no laboratory apparatus is able to reproduce the prescribed stress paths for soil elements out-of-plane with the lateral loading direction. Concerning the other soil elements, the Hollow Cylinder Torsional Apparatus (HCTA) is the only apparatus that may offer the control of four stresses (as briefly described in Section 2.4.2.2 in Chapter 2). However, as a degree of approximation, given the similarity of the stress paths in Fig. 3.11g and the element in line with the direction of loading (i.e. Fig. 3.11a), only the stress paths in the \( \left( \frac{t_{xy}}{p}, \frac{\sigma_x - \sigma_y}{2p}, \frac{\sigma_z - p'}{p} \right) \) space are considered for the soil elements located out of the lateral loading plane as such the HCTA could also be employed for these elements.

Nevertheless, when imposing HCTA loading conditions, it will be taken into account the restrictions of the availability of the stress space due to the sample geometry and boundary effects (Hight et al., 1983). Based on the 3D FE analysis, it has also been shown that the stress paths experienced by the soil elements around the pile shaft are fairly linear during monotonic pile loading. Therefore, consideration of linear stress paths in experiments is considered an acceptable approximation.

3.5.1 Evaluation of laboratory stress paths

The HCTA stress paths, that replicate FE predictions for four representative soil elements in different locations around the laterally loaded pile, are shown in Fig. 3.15 as follows:

- **CASE F**: soil element in front to the laterally loaded pile, representative element EF1.
• **CASE B**: soil element at the back of the laterally loaded pile, representative element EB.

• **CASE D**: soil element diagonal to the loading direction, representative element ES2.

• **CASE S**: soil element on the side of the laterally loaded pile, representative element ES3.

Fig. 3.15 shows the HCTA admissible stress paths according to the restriction on the applicable ratio of internal \((p_i)\) to external \((p_o)\) pressures to avoid major stress non-uniformities across the sample wall: \(0.9 < p_o/p_i < 1.2\) by Hight et al. (1983). The details will be discussed in Chapter 5. The stress paths for simpler triaxial conditions are also reported in Fig. 3.15, where considered appropriate.

Fig. 3.15 shows that the HCTA can well reproduce the stress paths in the main deviatoric projection plane \(\tau_{xz}/p' \sim \sigma_x - \sigma_z\) or \(\tau_{xy}/p' \sim \sigma_x - \sigma_y\) in Figs. 3.15b, e, h, k) but some differences on the location of the stress state at the early loading stage can be noticed in the 3D plots for all the cases (Figs. 3.15a, d, g, j). This is due to the initial value of the intermediate principal stress as shown in the \((\sigma_x - \sigma_z)/2p' \sim \sigma_y - p'\) or \((\sigma_x - \sigma_y)/2p' \sim \sigma_z - p'\) projection planes (Figs. 3.15c, f, i, l). Only higher ratios of \(p_o/p_i\) well outside the applicable limits would enable to match the FE predicted stress paths.

Imposing a ratio \(p_o/p_i = 1.2\) may help to approach the initial field conditions but the improvement appears limited and probably does not justify the increased testing complexity and risk of strain localisation. Nevertheless, it should also be remembered that the field stress path was determined considering wished-in-place generation and \(K_0\) stress conditions. Ashour et al. (1998) suggested the initial horizontal effective stress in triaxial compression test should take as \(\sigma_h = K \cdot \sigma_v\), where \(K = 1\) due to pile installation effect. The results from the pile penetration experiments carried out by Jardine et al. (2013) show that horizontal stress is larger than vertical stress for the soil in the vicinity of pile at the end of pile installation. The large increase in magnitude of radial stress after the pile installation was also observed in numerical simulation performed by Staubach et al. (2021a). In such conditions, the value of \((\sigma_y - p')\) or \((\sigma_z - p')\) for the initial field stress would approach zero or even change sign, approaching the HCTA stress paths.
Figs. 3.15a-c and 3.15d-f also show that a conventional triaxial test produces a rather different stress path in the multiaxial stress space. Clearly, this type of test can not reproduce the development of shear stresses due to the frictional pile-soil interaction and the continuous rotation of principal stress axes induced by the lateral loading.

Figs. 3.15j-l show that both HCTA and simple torsional loading (which can be associated to simple shear) can reproduce the predicted FE stress path in the \( \frac{\tau_{xy}}{p_t} \sim \frac{\sigma_x - \sigma_y}{2p_t} \) plane (Fig. 3.15k) at the early loading stage but does not capture the late stage. It is conceivable that a stress path with an orientation \( \beta_{xy} \) slightly lower than 90° may better capture the FE predicted stress path, but the application of a vertical stress path in the \( \frac{\tau_{xy}}{p_t} \sim \frac{\sigma_x - \sigma_y}{2p_t} \) plane may conversely enable to fully explore the soil behaviour up to pure torsional conditions.

Based on the above discussion, a summary of possible options to reproduce in the laboratory the stress paths of soil elements around monotonic laterally loaded pile is provided in Table 3.3. The table differentiates among four different soil element locations and provides suggestions for initial and incremental stress conditions to be applied in HCTA testing. The suggested testing conditions match those in the previous section by considering: 1) linearised stress paths; 2) equal outer and inner cell pressures (\( p_o = p_i \)); and 3) constant outer and inner cell pressures during shearing, which neglects the large increase in mean isotropic stress observed in Fig. 3.8 (and whose effect has been normalised throughout the paper), but this follows conventional practice in laboratory exploration of the shear behaviour of soils (i.e. triaxial shear tests are typically performed by maintaining constant cell pressure). Table 3.3 provides a schematic view of the stress paths in the main deviatoric stress plane only, since the account of the intermediate stress axis is fixed by the condition \( p_o = p_i \).
Figure 3.15 Comparison between idealised stress paths and those applicable in element testing in \( \frac{\tau_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_{xy}}{2p'} \) space, projection on plane \( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \), and on plane \( \frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_{xy}}{2p'} \) for soil elements Case F (a, b, c), Case B (d, e, f), and in \( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_{xy}}{2p'} \) space, projection on plane \( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \), and on plane \( \frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_{xy}}{2p'} \) for soil elements Case D (g, h, i), and Case S (j, k, l).
3.5.2 Note on rotation of stress coordinates between field and laboratory

Laboratory samples are typically obtained from vertical cylindrical cores (in cohesive soils) or prepared through vertical depositional and compaction procedures. During the mechanical testing in the HCTA (or triaxial, simple shear testing), the major variations are applied to the normal or shear stress to a horizontal face.

Therefore, unless special and unconventional sampling or sample preparation procedures are adopted (e.g. horizontal coring from block samples of clays or frozen sands, or laboratory reconstitution/preparation of cylindrical samples maintaining the axis of symmetry horizontal), the orientation between the stress direction and the material axis cannot be maintained between field and laboratory conditions. A relative $90^\circ$ rotation between the material axis and the stress direction must be applied, as shown in Fig. 3.16. This denotes one of the limitations of laboratory element testing in soils for laterally loaded piles.

Figure 3.16 Schematic of transition from stress state in field situation to that in the HCTA.
### Table 3.3 Summary of recommended experimental tests for the soil elements around the laterally loaded pile.

<table>
<thead>
<tr>
<th>Soil element position</th>
<th>Suggested HCTA testing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case F</strong></td>
<td><strong>Initial conditions</strong></td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{z0} = \gamma' \cdot z );</td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{x0} = \sigma_{y0} = K \cdot \sigma_{z0} ) [\text{( K ) is defined to consider pile installation effect}];</td>
</tr>
<tr>
<td></td>
<td>• ( 0 \leq \tau_{xz} \leq \sigma_{x0} \cdot \tan(\phi_{ps}) ).</td>
</tr>
<tr>
<td></td>
<td><strong>Incremental stress or strain conditions:</strong></td>
</tr>
<tr>
<td></td>
<td>• Apply vertical stress ((\Delta \sigma_x)) or strain ((\Delta \varepsilon_x)) increments;</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \sigma_x = \Delta \sigma_y = 0 ) [\text{constant inner and outer cell pressures}];</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \tau_{xz} = (\Delta \sigma_x - \Delta \sigma_z)/2 \cdot \tan(\beta_{xz}) ). [\beta_{xz} = 10^\circ \text{ to } 25^\circ]</td>
</tr>
<tr>
<td><strong>Case B</strong></td>
<td><strong>Initial conditions</strong></td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{z0} = \gamma' \cdot z );</td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{x0} = \sigma_{y0} = K \cdot \sigma_{z0} ) [\text{( K ) is defined to consider pile installation effect}];</td>
</tr>
<tr>
<td></td>
<td>• ( 0 \leq \tau_{xz} \leq \sigma_{x0} \cdot \tan(\phi_{ps}) );</td>
</tr>
<tr>
<td></td>
<td><strong>Incremental stress or strain conditions:</strong></td>
</tr>
<tr>
<td></td>
<td>• Apply vertical stress ((\Delta \sigma_x)) or strain ((\Delta \varepsilon_x)) increments;</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \sigma_x = \Delta \sigma_y = 0 ) [\text{constant inner and outer cell pressures}];</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \tau_{xz} = (\Delta \sigma_x - \Delta \sigma_z)/2 \cdot \tan(\beta_{xz}) ). [\beta_{xz} = -60^\circ]</td>
</tr>
<tr>
<td><strong>Case D</strong></td>
<td><strong>Initial conditions</strong></td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{z0} = \gamma' \cdot z );</td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{x0} = \sigma_{y0} = K \cdot \sigma_{z0} ) [\text{( K ) is defined to consider pile installation effect}];</td>
</tr>
<tr>
<td></td>
<td>• ( \tau_{xy} = 0 ).</td>
</tr>
<tr>
<td></td>
<td><strong>Incremental stress conditions:</strong></td>
</tr>
<tr>
<td></td>
<td>• Apply vertical stress ((\Delta \sigma_x)) or strain ((\Delta \varepsilon_x)) increments ((\theta &lt; 45^\circ)); shear stress ((\Delta \tau_{xy})) or strain ((\Delta \gamma_{xy})) increments ((45^\circ &lt; \theta &lt; 90^\circ));</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \sigma_x = \Delta \sigma_y = 0 ) [\text{constant inner and outer cell pressures}];</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \tau_{xy} = (\Delta \sigma_x - \Delta \sigma_y)/2 \cdot \tan(\beta_{xy}) ) [\beta_{xy} \sim \theta].</td>
</tr>
<tr>
<td><strong>Case S</strong></td>
<td><strong>Initial conditions</strong></td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{z0} = \gamma' \cdot z );</td>
</tr>
<tr>
<td></td>
<td>• ( \sigma_{x0} = \sigma_{y0} = K \cdot \sigma_{z0} ) [\text{( K ) is defined to consider pile installation effect}];</td>
</tr>
<tr>
<td></td>
<td>• ( \tau_{xy} = 0 ).</td>
</tr>
<tr>
<td></td>
<td><strong>Incremental stress conditions:</strong></td>
</tr>
<tr>
<td></td>
<td>• shear stress ((\Delta \tau_{xy})) or strain ((\Delta \gamma_{xy})) increments;</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \sigma_x = 0 );</td>
</tr>
<tr>
<td></td>
<td>• ( \Delta \sigma_x = \Delta \sigma_y = 0 ) [\text{constant inner and outer cell pressures}].</td>
</tr>
</tbody>
</table>
3.6 Summary and Conclusions

Using three-dimensional finite element analyses, this chapter has investigated the stress paths experienced by soil elements around the monotonically laterally loaded large-diameter pile in granular soils. The main aim of the chapter was to determine typical stress paths for soil elements at different locations around the pile and assess whether laboratory element testing procedures can mimic those stress paths and identify the main shortcoming.

Inspection of the full six-dimensional stress state for the selected soil elements demonstrated that either four or five stress components undergo consistent variation as the pile is laterally loaded, inducing variation of Lode’s angle and re-orientation of principal stress axes for these soil elements. While there is no experimental apparatus which enables to replicate the full six-dimensional stress state of soil elements around laterally loaded piles, the HCTA is the only laboratory testing equipment which permits the control of orientation and rotation of the principal stress axes. However, if a reduction of the stress-state to four dimensions by neglecting one of the shear stress components for the soil elements out-of-plane of lateral loading is applied, the HCTA could offer a reasonable replication of the numerically determined stress paths.

Typical linearised stress conditions for representative soil elements – front (top) and back (bottom) of the pile in plane with the loading direction as well as diagonal and on the side of the of the pile – have been determined and they are summarised in Table 3.3 of this paper alongside the conditions to be applied in HCTA element testing for their simulation. The analysis also identified two further crucial points for further considerations when assessing the relevance of laboratory element testing:

1) it appears that the HCTA would be unable to replicate the initial magnitude of the intermediate principal stress from FE predictions, due to the large difference between the initial vertical and horizontal stresses. The existing experimental tests and numerical analysis have confirmed that the horizontal stress in the soil would increase after experiencing pile penetration. Therefore, this limitation may be overruled or alleviated if the pile installation effects are considered.
2) there are difficulties in maintaining the relative orientation between material axis and the principal stress direction unless special and unconventional sampling or sample preparation procedures are adopted (e.g. horizontal coring from block samples of clays or frozen sands, or laboratory reconstitution/preparation of cylindrical samples maintaining the axis of symmetry horizontal). Nevertheless, this limitation applies to all laboratory testing equipment and may be critical when dealing with strongly anisotropic geomaterials.

It should be highlighted that the employment of HCTA testing is not seen as a replacement of the current conventional day-to-day testing practice (based on triaxial and direct shear testing) because of the complexity around sample preparation and testing, and the cost/time implications. Nevertheless, it is of upmost important to recognise the limitations of the current testing practice, which can then be accounted to optimise the current design procedures. Research work using HCTA and exploring the soil behaviour under the proposed stress paths could reveal these shortcomings. In addition, given the nature of the actions in offshore wind applications, the study of the stress paths under cyclic loadings will be presented in Chapter 4.
Chapter 4  Numerical evaluation of soil stress paths under cyclic lateral pile loading

Statement

Some contents presented in this chapter were extracted from the following publications by the author:


4.1 Introduction

Chapter 3 has demonstrated that the unidirectional lateral loading acting on monopile can cause a complex variety of stress paths with rotation of the principal stress axis in the surrounding soil domain. However, the analysis focused on monotonic response. Real environmental loads acting on monopile foundations are in nature of varying amplitude, average level, and frequency. Therefore, this chapter focuses on the evaluation of stress paths in soil elements surrounding the monopile foundation subjected to lateral cyclic loading. Similar to Chapter 3, the procedure detailed in this chapter involves a two-stage analysis:

1) firstly, 3D Finite element analysis will be employed to extract and derive typical stress paths for soil elements in different positions around the pile;
2) then, it will be analysed whether laboratory testing apparatus can simulate such stress paths and which simplifications have to be introduced.

4.2 FE Modelling of the soil-monopile system and overall outputs

4.2.1 3D FE modelling configuration

Figure 4.1 Finite element model and investigated soil elements.
This chapter applies the same FE model as that used in Chapter 3. The mesh and the representative soil elements are illustrated in Fig. 4.1. Their locations are defined in the table provided within the same figure.

The SANISAND-MS constitutive model, which has already been introduced in Chapter 3, will continue to be applied in this chapter. However, the scope of Chapter 3 was limited to the case of a monotonically loaded monopile. Compared to Chapter 3, additional memory surface parameters same as those proposed by Liu et al. (2019, 2021b) for Karlsruhe sand, are provided in Table 4.1.

**Table 4.1 Values of parameters used in the memory surface constitutive model (Liu et al., 2021).**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Karlsruhe sand</th>
<th>Pile-sand interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>Dimensionless shear modulus</td>
<td>$G_0$</td>
<td>110</td>
<td>73.3</td>
</tr>
<tr>
<td></td>
<td>Poisson's ratio</td>
<td>$\nu$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Critical stress ratio in compression</td>
<td>$M$</td>
<td>1.27</td>
<td>0.953</td>
<td></td>
</tr>
<tr>
<td>Critical state</td>
<td>Compression to extension strength ratio</td>
<td>$c$</td>
<td>0.712</td>
<td>0.712</td>
</tr>
<tr>
<td></td>
<td>Reference critical void ratio</td>
<td>$e_0$</td>
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<td>0.845</td>
</tr>
<tr>
<td></td>
<td>Critical state line shape parameter</td>
<td>$\lambda_c$</td>
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<td>0.049</td>
</tr>
<tr>
<td></td>
<td>Critical state line shape parameter</td>
<td>$\xi$</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>Yield surface</td>
<td>Yield locus opening parameter</td>
<td>$m$</td>
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<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Hardening parameter</td>
<td>$h_0$</td>
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<td>5.95</td>
</tr>
<tr>
<td></td>
<td>Hardening parameter</td>
<td>$c_t$</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>Void ratio dependence parameter</td>
<td>$n^b$</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Dilatancy</td>
<td>‘Intrinsic’ dilatancy parameter</td>
<td>$A_0$</td>
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<td>1.06</td>
</tr>
<tr>
<td></td>
<td>Void ratio dependence parameter</td>
<td>$n^d$</td>
<td>1.17</td>
<td>1.17</td>
</tr>
<tr>
<td>Memory surface</td>
<td>Ratcheting parameter</td>
<td>$\mu_0$</td>
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<td>260</td>
</tr>
<tr>
<td></td>
<td>Memory surface shrinkage parameter</td>
<td>$\zeta$</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td>Dilatancy memory parameter</td>
<td>$\beta$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

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The need for the SANISAND-MS constitutive model in order to capture the cyclic ratcheting behaviour of the soil under long-term cyclic loading and the limitations for such purpose of the classical SANISAND04 model can be well illustrated with the comparison of the typical cyclic stress-strain response predicted by the two models (Fig. 4.2). The response of anisotropically consolidated specimens in triaxial tests (Fig. 4.2a) clearly shows that the SANISAND04 model (without memory surface) overpredicts the strain accumulation (ratcheting) due to its limited soil stiffening under drained high cyclic loading, while the implementation of memory surface concept for SANISAND-MS effectively enables to reproduce the soil hardening response during cycles observed in experimental tests (Figs. 4.2b and c).

Figure 4.2 (a) Stress path in cyclic triaxial tests on anisotropically consolidated specimens; (b) influence of the memory surface formulation on the response to asymmetric drained triaxial loading, and (c) comparison to laboratory test results in terms of accumulated total strain (experimental data from Wichtmann, 2005) (Pisanò, 2019).

The capabilities of the SANISAND-MS for predicting the high number of cyclic behaviour of soils have been validated by Liu et al. (2019) against the experimental results of Wichtmann (2005) (Fig. 4.3). Figs. 4.3a-c show the accumulated strain with the number of cycles with different initial confining pressures $p_{in}$, initial void ratios $e_{in}$, cyclic stress amplitudes $q^{ampl}$ and average stress ratios $\eta^{ave}$, respectively. It clearly shows that the SANISAND-MS captures well the accumulated strains $\varepsilon^{acc}$, despite a slight overestimation of $\varepsilon^{acc}$ for very dense and very loose specimens in Fig. 4.3b.

The numerical simulation follows the same analysis steps in Chapter 3: (1) applies geostatic stress; (2) generates the pile in the ‘wished in place’ method; and (3) applies the lateral cyclic load at pile top (eccentricity $e = 20$ m) in a load-controlled manner. The detail of the loading scenarios will be discussed in the following.
4.2.2 Loading scenarios

Salient evidence of the effect of irregular stress on soil response has been reported in Chapter 2. At pile scale, a 3D FE modelling with the implementation of SANISAND-MS constitutive model, was analysed regarding the accumulated lateral pile displacement for the loading cases with regular parcels of multi-amplitude and different parcel sequences by Liu et al. (2022), as shown in Fig. 4.4. It was found that regardless of average load values \( (H_{\text{ave}}) \) or soil densities \( (D_i) \), larger accumulated horizontal deformation rates of pile were found for the case with increasing cyclic amplitude of subsequent load packages (Figs. 4.4a and d), while relatively lower lateral displacement is observed for load package with descending amplitude (Figs. 4.4c and f).
Figure 4.4 Influence of the cyclic loading sequence on the evolution of monopile tilt — pile head lateral displacement against the number of loading cycles. (a)–(c), $H_{ave} = 2310$ kN, $D_r = 30\%$; (e)–(f), $H_{ave} = 4020$ kN, $D_r = 70\%$ (Liu et al., 2022).

However, as described in Chapter 2, the irregular loading histories are usually translated to idealised packages of cycles with constant amplitude, average load and load frequency. In addition, as the first time to explore the stress paths around laterally loaded monopile, simple loading conditions are preferred in this research. The 3D FE model was performed according to the simulation programme in Table 4.2, considering the influence of different loading conditions and initial sand’s relative densities on the cyclic response of the monopile-sand interaction system.

Typically, the loading conditions are characterized by two independent parameters defined by LeBlanc et al. (2010):

$$\zeta_b = \frac{M_{\max}}{M_{\text{ref}}} \quad (4.1)$$
\[ \zeta_c = \frac{M_{\text{min}}}{M_{\text{max}}} \]  

(4.2)

where \( M_{\text{ref}} \) is the static moment pile capacity, and \( M_{\text{min}} \) and \( M_{\text{max}} \) are respectively the minimum and maximum values during the load cycle (as shown in Fig. 4.5a).

As mentioned in Chapter 3, arbitrary reference rotation of 1.5°, 2° and 4° at model scale have been respectively used by Arshad and O’Kelly (2017), Byrne et al. (2015) (PISA project) and Leblanc et al. (2010b), while a reference rotation of 0.25° based on the SLS criteria suggested by DNV (2018) was proposed by Bayton et al. (2018), and a reference rotation of 0.5° for a FE monopile with a diameter of 5 m and embedded length of 20 m was selected by Liu (2020). However, these values were not intended to represent a design rotation. For this research, the pile capacity was defined at the pile rotation of 1°. The parameter \( \zeta_c \) in loading cases C1-C5 was zero to simulate one-way cyclic loading, while the value of \( \zeta_c \) in loading case C6 is equal to -1 for two-way loading. The ratio \( \zeta_b \) varies from 0.15 to 0.5 to simulate the effect of cyclic loading amplitude.

<table>
<thead>
<tr>
<th>Case</th>
<th>( D_r )</th>
<th>( \zeta_b )</th>
<th>( \zeta_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>50%</td>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>C2</td>
<td>50%</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>C3</td>
<td>50%</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>C4</td>
<td>30%</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>C5</td>
<td>70%</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>C6</td>
<td>50%</td>
<td>0.25</td>
<td>-1</td>
</tr>
</tbody>
</table>

The pile response is computed incrementally cycle by cycle for this implicit 3D FE calculations. Considering the time-consuming computation, in total 100 cycles at the pile head were applied in the form of sinusoidal load, which resulted in a calculation time of approximately 70 h for each case. However, the limited number of loading cycles is enough for the pile-soil interaction system to reach a stable state, which will be discussed in the following section.
4.2.3 Overall FE outputs and inspection of soil behaviour

4.2.3.1 Pile response

The typical cyclic response recorded at the pile head in terms of moment against rotation is shown in Fig. 4.5a for the simulation cases C1, C2 and C3 in Table 4.2. The monotonic response is also reported as a reference, although it does not typically reach well-defined failure, and instead shows continued hardening to large rotations, as shown in Fig. 4.5a. The reference pile capacity $M_{\text{ref}}$ is equal to 800 MNm corresponding to pile rotation $\theta_{\text{ref}} = 1^\circ$. The plot in terms of accumulated pile rotation against the number of loading cycles is provided in Fig. 4.5b. These plots show the capability of the FE model to predict the trends of progressive accumulation of displacement and rotation as observed in model pile tests (LeBlanc et al., 2010b; Roesen et al., 2013). Larger accumulations of rotation are observed at the early cyclic stages, followed by a progressive stiffening of the pile repose and a decrease in the rate of displacement and rotation accumulation. This response can be predicted owing to the capabilities of the employed constitutive model, which has been proved to satisfactorily capture the ratcheting behaviour and the progressive soil stiffening under cyclic loading. Such global ratcheting behaviour can be related to the local ratcheting observed in laboratory element tests (Wichtmann, 2005; Liu et al., 2019). As expected, under pure one-way cyclic loading, higher $\zeta_b$ values lead to larger accumulation of pile rotation and larger accumulation rate.

![Figure 4.5 FE outputs: (a) Moment versus pile head rotation for cases C1, C2 and C3; (b) pile head rotation versus number of cycles.](image-url)
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The two-way cyclic response of monopile in terms of moment against pile head rotation for $\zeta_b = 0.25$ is provided in Fig. 4.6, which shows the progressive inclination of the hysteretic loops. The loop shape remains largely consistent but rotated with a gradual stiffening of the response cycles. Compared to the one-way cyclic case, less strain is accumulated in the two-way cyclic case, indicating that one-way cyclic loading has a more profound influence on surrounding soil. In this sense, the scope of this chapter will be limited to the soil response under one-way cyclic pile loading.

Figure 4.6 Simulated monopile response to symmetric cyclic loading – simulation case C6.

4.2.3.2 Surrounding soil behaviour

The contours in Fig. 4.7 reveal the development of the Lode’s angle ($\theta_L$) around the pile shaft for simulation case C2 ($\zeta_b = 0.25$) after ‘wished in place’ pile generation (Point T0 in Fig. 4.7a) and during the application of cycles ($\theta_L$ at the end of the $N = 1$ (Point T1 in Fig. 4.7a), $N = 50$ (Point T50 in Fig. 4.7a) and $N = 100$ (Point T100 in Fig. 4.7a) is presented). As shown in Fig. 4.7b, the soil’s stress state before the application of lateral loading is largely characterized by $\theta_L = -30^\circ$. As the lateral load is applied (Fig. 4.7c shows stress states after one cycle), the Lode’s angle $\theta_L$ shows rapid variation around the pile with transition to triaxial shearing or extension ($\theta_L \geq 30^\circ$) stress state in areas on the side of the pile and immediately at the back of the pile. However, these variations occur within the top third of the pile. After 50 loading cycles (Fig. 4.7d), the areas subjected to triaxial extension around the pile increase and mostly locates in front of the pile. Shearing ($\theta_L = 0^\circ$) is found to extend to deeper area close to the pile shaft and compression occurs at the back of the pile. Negligible variation of Lode’s angle is observed after 100 loading cycles.
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(Fig. 4.7e) compared with that after 50 loading cycles, suggesting a stable state is reached. This is also confirmed by the limited cyclic ratcheting after 50 loading cycles in Fig. 4.7a. This is contrary to the conclusion from the monotonic simulation in Chapter 3, where the compression was found in front of the pile and at the back toe of the pile, and extension occurred at the right back of the pile.

![Figure 4.7](image)

**Figure 4.7** (a) typical force and displacement relation for simulation case C2 ($\zeta_b = 0.25$); contours of Lode's angle at the end of (b) initial stage (after pile generation), (c) $N = 1$, (d) $N = 50$ and (e) $N = 100$.

The final distribution of Lode's angle at the end of the 100th cycle (Point $T_{100}$ in Fig. 4.7a) for different pile loading amplitudes ($\zeta_b$) and different relative densities ($D_r$) is provided in Figs. 4.8 and 4.9, respectively. By comparing the stress contours in Fig. 4.8, it is clear that larger $\zeta_b$ induces the larger extension zone around the pile, especially in the shallow soil layer. While the relative density shows a limited influence on the distribution of Lode's angle around the pile (Fig. 4.9).

![Figure 4.8](image)

**Figure 4.8** Contours of Lode's angle at the end of 100 loading cycles for different pile loading amplitudes: (a) $\zeta_b = 0.15$; (b) $\zeta_b = 0.25$; (c) $\zeta_b = 0.5$. 
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Figure 4.9 Contours of Lode's angle at the end of 100 loading cycles for different densities: (a) $D_r = 30\%$; (b) $D_r = 50\%$; (c) $D_r = 70\%$.

Fig. 4.10 presents cyclic stress paths in terms of mean effective stress ($p'$) and deviatoric stress ($q$) for all eight representative soil elements for case C2 ($\zeta_b = 0.25$), with the shaded sidebars indicating the variation of Lode's angle $\theta_L$ in the range between $-30^\circ$ - $30^\circ$ from triaxial compression to extension. The 1st cycle, 10th cycle and 100th cycle are particularly highlighted for better visualisation of the evolution of the $p'$-$q$ stress path. It can be clearly observed that the magnitude of $p'$-$q$ stress path decreases with the distance from the pile shaft (comparison of the results in Figs. 4.10a-c), depth (comparison of the results in Figs. 4.10a, d, e) and the orientation with respect to the pile loading direction (comparison of the results in Figs. 4.10a, f, g, h).

For the soil elements located at shallower position (elements EF1-3 and elements ES1-3 at depth $z/L = 0.15$), the mean effective stress ($p'$) increases with loading cycles, while the stress level and amplitude of deviatoric stress ($q$) remain almost constant. On the contrary, deeper soil elements (EF4 and EB) experience a visible reduction in deviatoric stress $q$ with a relatively stable stress level of $p'$. However, the cyclic amplitude of $p'$ seems to decrease as the cyclic loading proceeds (i.e. element EB).

The transition from triaxial compression ($\theta_L = -30^\circ$) to triaxial extension ($\theta_L = 30^\circ$) in Fig. 4.10 is not clear for almost all elements at all cycles due to the simultaneous variation of the normal and shear stresses in each cycle, in which the extension state appears at the lateral pile load $H = 0$ associated with a reduction of $q$. 
Figure 4.10 Stress paths in $p'-q$ stress plane for the represented soil elements (simulation case C2): (a) element EF1, (b) element EF2, (c) element EF3, (d) element EF4, (e) element EB, (f) element ES1, (g) element ES2, (h) element ES3.

The influence of the cyclic load amplitude ratio ($\zeta_b$) on the $p'-q$ stress paths for soil element EF1 is presented in Fig. 4.11. As expected, larger $\zeta_b$ induces larger variation in both $p'$ and $q$. Qualitatively similar results have been observed for $\zeta_b = 0.15$ and $\zeta_b = 0.25$ with the gradual increase in $p'$ (Figs. 4.11a and b), while ‘undrained-like’ $p'–q$ response loops are observed for $\zeta_b = 0.5$, as shown in Fig.
4.11c, which may be attributed to the local failure near the pile shaft induced by such large loading. Similar phenomenon was reported by Liu et al. (2021b) and Tsuha et al. (2012).

![Figure 4.11 Influence of the cyclic load amplitude ratio (ζ₀) on the stress paths in p'-q stress plane for soil element EF1: (a) ζ₀ = 0.15; (b) ζ₀ = 0.25; (c) ζ₀ = 0.5.](image)

The overall stress redistribution in the entire soil domain induced by the cyclic loading can be indicated by the contour of the \( p'/p'_{in} \) ratio, where \( p_{in} \) is the initial (after gravity loading) value of mean effective stress. Fig. 4.12 shows the evolution of the \( p'/p'_{in} \) ratio within the first cycle (Figs. 4.12a-c) and the final distribution of the \( p'/p'_{in} \) ratio after 100 cycles (Fig. 4.12d). A slight increase of \( p'/p'_{in} \) is observed near the ground surface and right bottom of the pile after pile generation (Fig. 4.12a, corresponds to Point T₀ in Fig. 4.7a). Fig. 4.12b (corresponds to Point P₁ in Fig. 4.7a) clearly shows the increase of \( p'/p'_{in} \) in the upper front area of the pile at the moment of peak load. Such a trend of \( p'/p'_{in} \) evolution agrees very well with the one presented by Liu and Kaynia (2021a) for the loading stages of the first cycle, although the evolution for two-way cyclic loads was shown. The \( p' \) in the reverse direction of the pile loading at the near back top layer increases (grey area) due to the compression on this area when the pile is pushed back (Fig. 4.12c, corresponds to Point T₁ in Fig. 4.7a). The grey area seems to be further extended by the long-term cyclic loading (Fig. 4.12d, corresponds to Point T₁₀₀ in Fig. 4.7a), while the non-negligible reduction in \( p'/p'_{in} \) occurs in the reverse direction of the loading near the pile shaft and in the front direction further away from pile top and near pile toe.

Overall, the shallow elements seem to be more sensitive to the variation of stress. One plausible reason is that the confining pressure acting on shallow elements is
very small and even minimal changes in one stress component can lead to large variation of the $p'/p'_{in}$.

Figure 4.12 Contours of $p'/p'_{in}$ for simulation case C2 ($\zeta_b = 0.25$) (a) initial stage (after pile generation), (b) at the peak load of $N = 1$, at the end of (c) $N = 1$ and (d) $N = 100$.

Figs. 4.13 and 4.14 respectively show the distribution of the $p'/p'_{in}$ ratio after 100 cycles (Point $T_{100}$ in Fig. 4.7a) for different pile loading amplitudes ($\zeta_b$) and different relative densities ($D_r$). The $p'/p'_{in}$ values are approximately symmetrically distributed in the vicinity of the pile (within the range of $2R$), with $p'/p'_{in}$ larger than 1 for $\zeta_b = 0.15$ (Fig. 4.12a), due to the gradual increase in $p$ revealed in Fig. 4.10.

The distribution of $p/p_{in}$ in Figs. 4.13b and 4.13c appears to be asymmetrical for larger $\zeta_b$ (i.e. $\zeta_b = 0.25$ and 0.5), where $p'/p'_{in}$ is found lower than 1 in front top area of the pile resulting from the decrease of $p'$, while the stress state at the back of the pile featured with $p'/p'_{in}$ larger than 1. Such asymmetry becomes more pronounced as $\zeta_b$ increases.
Figure 4.13 Contours of $p'/p'_{in}$ after 100 loading cycles for different pile loading amplitudes: (a) $\zeta_b = 0.15$; (b) $\zeta_b = 0.25$; (c) $\zeta_b = 0.5$.

The influence of relative density on the distribution of $p'/p'_{in}$ is illustrated in Fig. 4.14. The area with $p'/p'_{in}$ larger than 1 progressively enlarges with the increase of relative densities. It seems that $p'$ tends to increase in denser sand. The denser sand seems to be more prone to the development of $p'$ for shallower soil elements, while the $D_r$ has limited influence on the $p'/p'_{in}$ for deeper soil elements.

Figure 4.14 Contours of $p'/p'_{in}$ after 100 loading cycles for different densities: (a) $D_r = 30\%$; (b) $D_r = 50\%$; (c) $D_r = 70\%$.

4.3 Stress variation in soil elements around the pile

4.3.1 Generalised stress states

The variation of all six stress components for the eight representative soil elements (Fig. 4.1) under lateral cyclic loading is presented in Fig. 4.15. The inspection of the stress components variation for the investigated elements supports the following conclusions:
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(a) The vertical stress $\sigma_z$ in all soil elements decreases with the number of cycles within the first several cycles, then remains constant. While $\sigma_y$ increases slightly at the beginning and then reaches a stable state.

(b) For the soil elements collinear with the loading direction (Figs. 4.15a-d), obvious cyclic variation in the normal stress $\sigma_x$ and shear stress $\tau_{xz}$ is observed, while the two components of the shear stress ($\tau_{xy}$ and $\tau_{yz}$) outside the x-z plane remain constant with a value close to zero. As the soil element moves far away from the pile shaft (Figs. 4.15a-c), the stress cyclic amplitude progressively decreases. A similar decreasing trend is observed in soil elements at different depths (Figs. 4.15a, d, e).

(c) The comparison of the stress components of soil elements at different orientations (Figs. 4.15a and 4.15f–h) reveals the cyclic stress amplitude of the normal stress $\sigma_x$ and shear stress $\tau_{xz}$ which decreases with the increasing of the angle $\theta$. Again, shear stress $\tau_{yz}$ remains relatively close to zero during the whole loading process. Soil elements located orthogonally from the loading direction ($\theta = 90^\circ$) develop the least stress amplitude in all stress components but a significant cyclic variation of the shear stress $\tau_{xy}$ (Fig. 4.15h). It is interesting to be noticed that the cyclic amplitude of shear stress $\tau_{xy}$ increases with the increasing of the angle $\theta$.

Overall, the involvement of stress components induced by lateral pile cyclic loading is similar to that induced by lateral pile monotonic loading reported in Fig. 3.9 in Chapter 3. The shear stress $\tau_{yz}$ remains relatively negligible for all soil elements. The stress condition for soil elements in-plane with the direction of loading (EF1-4) can be simplified into four stress components - the three normal stresses ($\sigma_x$, $\sigma_z$, $\sigma_y$) and the vertical shear stress $\tau_{xz}$. Similarly, the stress condition for soil elements located orthogonally from the loading direction (ES3) can be simplified into four stress components - the three normal stresses ($\sigma_x$, $\sigma_z$, $\sigma_y$) and the horizontal shear stress $\tau_{xy}$. However, the stress state for soil elements diagonal to the loading direction (ES1-2) appears a bit more complex, and can be defined by five stress components, with the neglection of component $\tau_{yz}$.
4.3.2 Multiaxial stress paths

This section aims to investigate the effects of the radial distance from the pile, depth and orientation with respect to the loading direction of the representative soil elements on the multiaxial stress paths.

4.3.2.1 Effect of radial distance from the pile

The results in Figs. 4.15a-e imply the variation of only three normal stresses plus one shear stress for soil elements in the direction of loading $\theta = 0^\circ$. Such stress state is typically illustrated in the three-dimensional stress space $(\tau_{xz}, \sigma_x-\sigma_z, \sigma_y-p')$ recommended by Muir Wood (2017), where the deviatoric
stress axis \((\sigma_y - p')/p'\) considers the influence of the out of plane (intermediate) stress, as shown in Fig. 4.16. The Matsuoka-Nakai (1974) peak failure envelope for an assumed friction angle \(\varphi'\) of 37° (mentioned in Chapter 3) is provided for reference in the \((\tau_{xz}/p' \sim \sigma_x - \sigma_z/2p' \sim (\sigma_y - p')/p')\) stress plane (Figs. 4.16b, 4.16e and 4.16h), which is the maximum mobilised friction angle observed in the FE simulations.

Figs. 4.16a-c show the three-dimensional stress paths alongside two planar projections on elements EF1, EF2 and EF3 located at the right side of the pile and same depth \((z/L = 0.15)\) but with different horizontal radial distances from the pile. Fig. 4.16b clearly shows that the stress paths in \((\tau_{xz}/p' \sim \sigma_x - \sigma_z/2p')\) plane start from a very similar normalised stress state but reach different final asymptotic stress cycles. These stress paths are featuring: (a) the initial stress points lie on the negative side of the \((\sigma_x - \sigma_z/2p')\) axis as a result of the vertical stress being higher than the horizontal stress; (b) application of the cyclic lateral pile loading induces an inclined stress path characterised by the slope of each cycle \((\beta_{xz} = \Delta \tau_{xz}/p' \Delta \sigma_x - \sigma_z/2p')\), which progressively moves towards the positive side of the \((\sigma_x - \sigma_z/2p')\) as the cyclic loading proceeds but it also seems to evolve with a decreasing rate. As expected, the influence of the applied cyclic loading decreases with increasing distance from the pile, resulting in a lower amplitude of the cycle but also in a more limited transition towards the positive side of the \((\sigma_x - \sigma_z/2p')\).

The stress paths in the stress plane \((\sigma_x - \sigma_z/2p' \sim (\sigma_y - p')/p')\) in Fig. 4.16c also show similar variations and it is expected that the stress paths for soil elements closer to the pile shaft are more prone to move forward. However, the slope of each cycle defined by \(\zeta_{xz} = \Delta (\sigma_y - p')/p' \Delta \sigma_x - \sigma_z/2p'\) for different distances remains unchanged with \(\zeta_{xz}\) of around -53°.
Figure 4.16 Stress paths for simulation case C2 in \( \left( \tau_{xz} \sim \frac{\sigma_x - \sigma_z}{2\tau'} \sim \frac{\sigma_y - \sigma_z}{p'} \right) \) space, projection on plane \( \left( \tau_{xz} \sim \frac{\sigma_x - \sigma_z}{2\tau'} \sim \frac{\sigma_y - \sigma_z}{p'} \right) \) for soil elements at: different distance to pile (a, b, c), different depth below ground level (d, e, f) and different orientation with respect to loading direction (g, h, i). Stress paths in \( \left( \tau_{xy} \sim \frac{\sigma_x - \sigma_y}{2\tau'} \sim \frac{\sigma_z - \tau}{p'} \right) \) space, projection on plane \( \left( \tau_{xy} \sim \frac{\sigma_x - \sigma_y}{2\tau'} \sim \frac{\sigma_z - \tau}{p'} \right) \), and on plane \( \left( \tau_{yz} \sim \frac{\sigma_y - \sigma_z}{2\tau'} \sim \frac{\sigma_z - \tau}{p'} \right) \) for soil elements at different orientation with respect to loading direction (j, k, l).

4.3.2.2 Effect of depth along the pile

Figs. 4.16d-f report the variation of the stress paths for elements EF1, EF4 and EB located in line with loading direction and at the same horizontal distance from the pile \( (x/R = 1.375) \) but at different depths \( (z/L = 0.15, 0.5 \text{ and } 0.85, \text{ respectively}) \). Fig. 4.16d reveals that both the amplitude of the cyclic stress loops and the inclination of the stress paths are affected by the depth of the considered elements.

From the deviatoric projection plane \( \left( \tau_{xz} \sim \frac{\sigma_x - \sigma_z}{2\tau'} \right) \) in Fig. 4.16e, it can be seen that
elements EF1 and EF4 located above the pile rotation point are characterised by \( \beta_{xz} \) less than 90°, while element EB lies below the pile rotation point is characterised by an inclination \( \beta_{xz} \) of the final stress loop less than 0° due to the reversed value of shear stress \( \tau_{xz} \). The slope \( \beta_{xz} \) for EB soil element is about \(-75°\).

The depth has a significant influence on the stress paths magnitude in the stress plane \( \left( \frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_y - p'}{p'} \right) \) in Fig. 4.16f. The magnitude of stress path tends to decrease with the depth due to larger confining pressure in deeper soil elements, while the inclination of the stress path seems to be unaffected with a similar inclination \( \zeta_{xz} \) of about \(-53°\).

The inclination \( \beta_{xz} \) versus number of cycles for soil elements in line with lateral loading at various distances and depths are illustrated in Fig. 4.17a. The results in Fig. 4.17a are complemented in Figs. 4.16b and 4.16e, which explicitly show the negligible cyclic variation of inclination \( \beta_{xz} \) of stress paths after about 10 cycles. Nevertheless, the stress paths keep evolving towards the positive side of the horizontal axis, as shown in Fig. 4.17b. Around 80 loading cycles have been experienced by the soil element before it reaches a stable state. \( \beta_{xz} \) is also found to decrease with increase of the distance to pile and increase with depth in Fig. 4.17a. However, reversed evolution of the stress path is observed in the soil element EB already shown in Figs. 4.16d and 4.16e. The depth of the soil element has a more pronounced effect on the inclination of stress path if compared with the distance from the pile.

Figure 4.17 Stress paths for soil elements in line with lateral pile loading: (a) inclination \( \beta_{xz} \) versus number of cycles for soil elements at various distances and depths; (b) evolution of stress path with loading cycles (the colour bar indicates number of cycles).
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The distribution of the inclination $\beta_{xz}$ obtained from the final stress cycles (stable state) along horizontal distance from the pile shaft at several depths for different pile loading amplitudes and for different relative densities is given in Fig. 4.18a and Fig. 4.18b, respectively. For a given depth, it is noticeable that $\beta_{xz}$ continuously decreases with the increase of the distance to pile. The decrease of $\beta_{xz}$ with distance is much higher under strong loading than that for the weak case (Fig. 4.18a), while such a decrease is less for denser sand (Fig. 4.18b). Overall, the variation of $\beta_{xz}$ lies between 25° to 35° for soil element locations up to horizontal distance of $7R$.

Figure 4.18 Relationship between inclination $\beta_{xz}$ for soil elements in line with lateral loading and distances at various depths: (a) various amplitude ($\zeta_b$); (b) various relative density ($D_r$).

4.3.2.3 Effect of orientation with respect to the loading direction (same distance from the pile shaft)

Figs. 4.16g-l depict the stress paths for soil elements located at different orientations with respect to the loading direction. As observed in Figs. 4.15f-h, the application of lateral pile load leads to the variation of five stress components with neglect of shear stress $\tau_{yz}$ for the cases $\theta > 0^\circ$. Two 3D stress spaces $\left(\frac{\tau_{xz}}{p^r} \sim \frac{\sigma_x - \sigma_z}{2p^r} \sim \frac{\sigma_y - p^r}{p^r}\right)$ in Fig. 4.16g and $\left(\frac{\tau_{xy}}{p^r} \sim \frac{\sigma_x - \sigma_y}{2p^r} \sim \frac{\sigma_z}{p^r}\right)$ in Fig. 4.16j are therefore needed to illustrate the representative stress paths. This result is consistent with those derived from monotonic loading (Fig. 3.11 in Chapter 3).

The evolution of stress paths in the $\left(\frac{\tau_{xz}}{p^r} \sim \frac{\sigma_x - \sigma_z}{2p^r} \sim \frac{\sigma_y - p^r}{p^r}\right)$ space at different $\theta$ is found to be similar (Fig. 4.16g). Both the magnitude and inclination of stress paths in
(\frac{\tau_{xx}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'}) plane tend to decrease as the \( \theta \) increases (Fig. 4.16h). The stress path for the soil element perpendicular to the direction of the load (ES3, \( \theta = 90^\circ \)) shows a flat trend with shear stress \( \tau_{xz} \) gradually approaching zero as the vertical interface shear stress \( \tau_{xz} \) is not expected to develop at this orientation. Similar trends of the stress paths can also be observed in the \( \left( \frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_y - p'}{p'} \right) \) plane characterised by the inclination \( \zeta_{xz} = -53^\circ \), except for the nearly vertical inclination for soil element ES3, in which the normal stress \( \sigma_x \) and \( \sigma_z \) keep unchanged during cycling.

Figs. 4.16j-l reveals that the stress paths in the horizontal \( x-y \) plane are highly dependent on the orientations (\( \theta \)) of soil element: the inclination of stress path in \( \left( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \right) \) stress plane in Fig. 4.16k, defined by \( \beta_{xy} = \Delta \frac{\tau_{xy}}{p'} / \Delta \frac{\sigma_x - \sigma_y}{2p'} \) for each stress cycle (where \( \Delta \frac{\tau_{xy}}{p'} \) and \( \Delta \frac{\sigma_x - \sigma_y}{2p'} \) are the differences between the peak point and trough point of each cycle), seems to increase with \( \theta \). Such finding results from the gradual decrease of variation of \( \sigma_x \) and the large development of shear stress \( \tau_{xy} \) as \( \theta \) increases, as indicated in Fig. 4.15. In particular, the vertical stress path for soil element ES3 (\( \theta = 90^\circ \)) suggests a direct simple shear condition (Fig. 4.16k).

The projection of the stress paths in the \( \left( \frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_z - p'}{p'} \right) \) stress plane (Fig. 4.16l) also appears to be influenced by the element positions (\( \theta \)) with the inclination \( (\zeta_{xy} = \Delta \frac{\sigma_z - p'}{p'} / \Delta \frac{\sigma_x - \sigma_y}{2p'}) \) of the stress loops changing from \(-30^\circ\) to \(0^\circ\), as the soil elements move away from the loading direction. The soil element ES3 at \( \theta = 90^\circ \) instead shows a limited variation in magnitude due to the least change in normal stress \( \sigma_x \).

The evolution of stress paths for the soil elements at different \( \theta \) indicates that the stress variation in \( x-y \) plane progressively becomes more and more important than that in \( x-z \) plane.

Fig. 4.19a shows the positive correlation between the inclination of stress path \( \beta_{xy} \) and orientation of soil element \( \theta \), as well as a relative constant \( \beta_{xy} \) is reached after 10 loading cycles. However, from Fig. 4.19b, in which the evolution of stress path in \( \left( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \right) \) plane with load cycles is indicated by shaded sidebar, it can be clearly observed that the stress path is shifted with every cycle and becomes stable after 80 cycles.
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Figure 4.19 Stress paths for soil elements around the pile: (a) inclination $\beta_{xy}$ versus number of cycles for soil elements at various orientations; (b) evolution of stress path with loading cycles (the colour bar indicates number of cycles).

An insight into the variation of $\beta_{xy}$ extracted from the last stress path cycle for different $(\zeta_B)$ and different relative densities $(D_r)$ is given in Fig. 4.20 and Fig. 4.21, respectively. The $\beta_{xy}$ for soil elements at $\theta = 0^\circ$ changes slightly with the decreasing of both distance and depth, as shear stress $\tau_{xy}$ is not expected to develop at this orientation. For soil elements outside the direction of loading ($\theta > 0^\circ$), the variation of $\beta_{xy}$ values is visible for soil elements within $4.5R$ distance from the pile shaft, then the values stay nearly constant (Figs. 4.20a and 4.21a). While the $\beta_{xy}$ values show only little variations with the depth (Figs. 4.20b and 4.21b).

The variation of $\beta_{xy}$ with distance for $\zeta_B = 0.15$, 0.25 and 0.5 illustrated in Fig. 4.20a shows that the $\beta_{xy}$ changes obviously with higher $\zeta_B$ values for the soil elements in the vicinity of pile, while it is not affected by $\zeta_B$ when the soil elements move further distance from the pile. The $\beta_{xy}$ values seem to be higher in denser sand (Fig. 4.21).
Figure 4.20 Variation of inclination $\beta_{xy}$ for soil elements at various orientations with: (a) various distances and (b) various depths under different loading amplitudes.

Figure 4.21 Variation of inclination $\beta_{xy}$ for soil elements in different relative densities ($D_r$) various orientations with: (a) various distances and (b) various depths.

Fig. 4.22 highlights the relationship between the inclination $\beta_{xy}$ induced by one-way cyclic pile loading and orientation by displaying the numerical results for all the simulation cases except case C6 simulating two-way cyclic loading. A linear relationship between the $\beta_{xy}$ and $\theta$ (i.e. $\beta_{xy} = \theta$ for simplicity) can be reasonably assumed, although most of the points lie below the fitting line. The points with higher $D_r$ and $\zeta_b$ appear to be closer to the fitting line.
4.3.2.4 Influence of loading amplitude and relative density on evolution of stress paths

The previous sections mainly focus on the variation of inclination of stress paths. The progressive shift of each cycle also appears to be an important feature to be considered. In this section the evolution of stress paths concerning the influence of the loading amplitude ($\zeta_b$) and relative density ($D_r$) is discussed.

The stress paths under monotonic and cyclic loading are qualitatively compared in Fig. 4.23 for soil element EF1 in the $(r_{xz} \sim \frac{\sigma_x - \sigma_z}{2\sigma_t})$ stress plane (Fig. 4.23a) and for soil element ES2 in the $(\frac{r_{xy}}{\sigma_t} \sim \frac{\sigma_x - \sigma_y}{2\sigma_t})$ stress plane (Fig. 4.23b), respectively. Fig. 4.23 clearly shows that the stress path induced by the first monotonic loading of cycles is superimposed on that induced by pure monotonic loading. However, the subsequent cyclic stress paths in the $(\frac{r_{xz}}{\sigma_t} \sim \frac{\sigma_x - \sigma_z}{2\sigma_t})$ stress plane in Fig. 4.23a tend to move forward, while the stress paths in the $(\frac{r_{xy}}{\sigma_t} \sim \frac{\sigma_x - \sigma_y}{2\sigma_t})$ stress plane in Fig. 4.23b show hysteretic responses. Such changes would cause the differences between the monotonic and cyclic stress paths (obtained from stable state) in terms of initial point and inclination, which will be discussed in detail the following section.
Cyclic stress paths under different amplitude ratio $\zeta_b$ are also provided in Fig. 4.23. Obviously, higher $\zeta_b$ values lead to larger magnitude of stress path. In addition, it is found that the soil elements tend to experience a non-symmetric two-way cyclic stress path with higher $\zeta_b$, though the one-way cyclic loading acting on the pile.

![Cyclic stress paths comparison](image)

Figure 4.23 Comparison of evolution stress paths under monotonic and cyclic loading with different cyclic load amplitude ratio ($\zeta_b$): (a) stress paths in $\left(\frac{\tau_{xz}}{p'} - \frac{\sigma_x - \sigma_z}{2p'}\right)$ plane for soil element EF1; (b) stress paths in $\left(\frac{\tau_{xy}}{p'} - \frac{\sigma_x - \sigma_y}{2p'}\right)$ plane for soil element ES2 ($D_r = 50\%$).

Regarding the effect of the relative density ($D_r$), larger progressive forward drift of stress path in $\left(\frac{\tau_{xz}}{p'} - \frac{\sigma_x - \sigma_z}{2p'}\right)$ stress plane is clearly visible for looser sand (e.g. $D_r = 30\%$) in Fig. 6.24a, while the larger hysteretic loop of stress path in $\left(\frac{\tau_{xy}}{p'} - \frac{\sigma_x - \sigma_y}{2p'}\right)$ stress plane is observed in denser sand (e.g. $D_r = 70\%$) in Fig. 6.24b. The relative density seems to have negligible influence on magnitude of stress in both stress plane.
Figure 4.24 Comparison of evolution of stress paths under monotonic and cyclic loading with different relative density ($D_r$): (a) stress paths in $\left( \frac{\tau_{xy}}{p'} - \frac{\sigma_x - \sigma_z}{2p'} \right)$ plane for soil element EF1; (b) stress paths in $\left( \frac{\tau_{xy}}{p'} - \frac{\sigma_x - \sigma_y}{2p'} \right)$ plane for soil element ES2 ($\zeta_b = 0.25$).

4.4 Relevance of laboratory testing to reproduce the stress paths

In the previous chapter, it was shown that a conventional triaxial test does not capture the response of a soil element surrounding the laterally loaded pile, whereas the HCTA which possesses four degrees of freedom appears to be a preferred choice for simulating multiaxial response. Given the similarity of cyclic stress paths to the monotonic stress paths in terms of the involvement of stress components, this section continues to focus on assessing the applicability of the cyclic stress paths.

4.4.1 Evaluation of laboratory stress paths

Similar to previous Section 3.5.1 in Chapter 3, the soil elements at four representative positions are investigated in terms of the replication of the FE stress path in HCTA, as illustrated in Fig. 4.25, in which the stress path at first cycle, 10th cycle and 100th cycle are displayed. The grey area in Fig. 4.25 denotes the range of stress paths which can be applied using different $p_o/p_i$ ratios within the admissible limits (details were reported in Chapter 3). The stress paths for simpler triaxial conditions are also reported where deemed appropriate. The stress paths
in \( \left( \frac{\tau_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_y - p'}{p'} \right) \) stress plane for soil elements in line with pile loading direction and stress paths in \( \left( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_z - p'}{p'} \right) \) stress plane for soil elements at \( \theta > 0^\circ \) are considered herein for assessment.

**Case F (element in front of the pile – EF1):** Figs. 4.25a-c show that the HCTA can well reproduce the stress paths (100th cycle) in the main deviatoric plane \( \left( \frac{\tau_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'} \right) \) (Fig. 4.25b) but discrepancies can be noticed in the 3D plot due to the difference in the \( \left( \frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_y - p'}{p'} \right) \) stress plane (Fig. 4.25c). The HCTA is not able of reproducing the initial stress conditions but can well reproduce the later half of the predicted FE stress path. It should be noticed that the value \((\sigma_y - p')\) at the initial state would approach zero or be even positive with evolution of the cyclic stress paths in Fig. 4.25c due to the gradual decreasing in \(\sigma_z\), which therefore benefits the reproduction of the stress paths by HCTA. However, for this case, such evolution causes the FE stress path beyond the upper boundary \((p_0/p_i = 0.9)\) of available HCTA tests (Fig. 4.25c), while the imposition of a ratio \(p_0/p_i = 1.2\) (lower boundary) can help the HCTA conditions approach FE monotonic stress paths as indicated in the previous chapter. The stress path extracted between the 10th to 40th cycle appears to be perfectly reproduced in HCTA.

Although such a behaviour resembles the pile installation effect explicitly discussed in Chapter 3, which instead causes trend \((\sigma_y - p')\) to approach zero by increasing the horizontal stress \(\sigma_x\) and \(\sigma_y\), the pile installation also needs to be considered for an accurate assessment in future.

Figs. 4.25a-c also show how a conventional triaxial test produces a rather different stress path in the multiaxial stress space, neglecting the development of shear stresses due to the frictional pile-soil interaction. A triaxial test would also neglect the continuous rotation of principal stress axes induced by the lateral loading. It should be reminded the two-way cyclic stress paths in \( \left( \frac{\tau_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'} \right) \) plane is possible for large \(\zeta_b\) suggested in Section 4.4.2.4 or for the soil elements closer to the pile.

**Case B (element on the back of the pile - EB):** the FE stress paths in Figs. 4.25d-f show a reverse trend compared to those in Case F due to the reversal sign in the
shear stress $\tau_{xz}$ (Fig. 4.15e). The discrepancies between the FE stress paths and those available in HCTA are more pronounced in this case (Fig. 4.25f), because the initial difference between vertical stress ($\sigma_z$) and radial stress ($\sigma_r$) is larger as deeper soil elements are taken into account. The evolution of stress paths is limited, therefore considering the pile installation effect may further mitigate these discrepancies.

**Case D (element diagonal with respect to lateral loading direction – ES2):** obvious hysteresis of FE stress path cycle is observed in the Case D (Fig. 4.25h). Accurately reproducing the hysteretic response is not the aim of this research. In this sense, the FE stress path can be well reproduced in the $\left( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \right)$ plane by the HCTA in terms of the initial point and inclination of the stress path. Similar to Case F, the HCTA cannot completely reproduce the effect of intermediate principal stress, especially at the early stage of loading (Fig. 4.25i). However, the stress paths obtained from earlier cyclic stages or a more accurate account of pile installation may lead to a better agreement between field and laboratory conditions.

**Case S (element on the side of the pile – ES3):** despite the hysteretic response, both HCTA or simple torsional loading (similar to stress $p_0 = p_i$ in HCTA) appear to be reasonable to reproduce the predicted FE stress path in the $\left( \frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \right)$ plane (Fig. 4.25k), while some discrepancies can also be noticed in the plot involving the intermediate principal stress (Fig. 4.25l). However, in light of the previous considerations about stress path evolution and installation effects, it is expected that the initial condition for stress path after 100th cycle or less cycles may be closer to isotropic than predicted, thus enabling the real stress path to approach the HCTA conditions.
Chapter 4. Numerical evaluation of soil stress paths under cyclic lateral pile loading

Figure 4.25 Comparison between idealised stress paths and those applicable in element testing in \( (\sigma_{xy}/p') - (\sigma_{xz} - \sigma_{yz} - \sigma_{y'p'}) \) space, projection on plane \( (\sigma_{xy}/p') - (\sigma_{xz} - \sigma_{y'p'}) \), and on plane \( (\sigma_{zx} - \sigma_{xy}) - (\sigma_{yx} - \sigma_{y'p'}) \) for soil elements Case F (a, b, c), Case B (d, e, f), and in \( (\sigma_{xy}/p') - (\sigma_{yx} - \sigma_{y'p'}) \) space, projection on plane \( (\sigma_{xy}/p') - (\sigma_{yx} - \sigma_{y'p'}) \), and on plane \( (\sigma_{zx} - \sigma_{xy}) - (\sigma_{yx} - \sigma_{y'p'}) \) for soil elements Case D (g, h, i), and Case S (j, k, l).
Table 4.3 provides a summary of the laboratory stress paths suggested to investigate the mechanical response of soil elements around cyclic laterally loaded pile. The table discriminates between four different soil element locations and summarises the initial and incremental stress conditions to be applied in HCTA testing. Overall, one-way stress path cycle is recommended for Case F, while the stress path cycle for Cases B, D and S appear to be partial two-way. Such application in HCTA is based on the following simplifications:

(a) reduction to a four-dimensional stress state by neglecting the two lowest shear stress components of FE stress conditions;

(b) given the known HCTA sample stress and strain field nonuniformities caused by the difference between inner ($p_i$) and outer cell pressure ($p_o$), $p_o = p_i$ is imposed;

(c) mean isotropic stress remains constant by keeping outer and inner cell pressure constant, despite the progressive increasing and vanishing in $p'$ for small $\zeta_b$ and higher $\zeta_b$ respectively (Fig. 4.11).

### 4.4.2 Note on rotation of stress coordinates between field and laboratory

Chapter 3 has demonstrated the necessity of a relative 90° rotation between the material axis and the stress direction to impose the FE stress paths in HCTA. In this chapter, it is initially assumed that the sample is horizontally oriented as is shown in Fig. 3.16 in Chapter 3, in which the largest stress variation occurs in the normal $x$ direction ($\sigma_x$) and non-negligible changes are observed in the shear components that are normal to this direction ($t_{xz}$ and $t_{xy}$). Due to the same challenge in reproducing the loading direction with respect of the material axis as in Chapter 3, a relative 90° rotation between the two directions is also required in this chapter (Fig. 3.16).
Table 4.3 Summary of recommended experimental tests for the soil elements around the laterally loaded pile.

<table>
<thead>
<tr>
<th>Soil element position</th>
<th>Suggested HCTA testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case F</td>
<td><strong>Initial conditions</strong></td>
</tr>
<tr>
<td></td>
<td>$\sigma_z = \gamma' \cdot z$;</td>
</tr>
<tr>
<td></td>
<td>$\sigma_x = \sigma_y = K \cdot \sigma_z$; [$K$ depends on the soil conditions and pile installation effect]</td>
</tr>
<tr>
<td></td>
<td>$0 \leq \tau_{xz} \leq \sigma_x \cdot \tan(\phi_{ps})$. [$\beta_{xz} = 15^\circ$ to $45^\circ$]</td>
</tr>
<tr>
<td></td>
<td><strong>Incremental stress or strain conditions:</strong></td>
</tr>
<tr>
<td></td>
<td>Apply vertical stress ($\Delta\sigma_x$) or strain ($\Delta\varepsilon_x$) increments;</td>
</tr>
<tr>
<td></td>
<td>$\Delta\sigma_z = \Delta\varepsilon_y = 0$; [constant inner and outer cell pressures]</td>
</tr>
<tr>
<td></td>
<td>$\Delta\tau_{xz} = (\Delta\sigma_x - \Delta\sigma_z)/2 \cdot \tan(\beta_{xz})$.</td>
</tr>
</tbody>
</table>

| Case B                | **Initial conditions**  |
|                       | $\sigma_z = \gamma' \cdot z$; |
|                       | $\sigma_x = \sigma_y = K \cdot \sigma_z$; [$K$ depends on the soil conditions and pile installation effect] |
|                       | $0 \leq \tau_{xz} \leq \sigma_x \cdot \tan(\phi_{ps})$; |
|                       | **Incremental stress or strain conditions:** |
|                       | Apply vertical stress ($\Delta\sigma_x$) or strain ($\Delta\varepsilon_x$) increments; |
|                       | $\Delta\sigma_z = \Delta\varepsilon_y = 0$; [constant inner and outer cell pressures] |
|                       | $\Delta\tau_{xz} = (\Delta\sigma_x - \Delta\sigma_z)/2 \cdot \tan(\beta_{xz})$. [$\beta_{xz} = -75^\circ$] |

| Case D                | **Initial conditions**  |
|                       | $\sigma_z = \gamma' \cdot z$; |
|                       | $\sigma_x = \sigma_y = K \cdot \sigma_z$; [$K$ depends on the soil conditions and pile installation effect] |
|                       | $\tau_{xy} = 0$. |
|                       | **Incremental stress conditions:** |
|                       | Apply vertical stress ($\Delta\sigma_x$) or strain ($\Delta\varepsilon_x$) increments ($\theta < 45^\circ$); shear stress ($\Delta\tau_{xy}$) or strain ($\Delta\gamma_{xy}$) increments ($45^\circ < \theta < 90^\circ$); |
|                       | $\Delta\sigma_z = \Delta\varepsilon_y = 0$; [constant inner and outer cell pressures] |
|                       | $\Delta\tau_{xy} = (\Delta\sigma_x - \Delta\sigma_z)/2 \cdot \tan(\beta_{xy})$. [$\beta_{xy} \approx \theta$] |

| Case S                | **Initial conditions**  |
|                       | $\sigma_z = \gamma' \cdot z$; |
|                       | $\sigma_x = \sigma_y = K \cdot \sigma_z$; [$K$ depends on the soil conditions and pile installation effect] |
|                       | $\tau_{xy} = 0$. |
|                       | **Incremental stress conditions:** |
|                       | shear stress ($\Delta\tau_{xy}$) or strain ($\Delta\gamma_{xy}$) increments; |
|                       | $\Delta\sigma_z = 0$; |
|                       | $\Delta\sigma_x = \Delta\sigma_y = 0$. [constant inner and outer cell pressures] |
4.5 Conclusions

This chapter explored the stress paths induced by one-way cyclic lateral pile loading. The stress paths obtained from the 3D finite element model considering the application of 100 lateral loading cycles are analysed and clear instructions for the application of these stress paths are provided. Based on the observations in this chapter, the following conclusions can be drawn:

- The 3D finite element analysis has shown that soil elements in line with the pile loading direction undergo complex stress paths involving the cyclic variation of three normal stress plus one shear stress out of six stress components, while five stress components neglecting the shear stress $\tau_{yz}$ are found to be changed during cycling for soil elements out of the pile loading direction.

- Analysis of the cyclic stress paths in the $\left(\frac{\tau_{xz}}{p'}, \frac{\sigma_x - \sigma_z}{2p'}, \frac{\sigma_y - p'}{p'}\right)$ stress plane has revealed a quite complex evolution of the stress paths to reach an asymptotic final cyclic stress condition. The magnitude, location and inclination of the asymptotic stress conditions depend on the location (distance, depth and orientation) of the inspected elements with respect to the pile and pile loading conditions.

- The cyclic stress paths in the $\left(\frac{\tau_{xy}}{p'}, \frac{\sigma_x - \sigma_y}{2p'}, \frac{\sigma_z - p'}{p'}\right)$ stress plane highly depend on the orientation of soil elements, which are characterized by hysteretic loops. This is particularly evident for higher $\zeta_b$ value.

- Different monotonic stress paths, both stress paths in $\left(\frac{\tau_{xz}}{p'}, \frac{\sigma_x - \sigma_z}{2p'}, \frac{\sigma_y - p'}{p'}\right)$ and $\left(\frac{\tau_{xy}}{p'}, \frac{\sigma_x - \sigma_y}{2p'}, \frac{\sigma_z - p'}{p'}\right)$ stress planes are found associated with $D_r$ and $\zeta_b$.

- The cyclic stress paths in the main deviatoric projection plane ($\frac{\tau_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'}$ or $\frac{\tau_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'}$) can be well reproduced by HCTA, while some compromises are required in the $\left(\frac{\sigma_x - \sigma_z}{2p'} \sim \frac{\sigma_y - p'}{p'}\right)$ or $\left(\frac{\sigma_x - \sigma_y}{2p'} \sim \frac{\sigma_z - p'}{p'}\right)$ plane. Although the evolution of cyclic stress path can mitigate or overrule the magnitude of the intermediate principal stress from FE predictions, pile installation effect still needs to be considered.
• Idealised cyclic stress paths for key locations around the monopile, starting
from an initial anisotropic state and evolving with simultaneous variation of
axial and torsional stress, along with the corresponding stress conditions
are summarised in Table 3 to inform the HCTA testing.

Particularly, these tests may reduce to simple shear conditions for soil element on
the side of the laterally loaded pile but, for other element locations, the HCTA
enables a much better simulation, including the replication of the rotation of
principal stress axes if compared to element tests ordinarily used in practice such
as the conventional triaxial apparatus. The assumptions for simplification of the
stress paths can provide recommendations for current testing practices. For the
following chapters, the prescribed stress paths informed by this chapter will be
performed to explore the response of soil subjected to multiaxial stress conditions.
Chapter 5  Experimental setup and testing procedure

5.1 Introduction

A common feature of the stress paths derived from the numerical investigation in Chapters 3 and 4 is the variation of multiple normal and shear stress components associated with the re-orientation of the principal stress axes. Therefore, it appears that the HCTA can be the experimental apparatus to reproduce the proposed stress paths since (i) it is the only apparatus that enables the independent control of up to four stress components; (ii) allows to impose rotation of principal stress axes; and (iii) allows for the conventional triaxial and simple shear stress conditions by applying appropriate stress and strain boundary constraints.

This chapter introduces the HCTA at University of Bristol in detail. Detailed evaluation of the small strain measurement system is presented, followed by a description of the material used in this research. The final part of this chapter deal with the sample preparation technique as well as the overall testing procedure including the saturation, consolidation and shearing as designed for this study.
5.2 Description of HCTA facilities

5.2.1 Principles of HCTA

5.2.1.1 General

Chapter 2 has briefly mentioned that the HCTA provides an effective method for the investigation of soil anisotropy, which can be attributed to its independent control of four degrees of freedom: axial load $W$, torsional load $T$, and internal $p_i$ and external cell pressures $p_o$, as schematically shown in Fig. 5.1a. Accordingly, four different stress components are allowed for managing: the axial stress $\sigma_z$, shear stress $\tau_{\theta z}$, radial stress $\sigma_r$, and lastly circumferential stress $\sigma_\theta$ for the equilibrium of $\sigma_r$. The corresponding deformations of a soil element can be expressed by axial strain $\varepsilon_z$, shear strain $\gamma_{\theta z}$, radial strain $\varepsilon_r$ and circumferential strain $\varepsilon_\theta$. As a result, this equipment permits performing fundamental research considering the effect of re-orientation of principal stress axes.

*Disclaimer: Please note the different definitions of the coordinate system if compared to Chapters 3 and 4. This is to maintain the convention of symbols used for stress and strain for HCTA testing in the literature.*

Figure 5.1 Definition of forces and stress state in hollow cylinder specimen: (a) surface loads, (b) stress components and (c) main principal stresses on a representative element of the specimen’s wall (Mandolini, 2018).
5.2.1.2 Stress and strain distribution in hollow cylindrical specimens

As shown in Fig. 5.1, the stress state for a HCTA sample is defined using the cylindrical coordinate system, which can be expressed in the following:

\[
\sigma = \begin{bmatrix}
\sigma_r & 0 & 0 \\
0 & \sigma_\theta & \tau_{\theta z} \\
0 & \tau_{z \theta} & \sigma_z
\end{bmatrix}
\]  \hspace{1cm} (5.1)

Following either the equilibrium considerations or soil's constitutive law, Hight et al. (1983), Miura et al. (1986) and Vaid et al. (1990) suggested that the average axial \(\sigma_z\), shear \(\tau_{\theta z}\), radial \(\sigma_r\) and circumferential \(\sigma_\theta\) stresses can be calculated by the equations listed in Table 5.1.

<table>
<thead>
<tr>
<th>Axis direction</th>
<th>Stress equations</th>
<th>Strain equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Axial normal</strong></td>
<td>(\sigma_z = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{W}{\pi (r_o^2 - r_i^2)})</td>
<td>(\varepsilon_z = -\frac{\Delta H_S}{H_S})</td>
</tr>
<tr>
<td><strong>Radial normal</strong></td>
<td>(\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} )</td>
<td>(\varepsilon_{xr} = -\frac{\Delta r_o - \Delta r_i}{r_o - r_i})</td>
</tr>
<tr>
<td><strong>Circumferential</strong></td>
<td>(\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{2(p_o - p_i) r_o^2 r_i^2 \ln \left(\frac{r_o}{r_i}\right)}{(r_o^2 - r_i^2)^2})</td>
<td>(\varepsilon_\theta = -\frac{\Delta r_o + \Delta r_i}{r_o + r_i})</td>
</tr>
<tr>
<td><strong>normal</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Circumferential</strong></td>
<td>(\tau_{\theta z} = \tau_{z \theta} = \frac{3T}{2\pi (r_o^3 - r_i^3)})</td>
<td>(\gamma_{\theta z} = \frac{2\Delta \theta_S (r_o^3 - r_i^3)}{3 H_S (r_o^2 - r_i^2)})</td>
</tr>
<tr>
<td><strong>shear</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The state of stress of a representative soil element can be represented by the mean stress, \(p\), generalised deviatoric component of stress, \(q\), the angle between the major principal stress direction and vertical \(z\)-axis, \(\alpha_\sigma\), and the intermediate principal stress ratio, \(b\), parameters defined as:

\[
p = \frac{\sigma_z + \sigma_\theta + \sigma_r}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}
\]  \hspace{1cm} (5.2)
\( q = \sqrt{\frac{(\sigma_z - \sigma_r)^2 + (\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2}{2} + 3\tau_{\theta z}^2} \) \hspace{1cm} (5.3)

\[ \alpha_\sigma = \frac{1}{2} \tan^{-1} \left( \frac{2\tau_{\theta z}}{\sigma_\theta - \sigma_z} \right) \] \hspace{1cm} (5.4)

\[ b = \frac{\sigma_z - \sigma_r}{\sigma_1 - \sigma_3} \] \hspace{1cm} (5.5)

Similar to stress state, a matrix for strain state is defined as:

\[
E = \begin{bmatrix}
\varepsilon_r & 0 & 0 \\
0 & \varepsilon_\theta & \gamma_{\theta z} \\
0 & \varepsilon_{z \theta} & \varepsilon_z
\end{bmatrix}
\] \hspace{1cm} (5.6)

The axial strain (\( \varepsilon_z \)), radial strain (\( \varepsilon_r \)), circumferential strain (\( \varepsilon_\theta \)) and shear strain (\( \gamma_{\theta z} \)) can be defined by the equations listed in Table 5.1 proposed by Hight et al. (1983) and Vaid et al. (1990), where \( H_s \) denotes the sample height, \( \Delta H_s \) is the vertical displacement, \( \Delta r_o \) and \( \Delta r_i \) are the changes in the outer and inner radius respectively and \( \theta_s \) stands for the rotation relative to the top of the specimen.

The state of strain of a representative soil element can be represented by the volumetric strain, \( \varepsilon_v \) and deviatoric strain, \( \varepsilon_q \), defined as follows:

\[ \varepsilon_v = \varepsilon_z + \varepsilon_r + \varepsilon_\theta \] \hspace{1cm} (5.7)

\[ \varepsilon_q = \sqrt{\frac{2(\varepsilon_z - \varepsilon_r)^2 + 2(\varepsilon_r - \varepsilon_\theta)^2 + 2(\varepsilon_\theta - \varepsilon_z)^2 + 3\gamma_{\theta r}^2}{3}} \] \hspace{1cm} (5.8)

### 5.2.2 Stress non-uniformities in hollow cylinder specimens

Studies by Hight et al. (1983) and Sayao and Vaid (1991) suggested that one of the main disadvantages of the hollow cylindrical specimens is the non-uniformity of stresses and strains across the sample’s wall caused by end-restraint and sample’s curvature. The extent of such non-uniformity was found to be significantly affected by the constitutive law of the soil, the specimen geometries and stress conditions.

For cylindrical solid samples, enlarged lubricated ends are usually used to mitigate the end restraints effect, such as conventional triaxial tests. However, this is not
applicable for HCTA samples due to the involvement of torsional loading, which requires specific loading cap solutions.

However, such non-uniform distribution of stress and strain can be minimised through an appropriate selection of the specimen geometries and by avoiding certain stress conditions (e.g., Sayão and Vaid, 1991), details will be discussed in the following.

**Effect of stresses**

Either the imposition of torsional loading or different external and internal cell pressure can cause the stress and strain variation across the wall of the hollow cylindrical specimen regardless of the end restraint, as schematically shown in Fig. 5.2. The previous chapters have briefly mentioned the stress non-uniformity arising from the stress difference between outer and inner pressure.

It is challenging to measure the stresses or the strains distributed on the horizontal cross-section of the hollow cylindrical specimen. In this respect, numerical modelling provided a suitable method for its acquisition. Hight et al. (1983), for example, schematically evaluated the stress non-uniformity characterised by two parameters $\beta_1$ and $\beta_3$ using the FE modelling with a strain-hardening Modified Cam-Clay constitutive model. $\beta_1$ described the difference between the actual and calculated mean stresses as follows:

$$\beta_1 = \frac{|\bar{\sigma}^* - \bar{\sigma}|}{\sigma_L} \quad (5.9)$$

where $\bar{\sigma}^*$ is the actual average stress calculated by the Equation in Table 5.1, $\bar{\sigma}$ represents the calculated stress, and $\sigma_L$ stands for the mean value of the radial and circumferential stresses (relevant definitions used for stress uniformity are illustrated in Fig. 5.2), which is given by:

$$\sigma_L = \frac{|\bar{\sigma}_p| + |\bar{\sigma}_r|}{2} \quad (5.10)$$

The coefficient $\beta_3$ which measures the degree of the stress non-uniformity was proposed by:
\[
\beta_3 = \frac{\int_{r_i}^{r_o} |\sigma(r) - \bar{\sigma}^*| dr}{(r_o - r_i)\sigma_L}
\]

(5.11)

where \(\sigma(r)\) defines the distribution of the stress components along the horizontal cross-section of the hollow cylindrical sample, as shown in Fig. 5.2. According to Hight et al. (1983), the ratio of external to internal cell pressures should be limited to \(0.9 < \frac{p_o}{p_i} < 1.2\) (criterion for available stress path in HCTA mentioned in Chapters 3 and 4), and \(\beta_3\) should be kept below 0.11 to control the level of non-uniformity within an acceptable range.

Figure 5.2 Definitions used for stress non-uniformity (Modified after Hight et al., 1983).

However, Vaid et al. (1990) argued that the parameter \(\beta_3\) proposed by Hight et al. (1983) seemed to underestimate the stress non-uniformities for particular stress paths under consideration of the influence of the torsional stress \(\tau_{ez}\), which could lead to a large non-uniform distribution for stress ratio \(R\) \((R = \sigma_1^*/\sigma_3^*)\). A new parameter \(\beta_r\) therefore was proposed considering the variation of the effective principal stress for a better assessment of the degree of non-uniformity, which can be defined as follows:

\[
\beta_r = \frac{(R_{max} - R_{min})}{\bar{R}}
\]

(5.12)

where \(R_{max}\) and \(R_{min}\) represent the maximum and minimum principal stress ratios, respectively and \(\bar{R}\) represents the average level of stress ratio.
The author compared the variation of the two coefficients $\beta_3$ and $\beta_r$ with stress ratio $R$ for two arbitrary stress states: $b = 0.5$, $\alpha = 0$ and $b = 0$, $\alpha = 45^\circ$ as shown in Fig. 5.3, where rotation of principal stress axis $\alpha$ and the intermediate principal stress parameter $b$ can be calculated by Equations 5.4 and 5.5, respectively. The results show that both $\beta_3$ and $\beta_r$ increase with $R$. From Fig. 5.3, acceptable non-uniformities appear to be within the full range of $R$ under evaluation of $\beta_3$. However, most HCA tests showed that the non-uniformities are intolerant once $R$ exceeds 2.0-2.2 for the general specimen geometry ($R_o = 7.6$ cm, $R_i = 5.1$ cm). In this respect, $\beta_r$ seems to be a more rational index of stress nonuniformity across the wall of the specimen. An acceptable degree of the stress non-uniformity was found by the author when the difference between $R_{\text{max}}$ and $\bar{R}$ is less than 10% corresponding to $\beta_r \leq 0.2$. 
Figure 5.3 Effect of stress ratio level on non-uniformity coefficients (Vaid et al., 1990).

**Effect of sample geometry**

Another key factor affecting the stress non-uniformities is the specimen geometry. Sayão and Vaid (1991) summarised some HCA sample geometries used by various researchers, as listed in Table 5.2 and illustrated in the plot of wall thickness \((R_e-R_i)\) of the hollow cylindrical sample against \(R/R_e\) as well as \(H/2R_e\) in Fig. 5.4, where \(R_e\) and \(R_i\) are respectively the outer and inner radii. The number next to the points in Fig. 5.4 refers to the test number in Table 5.2. The points enclosed by the square in Fig. 5.4 highlights the recommended range of the dimensions of the specimens for an acceptable degree of stress non-uniformity.
Table 5.2 Summary of HCA geometries used by researchers (Sayão and Vaid, 1991).

<table>
<thead>
<tr>
<th>No.</th>
<th>Reference</th>
<th>Institution</th>
<th>Specimen Dimensions (mm)</th>
<th>Soil Type</th>
<th>Control Restrictions</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cooling &amp; Smith 1990</td>
<td>Building Research Station (ENR)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>2</td>
<td>Norton 1968</td>
<td>M.I.T.</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>3</td>
<td>Oden &amp; Kie 1985</td>
<td>S.M. laboratory, Dela</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>4</td>
<td>Nikiforov 1973</td>
<td>University of Wisconsin (UW)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>5</td>
<td>Sayao &amp; Leander 1985</td>
<td>University of Waterloo (UW)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>6</td>
<td>Sayao &amp; Leander 1985</td>
<td>University of Waterloo (UW)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>7</td>
<td>Wa et al. 1990</td>
<td>Michigan State University (MSU)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>8</td>
<td>Brown &amp; Ramesh 1990</td>
<td>Cornell University (CU)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>9</td>
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<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
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<tr>
<td>10</td>
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<td>Clay</td>
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<tr>
<td>11</td>
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<td>40-50</td>
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<tr>
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<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>18</td>
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<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>19</td>
<td>Brown &amp; Ramesh 1990</td>
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<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>20</td>
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<td>Cornell University (CU)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>21</td>
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<td>Cornell University (CU)</td>
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<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
<tr>
<td>22</td>
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<td>Cornell University (CU)</td>
<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
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</tr>
<tr>
<td>23</td>
<td>Brown &amp; Ramesh 1990</td>
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<td>50, 100</td>
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<tr>
<td>24</td>
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<td>40-90</td>
<td>50, 100</td>
<td>40-50</td>
<td>Clay</td>
</tr>
</tbody>
</table>

Figure 5.4 Collection of hollow cylinder sample dimensions by Sayão and Vaid (1991).

The results presented in Fig. 5.4 considering the influence of the end restraint and curvature ($\beta_r$) of the sample on stress non-uniformity from practical and theoretical perspectives. The following desirable geometries of the sample were recommended by the authors:
(a) wall thickness, 20 – 26 mm.
(b) ratio of inner to outer radius: 0.65 – 0.82.
(c) ratio of height to outer diameter: 1.8 – 2.2.

The first and third conditions are satisfied by the HCTA sample set-up, while the second one could be neglected because the ratio is 0.6, close to the lower acceptable limit. A compromise has to be made for the selection of magnitude of wall thickness and outer diameter, since the larger uniform shear strain was found at the low ratio of wall thickness to diameter, which led to an increasing radial stress gradient across the wall when the values of outer and inner pressures are not equal.

5.2.3 Influence of membrane

The specimen tested in hollow cylinder tests are typically enclosed by two rubber membranes, which may influence the boundary conditions of the specimen due to the penetrating effect on the volumetric measurement and the restraining effect on the stress calculation.

5.2.3.1 Membrane penetration

The membrane penetration is the deflection of rubber membrane into the interstices over the side of the cylindrical specimen whenever the cell pressure is elevated to begin the consolidation stage, as indicated in Fig. 5.5. Both the volumetric variation of the sample in drained condition and the build-up of the pore water pressure in undrained condition were found highly influenced by the membrane penetration. Therefore, it is important to minimise or correct such effect to accurately evaluate the soil strength or stress and strain behaviour of the soil.
Chapter 5. Experimental setup and testing procedure

Figure 5.5 Zoom on hollow cylinder wall: membrane penetration induced by internal ($p_i$) and external ($p_o$) pressure (Mandolini 2018).

Frydman et al. (1973) stated that membrane penetrability highly depends on cell pressure and particle size. Clear experimental evidence is provided in Fig. 5.6. Fig. 5.6a shows a strong positive correlation between the unit membrane penetration and the confining pressure in a logarithmic format. Similar linearity in the relationship between the slope of the penetration curves and the mean particle size $D_{50}$ found in Fig. 5.6b.

![Figure 5.6 Membrane penetration linear relationships: (a) effective lateral stress $\sigma_3$ and unit membrane penetration $\Delta V_m$; (b) particle size $d$ and slope $S$ of membrane penetration curves (Frydman et al., 1973).](image-url)
Apart from the confining pressure and grain sizes, the influence of the sample dimensions on membrane penetration was also considered by numerous experimental studies (e.g. Frydman et al., 1973; Baldi and Nova, 1984). The author estimated the membrane penetration volume \( (V_m) \) through the equation proposed by Baldi and Nova (1984):

\[
V_m = \frac{D_{50}V_0}{2D} \left[ \frac{D_{50}}{E_m t_m} \right]^{1/3} \left[ (\sigma_3')^{1/3} - (\sigma_{30}')^{1/3} \right] \tag{5.13}
\]

where the \( V_0 \) and \( D \) are the volume and diameter of the specimen. By considering the value of mean grain size (\( D_{50} \)) equating 0.38 mm, membrane elastic modulus (\( E_m \)) equating 2 MPa, thickness of the outer membrane (\( t_{m,o} \)) equating 0.5 mm, thickness of the outer membrane (\( t_{m,i} \)) equating to 0.3 mm, the initial confining pressure (\( \sigma_3' \)) of 20 kPa and final effective confining pressure (\( \sigma_{30}' \)) of 50 kPa, a total membrane penetration volume (\( V_m \)) for both the inner and outer membrane surfaces of 100.8 mm\(^3\) which represents 0.01% of the initial volume, were obtained. Such volumetric variations can be neglected in this research. Therefore, no correction of the calculation of the volume changes was applied considering the membrane penetration.

5.2.3.2 Membrane resistance

The measured strength of soil in triaxial tests was found increased due to the membrane resistance (Henkel and Gilbert, 1952). Numerous studies (Raghunandan et al., 2015; Greeuw et al. 2001; LaRochelle et al. 1988; Frydman et al. 1973; Bishop and Henkel 1962; Henkel and Gilbert, 1952) on the estimation of the membrane resistance indicated the significance of membrane stiffness \( E_m \), which relates to the dimensions of the membrane such as the diameter, \( d_m \) and thickness \( t_m \). Following the assumption of perfect cylinder shape of external and internal membranes with a Poisson’s ratio of 0.5 for rubber, suitable corrections of stress components based on elasticity theory for HCTA tests have been well documented by Tatsuoka et al. (1986), as provided in the following:

\[
\Delta \sigma_z = - \frac{4}{3} \frac{E_m t_m}{r_o^2 - r_i^2} \left\{ r_o \left[ 2(\varepsilon_{zm})_o + 2(\varepsilon_{\theta m})_o \right] + r_i \left[ 2(\varepsilon_{zm})_i + 2(\varepsilon_{\theta m})_i \right] \right\} \tag{5.14}
\]

\[
\Delta \sigma_\theta = - \frac{2}{3} \frac{E_m t_m}{r_o - r_i} \left[ \left( \varepsilon_{zm} \right)_o + 2(\varepsilon_{\theta m})_o \right] + \left[ \left( \varepsilon_{zm} \right)_i + 2(\varepsilon_{\theta m})_i \right] \tag{5.15}
\]
\[
\Delta \sigma_r = -\frac{2}{3} \frac{E_m t_m}{r_0 + r_i} \left\{ \left[ (\varepsilon_{z_m})_o + 2(\varepsilon_{\theta_m})_o \right] - \left[ (\varepsilon_{z_m})_i + 2(\varepsilon_{\theta_m})_i \right] \right\} 
\] (5.16)

\[
\Delta \tau_{\theta z} = -2E_m t_m \frac{r_o^3 + r_i^3}{(r_o^3 - r_i^3)(r_o - r_i)} \gamma_{\theta z}
\] (5.17)

where \((\varepsilon_{z_m})_o\) and \((\varepsilon_{z_m})_i\) denote the average axial strains for the external and internal membranes respectively, while \((\varepsilon_{\theta_m})_o\) and \((\varepsilon_{\theta_m})_i\) are the average circumferential strains for the external and internal membranes respectively. These strain components can be determined by:

\[
\varepsilon_{z_m} = \varepsilon_{z_m}^* + \varepsilon_{z_c} + \varepsilon_z 
\] (5.18)

\[
\varepsilon_{\theta_m} = \varepsilon_{\theta_m}^* + \varepsilon_{r_c} + \varepsilon_{\theta}
\] (5.19)

where \(\varepsilon_{z_m}^*\) and \(\varepsilon_{\theta_m}^*\) are the initial membrane strains, \(\varepsilon_{z_c}\) and \(\varepsilon_{r_c}\) are the axial and radial strains under compression and \(\varepsilon_z\) and \(\varepsilon_{\theta}\) represent the average axial and circumferential normal strains of the sample. The value of initial axial strain equal to 0 and the value of circumferential strains equal to -0.02 for both inner and outer membranes were adopted in this research. Therefore, based on the Equations 5.14 - 5.19 suggested by Tatsuoka et al. (1986), corrections accounting for the membrane’s resistances were applied to the calculations of the stress components summarised in Table 5.1.

5.3 Description of HCTA facilities

5.3.1 General

The HCTA device at University of Bristol is capable of performing controlled stress path testing under multiaxial monotonic and cyclic loading conditions. As schematically illustrated in Fig. 5.7, the HCTA consists of:

(a) a confining cell with an external diameter of 597 mm, an internal diameter of 521 mm and a height of 960 mm, which is capable of sustaining cell pressure up to 1 MPa;

(b) data acquisition and control system;

(c) loading system which allows the independent control of axial loading, torsional loading, inner cell pressure, outer cell pressure and pore pressure;
(d) measurement system including four stress components transducers, overall axial displacement and rotation measurements setup, and two volume change measurement setups. In addition, a small strain measurement system was employed to determine the quasi-elastic stiffness.

The photo of overall view of the apparatus is given in Fig. 5.8 (Ibraim et al., 2011).

![Diagram](image1)

**Figure 5.7** HCTA schematic representation (Yoon 2005).

![Photo](image2)

**Figure 5.8** Photo of the HCTA at University of Bristol (Ibraim et al., 2011).
5.3.2 Loading system

Two independent hydraulic loading systems for axial force and torque loading are available for the operation of the HCTA. The hydraulic actuators for both loading systems driven by hydraulic flow system were placed on the top of the cell (Fig. 5.9a). The hydraulic system was particularly designed to provide stable and accurate cyclic or dynamic loading with a frequency of up to 20 Hz. In addition, compared to other driving system (e.g. pneumatic loading system), the application of the hydraulic flow can effectively minimise the backlash-free when loading direction changing, which significantly affects the small strain of soil. The external and internal cell pressures, and the pore water pressure acting on the sample are supplied by a compressor unit and applied through three pneumatic air-water interface devices (Fig. 5.9b), which can be controlled manually from the main panel fitted with three corresponding accurate pressure regulators (Fig. 5.9c).

Figure 5.9 Loading system of HCTA: (a) axial force/torque load cell; (b) air water interface devices; (c) manual pressure regulator.

5.3.3 Overall measurement system

The measurement system including stress and strain measurements has been recently calibrated for the accuracy of the measurements. The detailed procedures for assessing the resolution of sensors, long-term stability, and calibration characteristics are provided by Yoon (2005).

Measurement of stresses

A submersible load cell placed inside of the confining cell, thus eliminating the effects of piston friction, is used to measure the axial and torsional loading (Fig.
5.10a). This load cell is capable of providing measurements for the axial load of up to 8 kN and the torsional load of up to 400 Nm. Electronic pressure transducers (Model PDCR 4010), which connect the pressure sensor blocks to the data acquisition system (Fig. 5.10b), were used to measure the internal and external cell pressures, and pore water pressures. The measurement of pressure can also be performed from the pressure gauges with an accuracy of 2.5 kPa and a range from 0 to 1000 kPa, as shown in Fig. 5.10c.

![Stress measurement](image)

Figure 5.10 Stress measurement: (a) submersible load cell; (b) electronic pressure transducer; (c) gauges for inner, cell and pore pressures.

**Measurement of volume/displacement**

Axial displacement is measured by linear variable displacement transducers (LVDT, Model: ATC1000, see Fig. 5.9a) and rotation was measured by a rotary capacitive displacement transducer (RCDT, Model: RCDT 300, see Fig. 5.11a). The two transducers are located outside the cell on its top surface. Two identical volume change devices to measure the volumetric change of the overall specimen and the inner cell variations were used, respectively (Fig. 5.11b). LVDTs connected with the floating piston of the volume change devices were employed for the monitoring of the volume variations. Combined with the height change of the specimen given by vertical LVDTs shown in Fig. 5.9a, the internal and external radial specimen displacements can be calculated by Equations 5.20 and 5.21, respectively:
\[ \Delta r_i = \sqrt{\frac{H r_i^2}{H - \Delta H} - \frac{\Delta V_i}{\pi (H - \Delta H)}} - r_i \]  
\[ \Delta r_o = \sqrt{\frac{H r_o^2}{H - \Delta H} + \frac{\Delta V_i + \Delta V_s}{\pi (H - \Delta H)}} - r_i \]

where the \( \Delta V_i \) are the volume change of the inner chamber while \( \Delta V_s \) indicates the sample volume variations.

### 5.3.4 Data acquisition and control system

The Instron multi-axis FastTrack 8800 Controller, integrated with GPIB and PCI cards and a 16 channels analogue/digital data acquisition system with a 19 bits resolution were used for the control of hydraulic system. Accompanied by the LabVIEW programme, the measurements collected from HCTA transducers can be recorded and displayed on the screen (Fig. 5.12). In addition, the LabVIEW programme can be developed for the specific interface tracking specific soil mechanics stress and strain variables.
5.3.5 Small strain measurement system

The small strain measurement consists of three pairs of non-contact displacement με transducers with a measurement range of 2 mm and high resolution better than 0.1 μm (Fig. 5.13a). Such a high-resolution measurement can be achieved due to the sensitive eddy current, which is generated by sensors and interacts with conductive targets (e.g. aluminium plate) according to the distance between them. Fig. 5.13b schematically illustrates the arrangement of the non-contact transducers and corresponding aluminium plate targets. All the sensors were fixed on the stainless-steel bars. The axial and circumferential deformations in the central part of the sample were given by sensors S1-S2 and S3-S4, respectively. The corresponding rectangular aluminium plate targets were attached to two 3D-printed rings at different elevations of the sample, as shown in Fig. 5.13b. Such rings were stuck to different levels along the wall of the sample using three 3D-printed flexible thin stripes glued to the outer membrane. The outer radius sensors S5 and S6 for the measurements of radial sample displacements, aimed towards aluminium foil targets attached to the internal side of the outer membrane (Fig. 5.13b). The inner radius can be measured directly by a single LVDT positioned inside of the inner cell of the sample, as shown in Fig. 5.14 (Ibraim et al., 2011). However, for this research, it is calculated by Equation 5.20.
Figure 5.13 Small strain measurement system: (a) installation of the system; (b) schematic representation of the system mounted around the specimen (after Mandolini, 2018).

Figure 5.14 Measurement of radial displacement by LVDT (Ibraim et al. (2011)).

Thanks to the small strain measurement system, local strains of the central part of the specimen were expected to be evaluated. Fig. 5.15 shows the deformations at axial ($z$), radial ($r$) and circumferential ($\theta$) directions of the central part of the sample with the height of $H_c$ equalling 100 mm. The central strain components (axial ($\varepsilon_{z,c}$), radial ($\varepsilon_{r,c}$), circumferential ($\varepsilon_{\theta,c}$) and shear ($\gamma_{\theta z,c}$)) can be calculated by the equations summarised in Table 5.3.
Figure 5.15 Deflection of the central part of the sample, in the circumferential (a and b), axial (c) and radial directions (d) (Mandolini, 2018).

Table 5.3 Equations of strain components obtained from small strain measurement system (Mandolini, 2018).

<table>
<thead>
<tr>
<th>Axis direction</th>
<th>Small strain equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial normal</td>
<td>( \varepsilon_{z,c} = -\frac{d_z}{H_c} ), where ( d_z = d_{S2} - d_{S1} ), and ( d_{S1} ) and ( d_{S2} ) are the measurements of sensors S1 and S2, respectively.</td>
</tr>
<tr>
<td>Radial normal</td>
<td>( \varepsilon_{r,c} = -\frac{\Delta r_{o,c} - \Delta r_{l,c}}{r_{o} - r_{l}} ), where ( \Delta r_{o,c} = \frac{d_{S5} + d_{S6}}{2} ), and ( d_{S5} ) and ( d_{S6} ) are the measurements of sensors S5 and S6, respectively.</td>
</tr>
<tr>
<td>Circumferential normal</td>
<td>( \varepsilon_{\theta,c} = -\frac{\Delta r_{o,c} + \Delta r_{l,c}}{r_{o} + r_{l}} )</td>
</tr>
<tr>
<td>Circumferential shear</td>
<td>( \gamma_{\theta z,c} = \frac{\theta_c r_{ave,c}}{H_c} ), where ( \theta_c = \frac{d_{S3} - d_{S4}}{r_{o} + dS} ), and ( d_{S3} ) and ( d_{S4} ) are the measurements of sensors S3 and S5, respectively; ((r_{o} + dS)) denotes the distance between the centre of the specimen to the aluminium target point (Fig. 5.15a).</td>
</tr>
</tbody>
</table>
5.4 Tested material

The experimental tests have been conducted at the University of Bristol on Hostun RF (S28) sand (Fig. 5.16), widely characterised in previous studies (e.g. Escribano et al., 2018; Mandolini et al., 2021). This sand is a standard European material for laboratory testing with a high siliceous content ($SiO_2$) of 98%, angular to sub-angular grains (Ibraim et al., 2012), mean grain size, $D_{50} = 0.38$ mm, coefficient of uniformity, $C_u = D_{60}/D_{10} = 1.9$ and coefficient of gradation $C_g = (D_{30})^2/(D_{10}D_{60}) = 0.97$. The maximum and minimum void ratios are $\epsilon_{\text{max}} = 1.00$, $\epsilon_{\text{min}} = 0.62$, respectively, while the specific gravity is $G_s = 2.65$.

\[\text{Figure 5.16 Particle size distribution for Hostun RF sand.}\]

\textit{Disclaimer: Please note that Hostun RF sand is slightly coarser than Karlsruhe sand, but they have similar grain shape, $\epsilon_{\text{min}}$ and $\epsilon_{\text{max}}$, shape of particle size distribution and absence of fines.}

5.5 Sample preparation

Given the degree of stress and strain non-uniformities, a hollow cylindrical sample with an outside radius ($r_o$) of 50 mm, inner radius ($r_i$) of 30 mm and height ($H$) of 200 mm was expected to be tested in the HCTA. The sample fabrication procedures are listed as follows.
5.5.1 Moulds assembly

The samples for the tests were fabricated outside the HCTA cell. Therefore, a small platform fixed on the top of a vibrator was used to place the base pedestal of the specimen (Fig. 5.17a). The base pedestal consists of an annular porous stone, an annular stainless-steel cap with an all-around O-ring attached inside and a stainless-steel plate at the bottom.

The inner membrane with a diameter of 60 mm was firstly secured in the base pedestal using a clamping ring (Fig. 5.17b). The inner mould is then fitted on the base pedestal and six pieces of filter paper are placed on the surface of the porous stone to avoid blocking by sand (Fig. 5.17c). Please note that all the membranes used in this research were lubricated by covering the surface with talcum powder to avoid adherence to the material or moulds.

The outer membrane having a diameter of 100 mm, was stretched over the base cap and sealed with two O-rings, followed by the mounting of the three-parts outer moulds (Fig. 5.17d). It should be noticed in Fig. 5.17 that the outer membrane was marked with grid lines for better visualisation of the deformation of the sample. A suction of 30 kPa was applied through the pipes connected to the outer moulds to ensure the external membrane was tightly attached to the outer mould walls. Two small foil targets of size 80 mm × 50 mm were attached to the internal side of the outer membrane, which guaranteed the conjunction with the proximity radial non-contact transducers (i.e. S5 and S6 in Fig. 5.13b).
5.5.2 Deposition of the soil

The initial fabric of the specimen, which highly relies on sampling techniques, is essential for determining the soil behaviour. The following three methods according to the depositional history of the soil in the field are typically used:
(a) Air pluviation (Fig. 5.18a). The specimen is fabricated by the free falling of sand through air, in which different densities of the specimen even including the minimum void ratio can be achieved by changing the fall height and rate of the sand grain. The methods such as tapping and vibrating post-pluviation are usually used for further densification of the sample, however, at the expense of causing the re-orientation of sand particles which is undesired for a replica of the field state of soil.

(b) Water pluviation (Fig. 5.18b). Different from air pluviation, the sand deposits are formed through water sedimentation in this method. It appears to be the best way for the simulation of the fabrics of sand deposited under water, especially for testing sands deposited in offshore environments. However, tapping or adjusting the drop height also is useful to obtain the different densities for this method. Compared to other methods, the largest initial saturation degree for sample can be generated with water pluviation.

(c) Moist tamping (Fig. 5.18c). This method is suitable for replicating the field compacted embankments, in which moist sand is compacted to the target density layer by layer. The overall density of the sample can be relatively easily controlled with the moist tamping method, even for the extreme loose specimen, whereas the inhomogeneous deposit is generated.

![Sample preparation methods](https://via.placeholder.com/150)

**Figure 5.18 Sample preparation methods: (a) air pluviation; (b) water pluviation; (c) moist tamping (after Wichtmann, 2021).**

The air pluviation method combined with high frequency and low amplitude vibrations was chosen for this study to ensure a uniform distribution of the sand specimen density. The dry sand was poured into the moulds with a modified funnel, in which an additional pipe was added to its spout to better control the pluviation height (Fig. 5.17e). The funnel was continuously moved around the samples’ thin
wall and simultaneously lifted up gently to ensure uniformity and keep a consistent falling height of the sand and minimise the denser layers at the bottom of sample. Different soil densities were achieved by vibration produced by a shaker. For this research, the vibration with acceleration of 2 g and a frequency of about 50 Hz was adopted. After flattening the top surface of the sample (Fig. 5.17f), the annular top cap alongside two 3D printed rings was placed on the top of the specimen (Fig. 5.17g) and the inner and outer membranes were sealed using this top cap and O-rings.

### 5.5.3 Mould disassembly

The previously applied vacuum of 30 kPa on outer moulds was then transferred to the sample to hold the specimen before removing the moulds (Fig. 5.17h). Once the suction remained stable, the moulds were detached. At this stage, it was important to identify the leaks both in inner and outer membranes by checking if the value of suction was stable. Liquid latex was available for repairing the membrane punctures, if necessary. The dimension of the specimen was then measured. Afterwards, the sample was carefully transferred inside the cell and fixed on the base of the cell. The inner cell was isolated with a stainless top cover by securing it on the top cap using clamping screws. After this step, the specimen was ready for the connection to the loading ram. Four hexagon al pots filled up with a strong resin called Araldite 2014-1 were fixed to the top cover. Correspondingly, four bolts attached to the bottom of the loading cell were progressively submerged in the resin as the loading cell slowly moved down (Fig. 5.17i). Such junction would be able to transmit load once the resin became hardened.

### 5.5.4 Small-strain measurement set up

As mentioned in the previous section, an additional small-strain measurement system will be used in this experimental investigation for the measurement of small strain elastic stiffness. All the non-contact sensors were secured on the stainless steel rods. Two 3D printed rings with the rectangular aluminium plate targets for the axial and circumferential transducers (S1-S2 and S3-S4 in Fig. 5.13b), were deployed outside of the outer membrane using three flexible suspended strips (Fig. 5.17k). The 3-D printed scaffoldings in Fig. 5-17j were placed to avoid the
inclination of the rings during installation and ensure an even distance of about 100 mm between the rings. It should be remembered that the foil targets for sensors S5-S6 have been stuck to the inside of the membrane before the deposition of sand. Once the targets were in position, the scaffoldings were removed. The HCTA sample was ready for the test (Fig. 5.17) after taking the measurements of the height between the two 3D printed rings, the distance between the centre of the sample and the centre of the S3 and S4 target. Thereafter, the cell was filled up with water.

5.6 Testing procedure

The tests were performed in drained conditions. All drained tests require fully water-saturated specimens. The saturation degree is essential for the evaluation of the sample volume variations.

1. **Saturation.** The suction acting on the sample to avoid the collapse of the sample, was progressively replaced by a confining cell pressure. However, the effective stress was maintained constant during this procedure. The saturation was followed. The sample was first flushed with carbon dioxide (CO₂) from the bottom to the top of the sample and then de-aired water was flushed through the sample. Back pressure up to 350 kPa with an effective stress 20 kPa was applied to achieve Skempton’s value $B$ of at least 0.95. It was assumed that the procedure of saturation wouldn’t cause the change of deformation of the specimen.

2. **Consolidation.** The effective isotropic stress was increased to 50 kPa and remained constant for at least half a day until no volumetric strains were detected.

3. **Shearing.** The main targets of the testing programme are to evaluate the stiffness evolution and accumulated strain under long-term multiaxial stress space. All the tests followed a consistent shearing procedure, as described in Fig. 5.19. After all the specimens were isotropically consolidated under the confining stress of 50 kPa and then, the sample was brought to the desired stress level by applying an additional deviatoric axial stress of 50 kPa with the constant inner and outer cell pressures (black lines in Fig. 5.19). After creep deformations (grey lines in Fig. 5.19) were allowed for several
hours, the specimens were imposed a series of one-way combined axial and torsional packages of cycles of up to around 30000 cycles with a loading frequency of 0.1 Hz (red lines in Fig. 5.19). At the end of each test, the specimen was brought to failure in the direction of the applied cyclic load. Ten consecutive axial cycles followed by ten consecutive torsional cycles with amplitudes of 5 kPa were applied at the beginning and at the end of each package of cycles for the evaluation of the Young’s and Shear moduli ($E_z$ and $G_{θz}$), respectively (blue lines in Fig. 5.19). A slower rate of 0.01 Hz for loading cycles was adopted to ensure a stable load control and sufficient recorded data of the stress-strain relation for an accurate measurement. Details of the testing programme will be presented in the following chapter.

Figure 5.19 Schematic of stress path in term with deviatoric stress ($q$) and time.

5.7 Conclusions

As highlighted in the previous chapters, the stress paths determined from the FE will be simulated through the HCTA, which possesses four degrees of freedom.

- This chapter explicitly described the HCTA including its general principals, loading system, measurement system and acquisition and control system, which enables the testing equipment to test granular soils under multiaxial monotonic and cyclic loadings.
- Complex small strain measurement system consisting of six high-resolution non-contact sensors was introduced for the accurate determination of the sand small strain stiffness.
• General testing procedure for HCTA was highlighted. In particular, different sample fabrication methods were listed and the air pluviation method appears to be a preferred method for this study.

• The primary aims of the experimental programme were to evaluate the quasi-elastic response of soil (e.g. strain accumulation and stiffness evolution) under long-term cyclic loading. The procedure to obtain these parameters was highlighted in this chapter.
Chapter 6  Evolution of stiffness and accumulated strain under multiaxial cyclic loading

6.1 Introduction

The recent growth of the offshore wind energy sector is driving the green industrial revolution to fight climate change through decarbonation and the global net zero emission target. In this process, it is vital that the offshore wind industry retains its competitiveness through cost reduction and increased productivity. Design improvement and optimisation is one of the avenues to increase cost-effectiveness and efficiency.

The support structure of an offshore wind farm typically accounts for 16% to 35% of the total development costs (Bhattacharya et al., 2021), with the monopile foundation having dominated the scene and being used in about 80% - 90% of the current offshore wind developments (Wind Europe, 2022). The design of monopiles
The chapter discusses the evolution of stiffness and accumulated strain under multiaxial cyclic loading. It has seen consistent improvement and optimisation in the last few years, with the progressive recognition of the importance of FE modelling and the development of new reaction curves (Burd et al., 2020; Jeanjean et al., 2017; Andersen, 2015). Among all the design criteria, cyclic rotation accumulation to comply with strict tilting tolerances and the evolution of the soil stiffness affecting the natural frequency of the wind turbines are two factors typically governing the foundation design.

Both in-situ testing (e.g. CPT, P-S wave logging) and laboratory investigations are crucial ingredients in the cyclic design process of monopiles. Laboratory tests are employed to inform and calibrate constitutive models used in FE analyses (Kementzetzidis et al., 2019; Burd et al., 2020; Staubach and Wichtmann, 2020) but also to generate soil reaction curves, using the procedure proposed by Andersen (2015) or through the similarity principles (Randolph, 2012; Lombardi et al., 2017). However, while most of these design procedures are based on simplified loading conditions involving cyclic triaxial and/or simple shear tests, more generalised multiaxial stress conditions are expected to occur in soil elements around laterally loaded pile (Arthur et al., 1980; Andersen et al. 2013). The literature has also provided that the application of more generalised stress conditions involving multi-directional loading or rotation of the principal stress axis can have a substantial effect on the cyclic soil response (Wichtmann et al., 2007b; Mandolini et al., 2021; Cai et al., 2015; Tong et al., 2010).

Therefore, the aim of this chapter is to investigate the mechanical soil response under cyclic stress paths truly representative of those experienced by soil elements around laterally loaded piles. The stress paths applied in the laboratory testing programme have been informed by the FE analysis presented in Chapter 4 (cyclic stress paths), briefly summarised in the next section. In particular, the effect of cyclic loading direction and rotation of the principal stress axis (a main feature of the FE identified stress paths) on both the cyclic strain accumulation and evolution of soil stiffness will be investigated at the element level.
6.2 FE informed stress paths and experimental programme

Chapter 4 (Cheng et al., 2021, 2023) investigated the cyclic stress paths experienced by soil elements in the vicinity of a laterally loaded monopile foundation based on advanced 3D finite element modelling. The FE model described in Chapters 3 and 4, considering a hollow steel monopile with $D = 8$ m and $L = 27$ m, was embedded in a medium-dense sand (Fig. 6.1a).

![Figure 6.1 FE results: (a) Finite element model and investigated soil elements, (b) stress paths of the investigated soil elements.](image)

Figure 6.1 FE results: (a) Finite element model and investigated soil elements, (b) stress paths of the investigated soil elements.

For the soil elements in front of the laterally loaded pile, it was recognised that, due to symmetry, only four stress components (three normal stresses, $\sigma_x$, $\sigma_y$, $\sigma_z$ and one shear stress $\tau_{xz}$) change during cyclic loading. Neglecting the effect of the intermediate principal stress axis ($\sigma_y$, which is off-plane with respect to the lateral load), the stress paths experienced by the four analysed elements are reported in the normalised plane $\tau_{xz}/p'$ versus $(\sigma_x-\sigma_z)/2p'$ stress plane in Fig. 6.1b (where $p'$ is the mean effective stress). The analysis of the stress paths reveals that:

- All the inspected elements are subjected to simultaneous variation of both normal and shear stresses, inducing re-orientation of the principal stress axis.
- The cyclic stress paths evolve towards a stable configuration, characterised by a linear one-way stress path in the $\tau_{xz}/p'$ versus $(\sigma_x-\sigma_z)/2p'$ plane.
Chapter 6. Evolution of stiffness and accumulated strain under multiaxial cyclic loading

- The inclination of the stress path in the $\tau_{xz}/p'$ versus $(\sigma_x-\sigma_z)/2p'$ stress plane depends on the location of the soil element around the monopile.
- The initial stress conditions are anisotropic.
- The extent of the stress paths is soil element position dependent but also affected by the amplitude of the monopile cyclic loading.

Therefore, using the HCTA, a systematic investigation to explore the effect of the re-orientation of the principal stress axes (simultaneous application of normal and shear stresses) and cyclic loading on the behaviour of granular soils has been carried out.

Following the stress path extracted from the last loading cycle (Fig. 6.1b), eight tests with different stress amplitude and re-orientation of the principal stress axes have been designed, as listed in Table 6.1 and shown in Fig. 6.2. After being isotropically consolidated to an effective confining stress of 50 kPa (Point 0 in Fig. 6.3), all the samples were brought to an anisotropic stress state by increasing the normal axial stress by 50 kPa (Point B in Fig. 6.2). Each of the samples was then subjected to loading-unloading cycles with a loading frequency of 0.1 Hz to study the influence of cyclic amplitude and rotation of principal stresses. The inner and outer cell pressures were maintained constant during shearing, both at an effective stress value of 50 kPa. However, these cycles have been split into several packages (namely package cycles, see Fig. 5.19 in Chapter 5) by the investigation points to allow for the measurement of small strain stiffness, as detailed in the following.
Chapter 6. Evolution of stiffness and accumulated strain under multiaxial cyclic loading

Figure 6.2 Schematic of the testing programme.

Table 6.1 List and details of tests performed in this experimental investigation.

<table>
<thead>
<tr>
<th>Test name</th>
<th>Stress path</th>
<th>$e_0$</th>
<th>$D_r$ (%)</th>
<th>$\Delta\sigma_z'$ (kPa)</th>
<th>$\Delta\tau_{\theta z}$ (kPa)</th>
<th>$\alpha_\sigma$ (°)</th>
<th>No. Cycles ($N$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{0A7}$</td>
<td>0-B-C1</td>
<td>0.835</td>
<td>43</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>28032</td>
</tr>
<tr>
<td>$\alpha_{0A21}$</td>
<td>0-B-C2</td>
<td>0.837</td>
<td>42.9</td>
<td>21</td>
<td>0</td>
<td>0</td>
<td>33004</td>
</tr>
<tr>
<td>$\alpha_{0A35}$</td>
<td>0-B-C3</td>
<td>0.797</td>
<td>53.4</td>
<td>35</td>
<td>0</td>
<td>0</td>
<td>30565</td>
</tr>
<tr>
<td>$\alpha_{15A7}$</td>
<td>0-B-C4</td>
<td>0.817</td>
<td>48.2</td>
<td>7</td>
<td>16.45</td>
<td>0-15</td>
<td>31485</td>
</tr>
<tr>
<td>$\alpha_{15A21}$</td>
<td>0-B-C5</td>
<td>0.829</td>
<td>45</td>
<td>21</td>
<td>20.50</td>
<td>0-15</td>
<td>30217</td>
</tr>
<tr>
<td>$\alpha_{15A35}$</td>
<td>0-B-C6</td>
<td>0.831</td>
<td>44.5</td>
<td>35</td>
<td>24.54</td>
<td>0-15</td>
<td>17563</td>
</tr>
<tr>
<td>$\alpha_{22.5A7}$</td>
<td>0-B-C7</td>
<td>0.840</td>
<td>42.1</td>
<td>7</td>
<td>28.5</td>
<td>0-22.5</td>
<td>27850</td>
</tr>
<tr>
<td>$\alpha_{22.5A21}$</td>
<td>0-B-C8</td>
<td>0.830</td>
<td>44.7</td>
<td>21</td>
<td>35.5</td>
<td>0-22.5</td>
<td>9958</td>
</tr>
</tbody>
</table>

153
In the name of the tests (Table 6.1), the number attached to ‘α’ represents the limit of the applied angle of the rotation of the principal stress axis, α, while the number after ‘A’ gives the limit value of the applied effective axial (A) stress, σ′\(_z\). For example, the tests α0A7, α0A21 and α0A35 considered only the application of effective axial stress cycles with amplitudes Δσ′\(_z\) = 7, 21 and 35 kPa respectively, without any application of torsional stress and thus in absence of re-orientation of the principal stress axes. In short, the three tests are triaxial tests from an initial anisotropic stress state. Tests α15A7, α15A21 and α15A35, were subjected to the same Δσ′\(_z\) with the combined application of torsional cycles of amplitude Δτ\(_{θz}\) to produce cyclic re-orientation of the principal stress axes Δα\(_σ\) between 0° and 15°.

The final two tests α22.5A7 and α22.5A21 imposed Δσ′\(_z\) = 7 and 21 kPa respectively, combined with the application of torsional cycles of amplitude Δτ\(_{θz}\) to produce cyclic re-orientation of the principal stress axes Δα\(_σ\) between 0° and 22.5°. A maximum number (N) of about 30000 cycles were applied for all tests, with exception of tests α15A35 and α22.55A21, which were prematurely terminated after 17563 and 9958 cycles, respectively, due to unforeseen technical problems of the hydraulic loading system.

Measurements of the small strain Young’s \(E_z\) and shear \(G_{θz}\) moduli, defined by the inclinations of the stress and strain loops, were monitored by the local measurement system consisting of six high-resolution non-contact displacement transducers described in Chapter 5. The first measurement was performed after the consolidation stage (Point 0 in Fig. 6.2), followed by the second measurement once the anisotropic stress state was reached (Point B in Fig. 6.2). As mentioned before, the small strain moduli were also evaluated at several investigation points during the cyclic stage (i.e. after 100, 500, 1000, and every 5000 cycles). The small strain moduli probing was always performed at the same anisotropic stress state (Point B in Fig. 6.2) by applying ten stress-controlled consecutive axial cycles with stress amplitude Δσ′\(_z\) = 5 kPa for the evaluation of Young’s modulus \(E_z\) and ten consecutive torsional cycles with stress amplitude Δτ\(_{θz}\) = 5 kPa for the evaluation of shear modulus \(G_{θz}\) (namely **measurement cycles**, see Fig. 5.19 in Chapter 5). These measurement cycles with a load frequency of 0·01 Hz were recorded at an acquisition rate of 30 measurement points per second.
After the application of the cyclic loading, all the samples have then monotonically sheared to failure, along the same direction of the applied cyclic loading. The Matsuoka–Nakai failure envelope of $\varphi' = 38^\circ$, whose applicability has been validated by Mandolini et al. (2019b) through the HCTA tests on Hostun sand, is also provided here as a reference (the details will be discussed in Section 6.3.3).

Chapters 3 and 4 mainly considered the numerical simulations of a medium-dense sand. In order to be consistent with the FE soil condition, a relative soil density $D_r = 50\%$ corresponding to a void ratio of 0.81 was considered for all the experimental tests in this chapter, although the actual void ratios with a range from 0.797 ($D_r = 53.4\%$) to 0.840 ($D_r = 42\%$) were detected after the samples were isotropically consolidated, as presented in Table 6.1.

The calculation of the average stresses (axial $\sigma_z$, radial $\sigma_r$, circumferential $\sigma_\theta$ and shear $\tau_{\theta z}$) and strains (axial, $\varepsilon_z$, radial, $\varepsilon_r$, circumferential $\varepsilon_\theta$, and shear $\gamma_{\theta z}$) have been provided in Chapter 5. Herein, following Wichtmann et al. (2005), the accumulated deformations $\varepsilon_{\text{acc}} (N)$ were recorded at the end of each cycle, without accounting for the very first irregular cycle, as illustrated in Fig. 6.3. The strain amplitude, $\varepsilon_{\text{amp}}$, is defined in Fig 6.3, which is also determined from the second cycle. The strain amplitude (Equation 6.1) and accumulated strain (Equation 6.2) are defined as:

$$
\varepsilon_{\text{amp}} = \sqrt{(\varepsilon_{z\text{amp}})^2 + (\varepsilon_{r\text{amp}})^2 + (\varepsilon_{\theta\text{amp}})^2 + (\gamma_{\theta z\text{amp}})^2} \quad (6.1)
$$

$$
\varepsilon_{\text{acc}} = \sqrt{(\varepsilon_{z\text{acc}})^2 + (\varepsilon_{r\text{acc}})^2 + (\varepsilon_{\theta\text{acc}})^2 + (\gamma_{\theta z\text{acc}})^2} \quad (6.2)
$$
Figure 6.3 Definition of accumulated strain and strain amplitude (Witchmann et al., 2005).

6.3 Overall mechanical response

6.3.1 Stress-strain response

Fig. 6.4 presents the deviatoric stress \( q \) versus deviatoric strain \( \varepsilon_q \) response for all the tests. Note these strains were measured by the overall external displacement and rotational transducers. The definitions of deviatoric stress and deviatoric strains are provided by the previous Equations 5.3 and 5.8 in Chapter 5. Point 0 represents the initial isotropic stress conditions, while Point B is the end of the anisotropic axial loading. Cyclic loading (amplitudes \( \Delta \sigma_z \) and \( \Delta \tau_{\theta z} \) as indicated in Table 6.1) is applied between Point B and Point C (C1-C8 correspond to eight tests respectively), after which the samples were brought to failure. The post peak behaviour of the samples is invisible due to the stress-controlled manner. The mobilised peak strength is reached at a strain of about 8% with exception of tests \( \alpha 15A7 \) (Fig. 6.4d) and \( \alpha 22.5A7 \) (Fig. 6.4g), in which shearing occurred at a smaller deviatoric strain level. The mobilised peak strength for tests \( \alpha 0A7 \), \( \alpha 0A21 \) and \( \alpha 0A35 \), which were loaded to failure along axial direction, is 180.4 kPa, 191.3 kPa and 184 kPa, respectively. It seems that the sample subjected to cyclic loading history with different amplitudes tends to exhibit a similar mobilised deviatoric stress. It is evident that the mobilised deviatoric strength decreases with the increase of the angle of principal stress direction. The results are consistent with those reported by Symes at al. (1988). During the B-C cyclic stage, the first stress-strain loop is featured with the largest strain amplitude compared with other stress-
strain loops, as shown in the close-up view of the cycles in Fig. 6.4. This feature was also observed by Niemunis et al. (2005). As the number of cycles increases, the stress-strain loops become smaller due to progressive loss stiffening and decreases in the accumulation of plastic deformation.

![Stress-strain responses for all the tests.](image)

Figure 6.4 Stress-strain responses for all the tests.

The axial and torsional stress–strain responses for all the tests are presented in Fig. 6.5. The tests α0A7, α0A21 and α0A35 were subjected to axial loading, therefore only the axial stress and strain trend was shown in Figs. 6.5a-c. For the rest of the tests, both axial and torsional stress-strain responses are visible due to
the application of combined axial and torsional stresses (Figs. 6.5d-h). Similar to that observed in the deviatoric stress-strain response, an increase of the inclination of the principal stress corresponds to a remarkable sample strength reduction for mobilised axial stress, while the changes in mobilised shear stress seem to be limited. Overall, the mobilised axial stress is larger than mobilised shear stress.

![Graphs showing axial and torsional stress-strain responses](image)

Figure 6.5 Axial and torsional stress-strain responses for all the tests.

It is worth noting that the comparison of the results for tests with the same axial cyclic stress amplitude but different rotation of principal stress axes (e.g. Figs. 6.5a, d, g; Figs. 6.5b, e, h; Figs. 6.5c, f) shows that the samples subjected to the larger additional torsional cycles tend to accumulate larger axial strain during the B-C cyclic stage. A plausible explanation of such occurrence is that torsional cyclic
loading can also generate axial strain. This conjecture can be confirmed through the multiaxial shear tests on a modified direct simple shear device (DSS) by Wichtmann et al. (2007b), in which the axial accumulated strain was recorded under pure shear loading. The conjugate axial strain in this research caused by the coupling effect of axial and torsional stresses is schematically shown in Fig. 6.6. Following the hypothesis proposed by Rechenmacher et al. (2010), Fig. 6.6 depicts two types of force chains under axial loading and torsional loading, in which the particles are numbered for a better illustration of the hypothesised kinematic activity at micro scale. Initially the force chain is aligned along the direction of major compressive principal stress (vertical direction in Fig. 6.6a). The torsional loading leads to the shearing and buckling of the force chains (Fig. 6.6b), which further causes the pronounced axial compression, as schematically illustrated by the movements at vertical direction of particles 2, 3, 9 and 10 from Fig. 6.6a to 6.6b. In this respect, the effect of the additional application of the torsional loading on axial strain in this research should be considered. Detailed discussion on the cyclic behaviour will be presented in the following.

Figure 6.6 Hypothesised force chain behaviour under: (a) axial loading and (b) torsional loading (after Rechenmacher et al., 2010).

6.3.2 Volumetric response

Fig. 6.7 presents the volumetric response in terms of deviatoric strain against volumetric strain for all the tests. In general, all the samples typically compress in
the initial loading stages then show a dilative response in the stage of loading to failure. The cyclic loading for most of the tests induces a compressive response except for tests α0A35, α15A7 and α22.5A7. This may be related to the different directions of cyclic loading with respect to the previous pre-cyclic monotonic consolidation. It is apparent that the volume response in the B-C cyclic stage becomes progressively more compressive with the increase of axial cyclic stress amplitude (e.g. Figs. 6.7a-c). In addition, the cyclic compression is also found to be influenced by the orientation angles (\(\alpha_0\)) of the principal stress axis, as shown in Figs. 6.7b, e and h, in which larger volumetric compression occurs in the tests with larger cyclic shear stress amplitude. A softer and more contractive behaviour was observed for larger imposed principal stress orientation angles (\(\alpha_0\)) in monotonic shearing by Yoshimine et al. (1998).
6.3.3 Mobilised peak strength

Upon completion of the cyclic loading stages, all the specimens were sheared to failure following the same direction of the applied stress cycle in the $(\tau_{\theta z}/p' - (\sigma'_z - \sigma'_\theta)/2p')$ stress plane. It should be reminded that the shear stage was performed in a stress-controlled mode, thus only peak strength conditions could be reached. The peak failure points for all the tested specimens are presented in Fig. 6.8 in the $(\tau_{\theta z}/p' - (\sigma'_z - \sigma'_\theta)/2p')$ plane. The multiaxial strength envelope proposed by Matsuoka and Nakai (1974) can satisfactorily capture all the experimental data by assuming a unique value of the effective peak friction angle of $\varphi' = 38^\circ$. The same
A multiaxial strength formulation was employed by Mandolini et al. (2019) to capture the ultimate failure envelope of monotonically sheared very loose Hostun sand samples to failure. The ultimate friction angle for Hostun sand was found to be in the range of 32° to 33° (Mandolini et al., 2019).

Figure 6.8 Matsuoka–Nakai (Matsuoka & Nakai, 1974) failure envelope with experimental strength envelopes.

6.4 Cyclic strain response

6.4.1 Cyclic strain amplitude

Fig. 6.9 presents the variation of the strain amplitudes $\varepsilon_{z \text{ ampl}}$, $\gamma_{\theta z \text{ ampl}}$, and $\varepsilon_{\text{ampl}}$ (see Equation 6.1 and Fig. 6.3) of package cycles with the number of cycles ($N$) for all the tests. Decreasing trends of the strain amplitude are observed within the first 1000 cycles following the application of combined axial and torsional stresses (e.g. tests with $\alpha_\sigma = 15^\circ$ and 22.5°) and then reach a stable state, while such trends are relatively small for tests without rotation of principal stress axes ($\alpha_\sigma = 0^\circ$). However, the values of $\gamma_{\theta z \text{ ampl}}$ are 0 for tests with $\alpha_\sigma = 0^\circ$ in Fig. 6.9b since no torsional loading was applied in these tests. Similar results on a subrounded-grains quartz sand in triaxial cyclic tests have been found by Wichtmann et al. (2005). Compared to the torsional strain amplitude $\gamma_{\theta z \text{ ampl}}$ in Fig. 6.9b, the axial strain amplitude in Fig. 6.9a
is smaller. Nevertheless, the decrease of the amplitude for the tests with same axial cyclic loading (e.g. tests \(\alpha 0A7, \alpha 15A7, \alpha 22.5A7\); tests \(\alpha 0A21, \alpha 15A21, \alpha 22.5A21\); tests \(\alpha 0A35, \alpha 15A35\)) shows an increasing trend as the re-orientation of principal stress axes \(\alpha\) increases. The largest decrease in strain amplitude is observed in the sample subjected to the largest combined axial and torsional amplitude (i.e. test \(\alpha 22.5A21\)). Since the strain amplitude decreases at the initial cyclic loading stage, for the sake of simplicity, the constant value of strain amplitudes after 1000 cycles in Fig. 6.9 will be used throughout the paper.

![Figure 6.9 Development of strain amplitude \(\varepsilon_{ampl}\) with number of cycles (N).](image)

Fig. 6.10 presents the axial strain amplitudes \(\varepsilon_{z ampl}\) and torsional strain amplitudes \(\gamma_{\theta z ampl}\) of package cycles after 1000 cycles for all the tests. The axial and torsional strain amplitudes respectively corresponding to the stress amplitude \(\Delta \sigma'_{z} = 5\) kPa and \(\Delta \tau_{\theta z} = 5\) kPa obtained from measurement cycles are also presented. Fig. 6.10a confirms that torsional strain amplitude is larger than axial strain amplitude under
the same stress condition. It can be seen that the axial strain amplitudes $\varepsilon_{z}^{\text{ampl}}$ are linearly proportional to the stress amplitude for $\Delta \sigma_{z}^{'} < 40 \, \text{kPa}$ (Fig. 6.10b). The result agrees well with that by Witchmann (2005). The relationship between the torsional stress and torsional strain amplitudes $\gamma_{z}^{\text{ampl}}$ appears to be linear but without passing zero point (Fig. 6.10b). The dramatic change of the slope for the torsional stress-strain amplitudes curve indicates that the combination of axial and torsional loading for package cycles seems to induce a larger decrease of torsional strain amplitudes than that under pure torsional loading for measurement cycles.

![Figure 6.10 Axial and torsional strain amplitudes after 1000 cycle for all the tests.](image)

### 6.4.2 Cyclic strain accumulation

A summary of the accumulated strain $\varepsilon^{\text{acc}}$ is plotted against the number of loading cycles in a logarithmical format in Fig. 6.11. As mentioned in Fig. 6.2, the accumulated strain recorded from the second cycle is of interest, which shows a proportional relationship with the logarithm of the number of cycles and then over-proportionally. It is clearly visible in Fig. 6.11 that higher stress amplitudes (either axial or torsional amplitude) cause larger accumulated strain.
Chapter 6. Evolution of stiffness and accumulated strain under multiaxial cyclic loading

Figure 6.11 Accumulation curves $\varepsilon^{\text{acc}}$ for all the tests.

Further insight into the accumulation of the four strain components with the number of cycles for all the tests is shown in Fig. 6.12. It can be seen that the changes in the vertical strain $\varepsilon_z$ and torsional strain $\gamma_{\theta z}$ dominate (Figs. 6.12a and 6.12b), whereas the radial and circumferential strains ($\varepsilon_r$ and $\varepsilon_\theta$) remain relatively close to zero except for tests with the principal stress orientation of 22.5° (Figs. 6.12c and 6.12d). Fig. 6.12a reveals an important coupling between cyclic torsional stresses and accumulated axial strains (e.g. the amount of permanent axial deformations increases with the applied torsional cyclic loading despite the same amplitude of the axial stress loop in tests $\alpha_0A7$, $\alpha_{15}A7$ and $\alpha_{22.5}A7$), which has been discussed in previous section. The results reported by Wichtmann et al. (2007b), Toyota et al. (2021) and Mandolini et al. (2021) have also demonstrated that axial strain accumulated even under pure torsional cyclic loading. However, it is interesting to note that the axial cyclic loading does not produce any torsional strain in Fig. 6.12b.
Figure 6.12 Accumulated strain components of all tests: (a) axial strains; (b) torsional strains; (c) radial strains; (d) circumferential strains.

Figs. 6.13a and 6.13b respectively present the accumulated deviatoric strain and volumetric strain after $10^4$ cycles with the axial stress amplitude. It is interesting to notice that, despite the similar axial stress amplitudes, the accumulated strain increases with an increasing rotation of principal stress axis $\alpha$ caused by the application of additional shear stress, as already explained by the results shown in Fig. 6.6. Similar result is also reported by Toyota et al. (2021), who found that the combination of cyclic vertical loading and torsional loading, which involves the re-orientation of principal stress axes, triggered the greatest axial strain, compared with the pure cyclic vertical loading, pure cyclic torsional loading and the combination of cyclic vertical loading, horizontal loading and torsional loading. Therefore, the significant increase in accumulated strain caused by the rotation of principal stress axes should be considered in strain accumulation model.
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Figure 6.13 (a) relationship between axial stress amplitude and deviatoric strains and (b) relationship between axial stress amplitude and volumetric strains.

6.4.2.1 Strain accumulation under triaxial condition

It has been noted in Chapter 2 that Niemunis et al. (2005) and Wichtmann et al. (2007a) have proposed and developed a high-cycle explicit model (HCA), which can be expressed by:

$$\varepsilon_{acc}(N) = f_{ampl}f_Nf_epf_Y$$

(6.3)

The function $f_{ampl}$ describing the influence of the strain amplitude $\varepsilon_{acc}$ depends on the strain amplitude $\varepsilon_{ampl}$ and coefficient $C_{ampl}$ with a reference amplitude $\varepsilon_{ref}^{ampl} = 10^{-4}$. The function $f_N$ accounting for the influence of the number of cycles ($N$) is determined by the coefficients $C_{N1}$, $C_{N2}$ and $C_{N3}$. The function $f_e$ is dependent on the void ratio $e$ and the coefficient $C_e$ with a reference void ratio $e_{ref}$ equating to $e_{max}$. The function $f_p$ is confirmed by the average mean pressure $p_{av}$ and the coefficient $C_p$ with a reference pressure of 100 kPa. While the influence of average stress ratio $Y$ can be described by function $f_Y$ with coefficient $C_Y$. The expressions of the functional components of the Equation 6.3 are summarised in Table 6.2.

A simplified calibration of Equation 6.3, based on approximately 350 drained cyclic triaxial tests on 22 clean quartz sands (14 sands with linear grain size distribution curves with $0.1 \text{ mm} \leq d_{50} \leq 3.5 \text{ mm}$ and $1.5 \leq C_u \leq 8$ and 8 sands with S-shaped grain size distribution curves with $0.15 \text{ mm} \leq d_{50} \leq 4.4 \text{ mm}$ and $1.3 \leq C_u \leq 4.5$), was proposed by Wichtmann (2016) by correlating the HCA model parameters with mean grain size ($d_{50}$), coefficient of uniformity ($C_u$) and minimum void ratio ($e_{min}$)
due to their easy determination by standard laboratory tests. Relevant correlations by Wichtmann (2016) are presented as follows:

\[
C_e = 0.95 \cdot e_{\text{min}} \quad (6.4)
\]

\[
C_p = 0.41 \cdot [1 - 0.34(d_{50} - 0.6)] \quad (6.5)
\]

\[
C_Y = 2.60 \cdot [1 + 0.12 \ln (d_{50}/0.6)] \quad (6.6)
\]

The HCA parameters for the Hostun sand derived from the Equations 6.4 - 6.6 are listed in Table 6.3.

**Table 6.2 HCA model functions implemented and reference values (Niemunis et al., 2005).**

<table>
<thead>
<tr>
<th>Function</th>
<th>Parameters</th>
</tr>
</thead>
</table>
| \[
\frac{e_{\text{ampl}}}{e_{\text{ref}}}^{C_{\text{ampl}}} \cdot \frac{100}{C_{\text{ampl}}}
\] \[
C_{\text{ampl}}, e_{\text{ref}}^{C_{\text{ampl}}} = 10^{-4}
\] | |
| \[
N = C_{N1}[\ln(1 + C_{N2}N) + C_{N3}N]
\] \[
C_{N1}, C_{N2}, C_{N3}
\] | |
| \[
\exp \left( -C_p \left( \frac{p_{av}}{p_{ref}} - 1 \right) \right)
\] \[
C_p, p_{ref} = 100kPa
\] | |
| \[
\exp \left( C_Y \frac{Y_{av}}{Y_{av}} \right)
\] \[
C_Y
\] | |
| \[
\frac{(C_e - e)^2}{1 + e_{\text{ref}}^{C_e - e_{\text{ref}}}^2}
\] \[
C_e, e_{\text{ref}} = 1
\] | |

**Table 6.3 The HCA parameters for Hostun sand estimated from the correlations proposed by Wichtmann (2016).**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{\text{ampl}})</td>
<td>2</td>
</tr>
<tr>
<td>(C_e)</td>
<td>0.589</td>
</tr>
<tr>
<td>(C_p)</td>
<td>0.44</td>
</tr>
<tr>
<td>(C_Y)</td>
<td>2.46</td>
</tr>
</tbody>
</table>

It should be noted that, no clear dependences of parameter \(C_{\text{ampl}}\) on \(d_{50}\) or \(C_u\) were observed by Wichtmann (2016). The HCA model assumes that the accumulated strain \(\varepsilon_{\text{acc}}\) is proportional to the square of the strain amplitude \((\varepsilon_{\text{ampl}})^2\) with \(C_{\text{ampl}} = 2\) and this proportionality is valid up to \(\varepsilon_{\text{ampl}} = 10^{-3}\) (Wichtmann, 2005). For this research, the accumulated strain after 10000 cycles, normalised with the void ratio function \(f_e\) (Table 6.2) to remove the effect of the slightly varying initial densities, is plotted versus the square of the strain amplitude in Fig. 6.14. Obviously, the linear...
result for tests with $\alpha_\sigma = 0^\circ$ in Fig. 6.14 supports the similar conclusion by Wichtmann (2005). Therefore, the same constant exponent $C_{\text{ampl}} = 2$ was adopted in this research. However, such quadratic correlation between the accumulated strain and the strain amplitude seems to be non-existent in the tests with $\alpha_\sigma$ equating to $15^\circ$ and $22.5^\circ$.

\begin{center}
\includegraphics[width=0.6\textwidth]{fig6_14}
\end{center}

\textbf{Figure 6.14 Relationship between strain amplitude and accumulated strain ($N = 10000$) for all the tests.}

Fig. 6.15 presents the strain accumulation curves $\varepsilon^{\text{acc}}$ normalised by the functions $f_{\text{ampl}}$, $f_e$, $f_p$ and $f_Y$ for all eight tests. It is clear that the accumulated strain curves obtained from tests $\alpha 0A7$, $\alpha 0A21$ and $\alpha 0A35$, which were subjected to pure axial cyclic loading under triaxial stress conditions, fall closely together, confirming the availability of HCA model. The strain accumulation curve can be well approximated by the formula:

$$f_N = C_{N1} \ln (1 + C_{N2}) + C_{N3} N$$

(6.7)

with the material constants $C_{N1} = 0.0025$, $C_{N2} = 0.38$ and $C_{N3} = 0.0043$.

In contrast, the normalised curves for tests with re-orientation of principal stress axes of $15^\circ$ and $22.5^\circ$ fall relatively apart from the HCA and between them too. The results in Fig. 6.14 and Fig. 6.15 indicate that the HCA model may not be suitable for the cases involving the re-orientation of principal stress axes.
6.4.2.2 Strain accumulation function under combined axial and torsional stresses

This section attempts to predict the $\varepsilon^{acc}$ under the combination of the axial and torsional cycles accounting for the influence of large strain amplitude and orientation of cyclic stress path. A strong correlation between accumulated strain and the positional relationship between the cyclic stress path and the critical state line (CSL) has been found by He et al. (2020). Following He et al. (2020), a similar parameter - coupling ratio $f_{coup}$ is proposed in this study, which combines the effects of the two factors. It can be defined as:

$$f_{coup} = \left(\frac{\varepsilon^{\text{ampl}}}{\varepsilon_{\text{ref}}^{\text{ampl}}}\right)^2 \frac{L_{DCYC}}{L_{DF}} \quad (6.8)$$

Where the value of reference strain amplitude $\varepsilon_{\text{ref}}^{\text{ampl}}$ is equal to $10^{-3}$, which is the lower boundary of large strain amplitude based on Wichtmann et al. (2005). $L_{DCYC}$ is the length of the cyclic stress path and $L_{DF}$ (reflecting the the deviatoric strength) is the linear length between the initial point of the cycle stress path and the failure point, as illustrated in Fig. 6.16. The two parameters ($L_{DCYC}$ and $L_{DF}$) can be expressed as:

Figure 6.15 Accumulated strain $\varepsilon^{acc}$ divided by the functions $f_e$, $f_P$, $f_Y$ and $f_{ampl}$, as a function of the number of cycles.
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\[ L_{DCYC} = \sqrt{(\Delta \tau_{\theta z})^2 + \left(\frac{\Delta \sigma'_z - \sigma'_\theta}{2}\right)^2} \quad (6.9) \]

\[ L_{DF} = \sqrt{(\tau_{\theta z,f} - \tau_{\theta z,ini})^2 + \left(\frac{\sigma'_z - \sigma'_\theta}{2}\right)_f - \left(\frac{\sigma'_z - \sigma'_\theta}{2}\right)_{ini})^2} \quad (6.10) \]

**Figure 6.16 Definitions of variables under cyclic loading.**

Since both the values of \( L_{DCYC} \) and \( L_{DF} \) consider the influence of the orientation of the principal stress axes, the ratio of \( L_{DCYC} \) to \( L_{DF} \) can well reflect the anisotropy of soil. Consequently, a new expression \( \varepsilon^{acc} (N) \) is obtained by keeping the functions \( f_e, f_p \) and \( f_Y \) unchanged and replacing the \( f_{ampl} \) with \( f_{coup} \) compared to the HCA expression:

\[ \varepsilon^{acc}(N) = f_{coup} f_N f_e f_p f_Y \quad (6.11) \]

Fig. 6.17a presents the relationship between the \( \varepsilon^{acc} \) normalized by the void ratio function \( f_e \) after \( 10^4 \) cycles and the coupling ratio \( f_{coup} \) (\( \alpha_c = 15^\circ \) and \( 22.5^\circ \)). Evidently, \( \varepsilon^{acc} (N = 10000) \) shows a good linear relationship with \( f_{coup} \), while the accumulated strain with \( f_{ampl} \) is relatively discrete for \( \alpha_c = 15^\circ \) and \( 22.5^\circ \) (Fig. 6.14). This indicates that accumulated strain \( \varepsilon^{acc} \) is related to \( f_{coup} \) more than the strain amplitude \( f_{ampl} \).

Fig. 6.17b shows the accumulated strain normalized by the functions \( f_e, f_p, f_Y \) and \( f_{coup} \) with the number of load cycles for tests with re-orientation of principal stress axes of \( 15^\circ \) and \( 22.5^\circ \). The new experimental data fit, \( f_N = C_{N1}\ln(1+C_{N2})+C_{N3}N \), is similar to Equation 6.7 but with different material constants \( C_{N1} = 0.063 \), \( C_{N2} = 0.17 \).
and $C_{N3} = 7.34 \cdot 10^{-5}$. The convergence of the results confirms the validity of the proposed function of coupling ratio for accumulated strains.

Figure 6.17 (a) Accumulated strain $\varepsilon^{acc}$ after $10^4$ cycles as a function of coupling ratio $f_{coup}$; (b) accumulated strain $\varepsilon^{acc}$ divided by the functions $f_e$, $f_p$, $f_Y$ and $f_{coup}$, as a function of the number of cycles.

6.4.3 Summary

In summary, the re-orientation of the principal stress axes causing the breakage of force chains of soil particles changes the underlying mechanism of strain accumulation. The HCA model calibrated by triaxial and direct simple shear tests was found to be not able of capturing such strain accumulation under multiaxial stress space. Therefore, a new empirical formula based on HCTA tests was proposed in this section to predict the accumulated strain under rotation of principal stress axes, although the tests were carried out on the same sand and under the same initial state.

6.5 Evolution of small strain soil stiffness

Wichtmann et al. (2004a, 2004b) has demonstrated that the accumulation of soil deformation not only depends on the void ratio, average stress and strain amplitude during cycles. The changes in fabric caused by re-orientation of soil particles should also be considered for the prediction of accumulated strain. Such changes further lead to the changes of small strain soil properties (i.e. small strain
stiffness $E_z$ and $G_{θz}$). This section focuses on evaluating the influence of large number of multiaxial loading cycles on small-strain stiffness.

### 6.5.1 Stress dependencies of stiffness

All the values of the Young’s and shear moduli before the start ($E_{z(i)}$ and $G_{θz(i)}$ at Point 0 and Point B in Fig. 6.2) at the end ($E_{z(f)}$ and $G_{θz(f)}$ at Point B in Fig. 6.2) of the cyclic stage are presented in Table 6.4 together with the results by Mandolini et al. (2021). All the initial values of Young’s and shear moduli ($E_{z(i)}$ and $G_{θz(i)}$) in Table 6.4 are displayed in Figs. 6.18a and 6.18b, respectively. Small variation of the initial moduli is observed for all the tests, demonstrating the repeatability of the tests.

According to Hardin and Blandford (1989) and Hoque and Tatsuoka (1998), the relationship between the Young’s modulus ($E_z$) and the effective axial stress shown in Fig. 6.18a is governed by:

$$E_z = C f(e) \left( \frac{\sigma'_z}{\sigma_{ref}} \right)^m$$  \hspace{1cm} (6.12)

where $\sigma_{ref}$ denotes the reference pressure of 1 kPa, the parameter $C = 5.6$ and the exponent $m = 0.52$ for Hostun sand as suggested by Mandolini et al. (2021). As shown in Fig. 6.18b, the shear modulus ($G_{θz}$) is dependent on the effective axial stress ($\sigma'_z$) and effective circumferential stress ($\sigma'_θ$) (Roesler, 1979; Di Benedetto et al., 1999), which can be expressed as:

$$G_{θz} = D f(e) \left( \frac{\sigma'_z}{\sigma_{ref}} \right)^{n1} \left( \frac{\sigma'_θ}{\sigma_{ref}} \right)^{n2}$$  \hspace{1cm} (6.13)

The experimental fitting equation with the coefficients $D$ of 2.3 and $n1$ (equal to $n2$) of 0.26 was obtained by Mandolini et al. (2021). Fig. 6.18 shows that the experimental data in this research agrees well with the fitted curves for both
Young's and shear moduli reported by Mandolini et al. (2021). Such agreement validates the performed tests in this research.

Table 6.4 Summary of the elastic small strain stiffness for all the tests.

<table>
<thead>
<tr>
<th>Test name</th>
<th>$\sigma'^z$ (kPa)</th>
<th>$\sigma'^\theta$ (kPa)</th>
<th>$E_{z(i)}$ (MPa)</th>
<th>$E_{z(f)}$ (MPa)</th>
<th>$G_{\theta z(i)}$ (MPa)</th>
<th>$G_{\theta z(f)}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0A7$</td>
<td>50 100</td>
<td>50 50</td>
<td>128.5 179.7</td>
<td>- 173.1</td>
<td>46.3 49.0</td>
<td>- 47.5</td>
</tr>
<tr>
<td>$\alpha_0A21$</td>
<td>50 100</td>
<td>50 50</td>
<td>121.4 154.3</td>
<td>- 131.5</td>
<td>45.1 51.9</td>
<td>- 46.6</td>
</tr>
<tr>
<td>$\alpha_0A35$</td>
<td>50 100</td>
<td>50 50</td>
<td>122.3 158.7</td>
<td>- 131.7</td>
<td>43.4 50.2</td>
<td>- 40.7</td>
</tr>
<tr>
<td>This research</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_{15}A7$</td>
<td>50 100</td>
<td>50 50</td>
<td>125.1 176.1</td>
<td>- 164.9</td>
<td>46.8 55.6</td>
<td>- 49.4</td>
</tr>
<tr>
<td>$\alpha_{15}A21$</td>
<td>50 100</td>
<td>50 50</td>
<td>130.4 162.3</td>
<td>- 147.0</td>
<td>48.3 54.4</td>
<td>- 47.1</td>
</tr>
<tr>
<td>$\alpha_{15}A35$</td>
<td>50 100</td>
<td>50 50</td>
<td>110.2 151.2</td>
<td>- 133.0</td>
<td>42.5 49.8</td>
<td>- 43.0</td>
</tr>
<tr>
<td>$\alpha_{22.5}A7$</td>
<td>50 100</td>
<td>50 50</td>
<td>135.2 188.9</td>
<td>- 168.5</td>
<td>47.1 53.8</td>
<td>- 48.0</td>
</tr>
<tr>
<td>$\alpha_{22.5}A21$</td>
<td>50 100</td>
<td>50 50</td>
<td>131.3 163.3</td>
<td>- 136.8</td>
<td>45.5 53.4</td>
<td>- 46.4</td>
</tr>
<tr>
<td>Mandolini et al. (2021)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A-\beta 0$</td>
<td>50 100 180 250</td>
<td>100 100 100 100</td>
<td>107.0 184.7 250.0 336.7</td>
<td>102.1 176.0 244.7 332.9</td>
<td>54.9 59.6 80.7 94.5</td>
<td>53.0 56.9 78.9 93.5</td>
</tr>
</tbody>
</table>
6.5.2 General soil stiffness change with number of cycles

Figs. 6.19 and 6.20 compare the typical axial and shear stress-strain loops extracted from measurement cycles after package cycles $N = 0, 1000, 10000$ and $30000$ for all the tests, where $N = 0$ denotes the initial measurement of small strain moduli made prior to the first package of cycles. However, exceptions of the stress-strain loops recorded after $N = 0, 1000, 10000$ and $17500$ for tests $\alpha 15A35$ (Figs. 6.19f and 6.20f) and stress-strain loops after $N = 0, 1000, 5000$ and $10000$ for tests $\alpha 22.55A21$ (Figs. 6.19h and 6.20h) should be noted due to the interruptions of the hydraulic loading supply. The loops have been manually translated to start at the origin of the stress and strain axes in order to better evaluate the changes in the stress-strain loops during the cyclic loading. All these axial and shear strains were obtained through the local measurement system. The linear axial stress-strain relationships for the small measurement cycles ($\Delta \sigma_z = 5\text{kPa}$) in Fig. 6.19 indicate an elastic behaviour of the sample at this stress level. In spite of some scatters in Fig. 6.20e, similar observations also can be made on the torsional stress-strain probing shown in Fig. 6.20. The small strain Young’s, $E_z$, and shear, $G_{\theta z}$, moduli were evaluated from the slopes of these stress-strain relations, as described in Section 6.2. A slight reduction of the slope for both axial and torsional stress-strain loops, corresponding to Young’s ($E_z$) and shear ($G_{\theta z}$) moduli respectively, is recorded.
Figure 6.19 Axial stress-strain loops of measurement cycles for all the tests.
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Figure 6.20 Torsional stress-strain loops measurement cycles for all the tests.

For the purpose of comparison, the package cycles at \( N = 1, 1000, 10000 \) and 30000 for axial and torsional stress-strains are highlighted in Figs. 6.21 and 6.22, respectively. Please note that torsional stress-strain loops for tests \( \alpha 0A7, \alpha 0A21 \) and \( \alpha 0A35 \) are not displayed in Fig. 6.22 due to the triaxial stress conditions. Again, all these axial and shear strains were obtained through the local measurement system. Different from measurement cycles, the hysteretic responses induced by larger cyclic stress amplitudes can be clearly seen in axial package cycles in Fig. 6.21, indicating the potential occurrence of plastic straining. In general, the hysteresis of axial package cycles in Fig. 6.21 becomes more prominent as the axial cyclic amplitude \( \Delta \sigma_x' \) increases. Conversely, less obvious hysteresis or more linearity in torsional stress-strain loops in Fig. 6.22 is observed.
Chapter 6. Evolution of stiffness and accumulated strain under multiaxial cyclic loading

It is worth noting in Figs. 6.21a, d and g that, although similar axial cyclic amplitudes $\Delta\sigma'_z = 7$ kPa are applied in the tests $\alpha0A7$, $\alpha15A7$ and $\alpha22.5A7$, different axial stress-strain responses can be observed. The response in Fig. 6.21a shows a typical linear relationship in triaxial stress condition that is consistent with those observed for measurement cycles, while the stress-strain curves in Figs. 6.21d and 6.21g appear to be exponential, possibly due to the influence of additional torsional loading cycles.

An interesting feature outlined in Fig. 6.21 is that the initial cycle loops never close for tests with larger cyclic amplitude (e.g. Figs. 6.21c, e, f, h), but eventually becomes closed as the cycles proceed, which indicates a hardening response to large cycle number. Such observation has also been experimentally reported by Tong et al. (2010).

The secant axial ($E_{sz}$) and shear ($G_{sz}$) moduli were calculated by the slope of the line connecting the peak and trough point limits of each hysteretic loop.
Figure 6.21 Axial stress-strain loops of package cycles for all the tests.
Figure 6.22 Torsional stress-strain loops of package cycles for all the tests.

The development of the secant axial and shear moduli of packages cycles ($E_{sz}$ and $G_{sθz}$) and elastic small strain Young's and shear moduli ($E_z$ and $G_{θz}$) obtained from the measurement cycles are compared in Fig. 6.23. For the purpose of eliminating the effect of the void ratio (e.g. densification), the stiffness values were normised by the void ratio function $f(e) = (3.01 - e)^2/(1 + e)$ used by Ibraim et al. (2011) for the same sand. Please note that the small strain Young’s $E_z$ and shear $G_{θz}$ moduli were evaluated at regular intervals. The measurement of the initial small strain moduli was made before the first package of cycles. However, it is not shown here as the horizontal $N$-axis is scaled logarithmically. Once again, the secant shear modulus ($G_{sθz}$) is not provided in Figs. 6.23a-c as no torsional loading was applied on the specimens in tests α0A7, α0A21 and α0A35.

From Fig. 6.23, decreasing trends of corrected small strain Young's and shear moduli ($E_z$ and $G_{θz}$) can be systematically observed for all the tests. The secant axial stiffness of package cycles is found to slightly increase within the first 100 cycles and then decrease in the successive applied cycles. This observation is in accordance with the results by He et al. (2020), which is different from the typical progressively increasing trend of secant modulus under drained cyclic loading due
to the cyclic densification (Sun et al., 2016; Han and Vanapalli, 2016; Cao et al., 2017).

Figure 6.23 Evolution of soil stiffness for all the tests.

Overall, the cyclic stress amplitudes have limited influence on small strain elastic moduli \( E_z \) or \( G_{θz} \). Wichtmann & Triantafyllidis (2004a and b) found that variations of Young’s modulus of sand provided by the bender elements during the application of high number cycles in conventional triaxial tests are small and are independent on the cyclic stress amplitude and average stress level. Similar results obtained from the resonant column tests on sand were confirmed by Tatsuoka et al. (1979) and Lo Presti et al. (1993), and also by Escribano and Nash (2015) based on bender elements.
The small strain Young’s modulus \((E_z)\) shows a similar trend with secant axial stiffness \((E_{sz})\) for tests \(\alpha0A7\), \(\alpha0A21\) and \(\alpha0A35\) (Figs. 6.23a-c). The difference between \(E_z\) and \(E_{sz}\) appears when the torsional loading is applied. On the contrary, the trend of \(G_\theta z\) and \(G_{\theta0z}\) remains similar for all the tests.

### 6.5.3 Influence of stress amplitudes

A proportional relationship between the stiffness degradation and the cyclic amplitude was found by Mandolini et al. (2021), who assessed the change of soil stiffness under different torsional cyclic loading through HCTA tests. This research evaluated the influence of different axial stress amplitudes on soil stiffness evolution. Fig. 6.24 presents the development of elastic Young’s and shear moduli with the logarithmical number of cycles for axial cyclic stress amplitudes of \(\Delta\sigma'_z = 7\text{kPa}, 21\text{kPa} \) and \(35\text{kPa}\), respectively (tests \(\alpha0A7\), \(\alpha0A21\) and \(\alpha0A35\)). The measured values are divided by the values measured before the cyclic loading stage \((E_z(0))\). A clear trend of stiffness degradation can be observed for both Young’s modulus \(E_z\) and shear modulus \(G_\theta z\). Such degradation is more remarkable following the application of higher axial stress amplitudes. From Fig. 6.24, it is found that the degradation of small strain modulus marginally increases as the axial stress amplitude increases.

![Figure 6.24 Evolution of corrected Young’s and shear moduli for different axial stress cycles in tests C1, C2 and C3.](image)

### 6.5.4 Influence of rotation of principal stress axes

Fig. 6.25 compares the evolution of the corrected small strain moduli \(E_z\) and \(G_\theta z\) of samples in the application of the same axial stress amplitudes but different shear
stress amplitudes. The horizontal axis represents the number of cycles and is scaled logarithmically. The results reported in Fig. 6.25 demonstrate that re-orientation of principal stress axes leads to a more pronounced degradation of soil stiffness with one exception shown in Fig. 6.25c, in which the degradation of Young’s modulus $E_z$ in test $\alpha15A21$ is smaller than that in test $\alpha0A21$.

Figure 6.25 Evolution of corrected Young’s and shear modulus moduli for combined axial and torsional stress cycles.

The results of cycles-related stiffness degradation for all the tests are summarised in Fig. 6.26, in which the normalized moduli after 10000 cycles are plotted against the mobilised friction angle. The variations of the corrected Young’s and shear
moduli in Fig. 6.26 present a clear decreasing trend of stiffness degradation with mobilised friction angle. The degradation rate of the normalised shear modulus decreases with the increase of mobilised friction angle, while for the Young’s modulus, a linear relationship seems to be assumed. Both Young’s and shear moduli are found to degrade up to around 85% with larger mobilised friction angle.

![Graph showing the relationship between stiffness degradation and mobilised friction angle.](image)

**Figure 6.26** Relationships between (a) ratio $E_z/E_{z(0)}$ after $10^4$ cycles and mobilised friction angle; (b) ratio $G_{θz}/G_{θz(0)}$ after $10^4$ cycles and mobilised friction angle.

### 6.6 Conclusions

This chapter provided an insight of the soil accumulated strain and variation of soil stiffness induced by the application of the high number of multiaxial stress path cycles informed by the FE analysis in Chapter 4, using HCTA. The main conclusions of this experimental investigation are summarised below.

(a) The HCTA tests in triaxial condition revealed that the specimen experiencing cyclic load history with different stress amplitudes tends to show a similar final soil strength.

(b) Due to the breakage of force chains of soil particles induced by the application of combined axial and torsional stress cycles, prominent increase of accumulated strain was found with a large amount of cyclic rotation of principal stress axes. This suggests the importance of considering the multiaxial nature of the load, which should be captured by strain accumulation model.

(c) Current available accumulation model HCA (Niemunis et al., 2005) provides good empirical relations to qualitatively describe the influence of cyclic strain
amplitude and number of load cycles on strain accumulation for soil under triaxial condition. However, it seems not to be suitable to predict the accumulated strain for samples subjected to the rotation of principal stress axes.

(d) A new parameter, coupling effect ratio $f_{coup}$ was proposed. Compared with $f_{ampl}$, $f_{coup}$ can better reflect the development of accumulated strain with large strain amplitude and orientation of cyclic stress path. In addition, empirical formula for predicting the accumulated strain for this case was established.

(e) The secant axial stiffness obtained from package cycles was increased at the beginning and then reduced. It was found that the evolution of secant stiffness and small strain elastic stiffness for both axial and shear direction show similar tendencies under pure axial cyclic loading. Application of additional shear stress cycles caused the difference between axial modulus for package cycles and measurement cycles, while the similarity between the evolution of shear modulus for both two types of cycles seemed to be independent on the stress conditions.

(f) Application of large numbers of axial cycles invoked degradation trends of small strain soil stiffness (obtained from measurement cycles), which are proportional to cyclic stress amplitude. Additional shear stress cycles may further aggravate this trend.

(g) Degradation of soil stiffness was found to increase at higher mobilised friction angles for both Young’s and shear moduli.
Chapter 7  Conclusions

7.1 Introduction

This thesis presented a detailed analysis of 3D FE-informed laboratory soil testing for the design of offshore wind turbine monopiles. It consisted of two parts. Part 1 included Chapters 3 and 4, in which 3D finite element analysis was employed for the systematic identification of the actual stress paths induced by lateral monotonic and cyclic pile loading and provided recommendations on stress paths to be applied in the laboratory to more appropriately reproduce field conditions. Chapters 5 and 6 formed part 2 of the work and reported the campaign of experimental investigation undertaken using HCTA to gain insight into the soil response under the obtained FE stress paths. A comparison between soil behaviour observed by imposed conventional and new stress paths was also carried out. While summary sections at the end of each chapter have presented detailed conclusions, this final chapter distils the key contributions and makes suggestions for future work.

7.2 Stress paths around large diameter laterally loaded piles in sand

Chapters 3 and 4 used 3D numerical analyses to investigate and identify the stress path experienced by soil elements around large-diameter piles in sand subjected
to monotonic and cyclic drained lateral loading, respectively. The performed 3D finite element model employed the latest developments in cyclic soil constitutive modelling proposed by Liu et al. (2019), named SANISAND-MS, which can accurately predict the expected soil response under a large number of loading cycles using the concept of hardening memory surface (Corti et al., 2016).

Inspection of the loading-induced stresses in the soil revealed the multiaxial nature of these stress paths, which are characterised by the rotation of one or more principal stress axes. Based on the outcome of the finite element analyses, typical stress paths for different soil elements around the piles are extracted. Such stress paths are then evaluated against those enabled by conventional and advanced laboratory soil element testing and some guidance for the implementation of these stress paths into practice has been advanced. The main conclusions for these two chapters, dealing with both monotonic and cyclic pile loading, are as follows:

- **Common conclusions for monotonic and cyclic lateral pile loading:**
  - The application of either monotonic or cyclic lateral pile loading induced the variations of four stress components (three normal stresses and one shear stress) for the soil elements aligned with the direction of lateral loading, and five stress components (the shear stress $\tau_{yz}$ is negligible) for soil elements outside the direction of loading.
  - For all analysed elements, the HCTA offered the best simulations of the typical stress paths if compared to other apparatus due to the possibility to independently control four stress variables and the possibility to impose continuous rotation of principal stress axes.
  - It should be highlighted that while HCTA testing appears to offer the best simulation of the extracted stress paths, it could not fully capture the full variation of the six-dimensional stress states for some soil elements. The reduction to a four-dimensional stress state (maximum number of stress variable controllable in element testing) by neglecting the two lowest shear stress components were required.
  - While the HCTA offered the control of four degrees of freedom, available stress conditions for HCTA should satisfy one main
constraint relates to the applicable ratio between the internal \( p_i \) and external pressure \( p_o \), which is in the range \( 0.9 < p_o/p_i < 1.2 \) to avoid major non-uniformities across the sample wall.

- It was impossible for laboratory element testing to maintain the orientation between the stress direction and the material axis between field and laboratory conditions, unless special sampling or sample preparation procedures (e.g. horizontally coring from block samples for clays or preparation of cylindrical samples maintaining the axis of symmetry horizontal) were adopted. Therefore, a relative 90° rotation between the material axis and the stress direction must be applied.

- Specific conclusions for monotonic lateral pile loading
  - The monotonic stress paths can be well reproduced by HCTA in the \( \left( \frac{\tau_{xz}}{p'_t} \sim \frac{\sigma_x - \sigma_z}{2p'_t} \right) \) or \( \left( \frac{\tau_{xy}}{p'_t} \sim \frac{\sigma_x - \sigma_y}{2p'_t} \right) \) stress plane, while the differences at the initial loading stage in \( \left( \frac{\sigma_x - \sigma_z}{2p'_t} \sim \frac{\sigma_y - p't}{p'_t} \right) \) or \( \left( \frac{\sigma_x - \sigma_y}{2p'_t} \sim \frac{\sigma_z - p't}{p'_t} \right) \) stress plane caused by the \( K_0 \) stress condition, can be overruled or alleviated if the pile installation effects were considered.
  - The inclination of monotonic stress paths in \( \left( \frac{\tau_{xz}}{p'_t} \sim \frac{\sigma_x - \sigma_z}{2p'_t} \right) \) or \( \left( \frac{\tau_{xy}}{p'_t} \sim \frac{\sigma_x - \sigma_y}{2p'_t} \right) \) stress plane was related to the distance, depth and orientation of the soil elements, while the inclination of the stress paths in \( \left( \frac{\sigma_x - \sigma_z}{2p'_t} \sim \frac{\sigma_y - p't}{p'_t} \right) \) or \( \left( \frac{\sigma_x - \sigma_y}{2p'_t} \sim \frac{\sigma_z - p't}{p'_t} \right) \) stress plane remained relative stable except that for soil element at the side of the pile.
  - The relative density showed negligible influence on stress paths of soil.
  - The suggested monotonic HCTA testing shown in Table 7.1 has already been presented in Table 3.3 in Chapter 3. As the most important findings for monotonic stress paths, they are re-presented here for the sake of completeness.
Table 7.1 Summary of recommended experimental tests for the soil elements around the laterally loaded pile (same with Table 3.3).

<table>
<thead>
<tr>
<th>Soil element position</th>
<th>Suggested HCTA testing</th>
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</thead>
<tbody>
<tr>
<td><strong>Case F</strong></td>
<td><strong>Initial conditions</strong></td>
</tr>
<tr>
<td></td>
<td>• $\sigma_{z0} = \gamma' \cdot z$;</td>
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<tr>
<td></td>
<td>• $\sigma_{x0} = \sigma_{y0} = K \cdot \sigma_{z0}$ [$K$ is defined to consider pile installation effect];</td>
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<td></td>
<td>• $0 \leq \tau_{xz} \leq \sigma_{x0} \cdot \tan(\phi_{ps})$.</td>
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<td></td>
<td><strong>Incremental stress or strain conditions:</strong></td>
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<tr>
<td></td>
<td>• Apply vertical stress ($\Delta \sigma_x$) or strain ($\Delta \varepsilon_x$) increments;</td>
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<tr>
<td></td>
<td>• $\Delta \sigma_z = \Delta \sigma_y = 0$ [constant inner and outer cell pressures];</td>
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<tr>
<td></td>
<td>• $\Delta \tau_{xz} = (\Delta \sigma_x - \Delta \sigma_z) / 2 \cdot \tan(\beta_{xz})$. [$\beta_{xz} = 10^\circ$ to $25^\circ$]</td>
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<tr>
<td><strong>Case B</strong></td>
<td><strong>Initial conditions</strong></td>
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<td></td>
<td>• $\sigma_{z0} = \gamma' \cdot z$;</td>
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<td></td>
<td>• $\Delta \tau_{xz} = (\Delta \sigma_x - \Delta \sigma_z) / 2 \cdot \tan(\beta_{xz})$. [$\beta_{xz} = -60^\circ$]</td>
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<td><strong>Case D</strong></td>
<td><strong>Initial conditions</strong></td>
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<td></td>
<td>• $\sigma_{z0} = \gamma' \cdot z$;</td>
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<td></td>
<td>• $\tau_{xy} = 0$.</td>
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<td><strong>Incremental stress conditions:</strong></td>
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<td></td>
<td>• Apply vertical stress ($\Delta \sigma_x$) or strain ($\Delta \varepsilon_x$) increments ($\theta &lt; 45^\circ$); shear stress ($\Delta \tau_{xy}$) or strain ($\Delta \gamma_{xy}$) increments ($45^\circ &lt; \theta &lt; 90^\circ$);</td>
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<td></td>
<td>• $\Delta \sigma_z = \Delta \sigma_y = 0$ [constant inner and outer cell pressures];</td>
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<td></td>
<td>• $\Delta \tau_{xy} = (\Delta \sigma_x - \Delta \sigma_y) / 2 \cdot \tan(\beta_{xy})$. [$\beta_{xy} \sim \theta$].</td>
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<td><strong>Case S</strong></td>
<td><strong>Initial conditions</strong></td>
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<td></td>
<td>• $\Delta \sigma_x = 0$;</td>
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• Specific conclusions for cyclic pile loading
  - The cyclic lateral pile loading induced a progressive forward drift of stress paths in \( \frac{r_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'} \) stress plane, while hysteretic loops of stress paths in \( \frac{r_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \) stress plane were triggered.
  - The relative density combined with the amplitude of pile loading seemed to determine the stress paths of soil elements located at different locations.
  - Compared with monotonic stress paths, besides the perfect reproduction of the stress paths in \( \frac{r_{xz}}{p'} \sim \frac{\sigma_x - \sigma_z}{2p'} \) or \( \frac{r_{xy}}{p'} \sim \frac{\sigma_x - \sigma_y}{2p'} \) stress plane, the evolution of FE cyclic stress paths helped to approach the available HCTA stress conditions due to the gradual decreasing of vertical stress \( \sigma_z \). In this respect, it seemed to be easier for the cyclic stress paths extracted from the stable state to be reproduced by HCTA than the monotonic stress path.
  - Once again, although they have been reported in Chapter 4, the important concluding remarks in terms of suggested cyclic HCTA testing are summarised in Table 7.2 (same as Table 4.3).

It should be stressed that the aim of this research is not to promote the HCTA. It is unrealistic to completely replace conventional testing in the day-to-day engineering practice. This is due to (i) the complexity of HCTA sample preparation and testing, (ii) the large amount of tests to be performed within an offshore wind farm development, (iii) the increased cost, and (iv) the current availability of HCTA. Nevertheless, it is important that design engineers understand the limitation and simplification adopted in their design process. It is also expected that HCTA could be used in current/future research work to explore in depth the multiaxial soil behaviour and develop appropriate corrections/modifications to the design process based on triaxial and DSS tests.
Table 7.2 Summary of recommended experimental tests for the soil elements around the laterally loaded pile (same with Table 4.3).

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<tr>
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<td>• $0 \leq \tau_{xz} \leq \sigma_{x0} \cdot \tan(\phi_{ps})$. [$\beta_{xz} = 15^\circ$ to $45^\circ$]</td>
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<td><strong>Incremental stress or strain conditions:</strong></td>
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<td></td>
<td>• $\Delta \tau_{xz} = (\Delta \sigma_x - \Delta \sigma_z)/2 \cdot \tan(\beta_{xz})$. [$\beta_{xz} = -75^\circ$]</td>
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<td><strong>Case D</strong></td>
<td><strong>Initial conditions</strong></td>
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<tr>
<td></td>
<td>• $\sigma_{z0} = \gamma'z$;</td>
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<td></td>
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<td>• $\Delta \tau_{xy} = (\Delta \sigma_x - \Delta \sigma_y)/2 \cdot \tan(\beta_{xy})$. [$\beta_{xy} \approx \theta$]</td>
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<tr>
<td><strong>Case S</strong></td>
<td><strong>Initial conditions</strong></td>
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<td></td>
<td>• $\Delta \sigma_z = \Delta \sigma_y = 0$. [constant inner and outer cell pressures]</td>
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</table>
7.3 Soil behaviour under the FE informed stress paths

Complementary laboratory testing using the HCTA was carried out on granular soil samples to investigate soil response under multiaxial cyclic loading (with up to about $3 \times 10^4$ cycles). The HCTA tests employed stress conditions starting from an initial anisotropic purely deviatoric state and featuring stress cycles characterised by simultaneous variation of axial and torsional stress, which resembled those experienced by soil elements in front of cyclic laterally loaded pile obtained from FE analysis. This would result in a cyclic re-orientation of principal stress axes. The main findings were:

- The cyclic load history seemed to have limited influence on the final soil strength.
- Comparison of HCTA tests imposing different amounts of cyclic rotation of principal stress axes (including the case of no rotation, typically of conventional triaxial testing) revealed an important influence of this rotation on the magnitude and direction of accumulated plastic strains.
- The accumulated strain under triaxial condition was found to fit well with the high cycle accumulation model (HCA) by Niemunis et al. (2005). However, the HCA model may not be suitable for predicting the accumulated strain for samples subjected to additional large torsional strain amplitudes. In this respect, a new parameter coupling effect ratio $f_{\text{coup}}$ as a replacement of parameter $f_{\text{ampl}}$, governing the development of accumulated strain with large strain amplitude and orientation of cyclic stress path, was therefore proposed and showed a good agreement with the modified HCA fitting curve.
- The secant stiffness obtained from package cycles was increased at the beginning and then reduced. It was found that the evolution of secant stiffness and small strain elastic stiffness showed similar tendencies under pure axial cyclic loading. Additional shear stress cycles caused the difference between vertical modulus for package cycles and measurement cycles, while the evolution of shear modulus seems to be independent to the stress conditions.
• Both axial and torsional cyclic amplitudes were found to influence the size of the small strain stiffness drop. The combination of them seemed to have a more profound influence on the degradation of small strain stiffness. However, the comparison of all the results in terms of stiffness degradation suggested a clear decreasing trend with increase of mobilised friction angle either for both Young’s and shear moduli.
• Greater stiffness degradation was observed under the larger rotation of principal stress axes, even though under similar mobilised friction angle.

7.4 Potential impact of results on monopile design

As described in Chapter 2, the design of the foundation must guarantee safe performance following four design checks (DNV, 2018), in which the pile capacity is governed by the ULS and ALS checks, the SLS check ensures the accumulated rotation and displacements within an acceptable range, while the natural frequency is determined by FLS check. Performing these design checks is challenging due to complex environmental cyclic load conditions. Given the importance of considering the effects of multiaxial loading on soil response (either this research or other works by Nakata et al. 1998; Chaudhary and Kuwano, 2003), potential impact accounting for multiaxial soil behaviour on pile design are summarised as follows:

• **ULS and ALS design – pile capacity.** The experimental work presented in Chapter 6 indicated that the soil strength seems to be independent on multiaxial cyclic loading history. Fig. 7.1 compares the response of specimen under pure monotonic loading and under combined monotonic and cyclic loadings. The almost superposition of the two monotonic curves suggests that the soil is not affected by the past cyclic loading history in terms of final strength. This phenomenon may shed light on ULS and ALS design. Nevertheless, due to the particle convection around the pile as shown by Cuéllar et al. (2012), other aspects such as density changes and re-arrangements of stress, should be taken into account as they may affect the overall pile capacity.
**Figure 7.1 Comparison of stress-strain response of soil under monotonic loading and combined monotonic and cyclic loadings.**

- **SLS design – accumulated deformation.** As shown in Fig. 7.2, compared to the response of specimen subjected to triaxial or direct simple shear loading ($\alpha_{sc} = 0^\circ$), significant strain accumulation was experimentally observed under multiaxial loading ($\alpha_{sc} = 15^\circ$ and $22.5^\circ$). These tests are expected to be used to calibrate the high cycle accumulation model to predict larger deformation under complex cyclic loading. However, concerning the overall response of pile-soil interaction system, such feature should be proved by testing the whole procedure (e.g. field testing and physical modelling) since other mechanisms such as local densification or loosening of the granular soil and subsequent re-distribution of stresses may play an essential role in pile displacement and rotation.
Figure 7.2 (a) relationship between axial stress amplitude and deviatoric strains and (b) relationship between axial stress amplitude and volumetric strains (same with Fig. 6.13).

- **FLS design - natural frequency.** One of the main implications for design derived from soil element testing is that the stiffness degradation under long-term multiaxial cyclic loading shown in Fig. 7.3 may lead to a reduction of stiffness of soil spring around monopile. The natural frequency of the turbine is highly sensitive to the surrounding soil stiffness is likely to change with cyclic loading. Such time-dependent natural frequency needs to be considered in monopile design to avoid dynamic resonance phenomena. Once again, local loosening and densification of sand around the pile foundation may occur. This therefore poses the important question: how will the natural frequency change under long-term cyclic loading? Decrease or increase?

Figure 7.3 Evolution of corrected Young’s and shear moduli for combined axial and torsional stress cycles (extract from Fig. 6.22).
Chapter 7. Conclusions

Overall, the main findings of this research highlighted the need to accurately assess the influence of multiaxial loading before conducting cyclic design for monopile foundations. However, it is necessary to well link the response at soil element scale to overall behaviour monopile-soil interaction system by considering more governing factors such as the change of soil density and re-arrangement of stress around the pile. The aforementioned potential impacts on pile design are not well-understood yet and therefore should be checked and verified by further research.

7.5 Suggestions for future research

7.5.1 Future research of pile design

Fig. 7.4 depicts the usefulness of HCTA tests and their application in developing design methods. As a first step, the HCTA was employed to identify some important features of soil response to multiaxial loading in this research (Fig. 7.4a). These experimental tests can subsequently be used to inform or calibrate the constitutive model implemented in 3D FE modelling to guide the monopile design (Figs. 7.4b and 7.4d). However, as informed by Section 7.4, the soil conditions around the laterally loaded pile are more complex (e.g. the convective regions and the local densification revealed by Cuéllar et al. (2012), overall pile tests (Fig. 7.4c) seem necessary to validate the FE modelling framework. Therefore, future research will focus on the field tests/monitoring or physical modelling (1-g and centrifuge tests) to investigate the influence of multiaxial soil response on pile behaviour including pile capacity, displacement and natural frequency. The outcome of this research in combination with the future findings will be beneficial for a more accurate pile design.
7.5.2 Suggestions for numerical modelling

- **Pile installation effect.** The assumption of ‘wished in place’ pile generation method in the FE appeared to be a major constraint in assessing the suitability of the FE stress paths in HCTA in this research, due to the challenges of using the FE modelling method for accumulated strain calculation to simulate the large deformations and stress changes induced by pile penetration. In light of the known issue, the Eulerian-Lagrangian approach can be used to generate the post pile installation stress condition (e.g. Fan et al., 2021a, 2021b and 2021c; Staubach et al., 2020), which then can be set as the initial soil conditions for the FE models involving high number of cycles.

- **Development of constitutive model accounting for multiaxial response.** It is clear that the informed stress paths rely on the constitutive model adopted in the FE modelling. For this research, despite the implementation of an advanced constitutive model capturing the ratcheting response in FE modelling, it was only calibrated against cyclic triaxial and shear tests. More complex cyclic constitutive models considering the influence of re-orientation of principal stress axes may be needed for reliable predictions of pile performance and pile-soil interaction (e.g. estimate the change of natural frequency with time). Future work will focus on the refinement of the
models to capture such feature. Although researchers (e.g. recently Petalas et al. 2020) have been developing constitutive models incorporated the influence of the principal stress axes rotations, it gives rise to models that are complicated and difficult to calibrate. Therefore, the development of the advanced constitutive model must be informed by more advanced testing devices, such as HCTA.

- **Soil conditions.** Although the saturated sand under high number of cycles is generally assumed to have a drained behaviour such as API (2014), the performance of monopile in saturated sand is practically undrained during high-frequency cycles. The predicted stress paths are somehow in relation to the pore pressure buildup and reduction in cyclic shear modulus. The assessment of the stress paths in undrained condition would need to be included. The recent developed SANISAND-MSu constitutive model based on SANISAND-MS to simulate the undrained behaviour of sand appears to be a good choice (Liu et al., 2020). In addition, the influence of partial drainage condition can’t be ignored, whereas it is difficult to apply in the lab (tests are typically conducted in either drained or undrained condition). The PDCAM model proposed by Jostad et al. (2015) is expected to be used to account for the partially drained cyclic response.

### 7.5.3 Suggestions for HCTA testing

This research presented some new experimental data which investigated the influence of cyclic loading amplitudes and angle of principal stress rotation on granular soil. However, it could be further enhanced in many ways:

- **Stress conditions.** The new proposed formula to predict the multiaxial strain accumulation in Chapter 6 was based on a limited number of HCTA tests in the same initial stress condition, in which the influence of confining pressure, void ratio, and stress ratio were considered according to the empirical correlations by Wichtmann (2016). The next testing programme should be designed to obtain more rigorous experimental evidence in terms of the influence of these factors on multiaxial soil response.
– **Soil conditions.** As discussed in Section 7.5.2, the granular soil used in this research was a clear limitation for many European ocean sites. Different soil types such as clay therefore need to be investigated for the next step. The laboratory exploration at the soil element scale should also accommodate the cases for undrained conditions. These experimental observations in different types of soil and stress conditions through HCTA will benefit the validation of the constitutive models to build confidence in their application for design.

### 7.6 Closure

This thesis, for the first time, shed light on the soil response under FE-identified actual stress paths to inform monopile foundation design of offshore wind farms. Optimisation of the soil testing procedure has been conducted and the importance of considering the rotation of principal stress axes was highlighted. The main outcome at current stage of this research along with the extended work as suggested in Section 7.5, will make great contribution to the improvement of monopile design.
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