Sensor Networks

Distributional Substitution for Inter-Sensor Distances in Random Fields

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Abstract—The distance between wireless sensors in random fields is crucial for performance analysis and sensor network deployment. However, the exact distribution models are normally of great complexity and can hardly lead to closed-form analytics for most cases. In this study, we investigate the inter-sensor distance distribution in random fields, propose a polynomial inter-sensor distance distributional substitute, and develop two strategies for distributional parameter mapping for different application scenarios. Simulation results presented in this paper verify the effectiveness and efficiency of the low-complexity distributional substitution technique. The verified analyses given in this paper can help to provide mathematically tractable performance metrics for wireless sensor networks (WSNs) where sensors are randomly distributed over two-dimensional space.

Index Terms—Inter-point distance distribution, distributional substitution, wireless sensor network, random field.

I. INTRODUCTION

Wireless sensor networks (WSNs) introduce unprecedented dynamism, driven by the mobility and agility of wireless sensors, leading to frequent fluctuations in network topology. This dynamism is particularly accentuated in short-range WSNs and drone-assisted WSNs. The associated complexity necessitates a modeling approach that is both flexible and adaptive. In this context, stochastic geometry has been proven to be an indispensable tool, offering unique capabilities to tackle the distinctive analytical challenges of WSNs [1]. It provides a flexible and accurate framework for modeling, analyzing, and optimizing the performance of WSNs [2]. Among the broad range of studies on stochastic WSNs, investigations into distance distributions of point processes are particularly significant for advancing research in remote sensing and data communications [3]. The inter-sensor distance distribution of wireless sensors distributed in random fields dominates the reliability of WSNs and suggests how wireless sensors shall be deployed. The pioneering examination of distances in uniformly random networks is documented in [4], where the distance distribution of the nearest neighbor to a random point in the Poisson field is elucidated. Exploring inter-node distance, [5] considers a fixed number of nodes independently distributed over a ball of arbitrary dimensions. Additionally, [6] delves into the distance distribution towards a reference point in the Poisson field. As part of a comprehensive investigation, [7] surveys and summarizes the distance distributions inherent in commonly used spatial point processes.

While closed-form probability density functions (PDFs) and cumulative distribution functions (CDFs) for the exact distance distribution model between two random points have been derived for some stochastic point processes [8], leveraging these exact distribution models for performance analyses and optimization for WSNs is rather tricky and often leads to mathematical intractability [9]. The inherent difficulty arises from the intricate forms of the PDFs and CDFs of these exact distance distributions, which incorporate inverse trigonometric functions and other specialized mathematical functions. Consequently, the integral forms of analytical results are seldom closed. Therefore, they can hardly yield substantial insights into the studied WSNs and fail to explicitly unveil the relation between network performance and sensor deployment strategies. To address these challenges, approximation techniques, e.g., Taylor series expansions and asymptotic analysis have been widely adopted [3], [10]. However, Taylor series based approximations necessitating numerical methods to determine the number of summative terms are less feasible for modeling the real-time dynamism of practical WSNs, while asymptotic analysis often fails in modeling moderate-sized networks due to edge effects and may produce invalid probability measures, limiting their practical utility.

In this regard, this paper aims to enhance the mathematical tractability of analyses and optimization for stochastic WSNs by proposing an approximate distance distribution model in the polynomial form, which can effectively capture the statistical characteristics of a variety of exact distributions of inter-sensor distances. Unlike asymptotic analysis that relies on idealized conditions and may lose accuracy in finite-scale scenarios, the proposed polynomial model directly approximates exact distributions across the entire parameter range. For the polynomial model, we propose two strategies tailored to different application scenarios for distributional parameter mapping.

II. PRELIMINARIES OF INTER-SENSOR DISTANCE DISTRIBUTION AND ITS SUBSTITUTION

In this paper, we assume that an undetermined number of random points, representing wireless sensors, are distributed over a finite disk centered at the origin with radius R, denoted as $\mathbb{C}_R \subset \mathbb{R}^2$. The location generation of these random points abides by a homogeneous

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two-dimensional Poisson point process (PPP) with constant density λ, which is denoted as $\Pi(\mathbb{C}_R)$ and is a first-order approximation of the spatial distribution of nodes in WSNs where the heights of deployed sensors are negligible compared to the perimeter of WSN. For simplicity, we denote the undetermined number of random points by N, which is a random number distributed by the Poisson distribution: $\mathsf{Poisson}(\lambda\pi R^2)$. For $n \in \mathcal{N} = \{1, 2, \ldots, N\}$, we denote the location of the nth point as $\mathbf{p}(r_n, \theta_n)$ through the two-dimensional polar coordinate, where $r_n \in [0, R]$ is the distance from point n to the origin, and $\theta_n \in [0, 2\pi)$ is the angle of the link connecting point n and the origin relative to the horizontal. In accordance with the homogeneity and isotropy of PPP, the PDFs and CDFs of both distance r_n and angle θ_n to the origin are given by $f_r(r) = 2r/R^2$ and $F_r(r) = r^2/R^2$, where $r \in [0, R]$, and $f_\theta(\theta) = 1/(2\pi)$ and $F_\theta(\theta) = \theta/(2\pi)$, where $\theta \in [0, 2\pi)$ [6].

From the formulations given above, for two randomly selected points in $\mathbb{C}_R$, say point n and k, the Euclidean distance between them can be expressed by the law of cosines as $L_{n,k} = L_{k,n} = ||\mathbf{p}(r_n, \theta_n) - \mathbf{p}(r_k, \theta_k)|| = \sqrt{r_n^2 + r_k^2 - 2r_n r_k \cos(\theta_n - \theta_k)}$, which is termed the inter-point distance and is obviously a dependent random number distributed over $[0, 2R]$ on the other two independent random numbers involved. The distribution of distance L between a pair of random points has been extensively studied in different subjects. Resorting to Croften's fixed-point theorem and the mean value theorem, the exact CDF and PDF of L in the homogeneous Poisson field have been derived to be [11]

$$\begin{cases} F_L(L) = \frac{2}{\pi}\arcsin\left(\frac{L}{2R}\right) - \frac{L\sqrt{4R^2-L^2}}{2\pi R^2} - \frac{L^3}{4\pi R^3}\sqrt{4 - \frac{L^2}{R^2}} + \frac{2L^2}{\pi R^2}\arccos\left(\frac{L}{2R}\right) \\ f_L(L) = \frac{4L}{\pi R^2}\left[\arccos\left(\frac{L}{2R}\right) - \frac{L}{2R}\sqrt{1 - \left(\frac{L}{2R}\right)^2}\right] \end{cases} \tag{1}$$

for $L \in [0, 2R]$. Due to the complexity of the inter-point distance distribution given in (1), it is a great challenge to have closed-form performance analysis and optimization when applying these exact expressions. Thus, it brings difficulties in revealing the relation between performance metrics and network parameters in WSNs.

Enlightened by the observation that most integrable functions multiplying a polynomial will still be integrable [12], we employ the distributional substitution technique and propose polynomial forms of CDF and PDF to approximate the original inter-point distance distribution formulated in (1). The polynomial approximation is developed from the perspective of CDF and PDF in the forms of

$$\begin{cases} \hat{F}_L(L) = \sum_{t=1}^{T} \varphi_t \left(\frac{L}{2R}\right)^t \\ \hat{f}_L(L) = \frac{\mathrm{d}\hat{F}_L(L)}{\mathrm{d}L} = \sum_{t=1}^{T} \frac{t\varphi_t}{2R}\left(\frac{L}{2R}\right)^{t-1} \end{cases}, \tag{2}$$

for $L \in [0, 2R]$, and $\{\varphi_t\}_{t=1}^T$ are T constrained parameters subject to $\sum_{t=1}^T \varphi_t = 1$ to guarantee the normativity of a legitimate distribution model. These T constrained parameters can be adapted in a way that fits different distribution models with closed set $[0, 2R]$. The number of constrained parameters T in itself can also be used to manage the trade-off between approximation accuracy and computational complexity. It is apparent that a larger T results in more accurate results, but requires more computing resources as there exist more polynomial terms that need to be processed. However, owing to the special feature of polynomial, increasing T does not affect the mathematical tractability of the distributional substitute. Therefore, with the tractable approximate distribution, the researchers interested in the inter-sensor distances will only need to consider how to figure out the T constrained parameters $\{\varphi_t\}_{t=1}^T$ to approximate the exact distributions more accurately.

III. DISTRIBUTIONAL PARAMETER MAPPING

The accuracy of substitution in our formative polynomial PDF and CDF is contingent upon the judicious configuration of T constrained parameters $\{\varphi_t\}_{t=1}^T$. In the sequel, we introduce two strategies for different application scenarios to establish the mapping relationship between the original distribution and the substituted distribution.

A. Moment Matching Method

We can determine $\{\varphi_t\}_{t=1}^T$ by the moment matching method [13]. Specifically, from (1), we can first derive the mth moment of L abiding by the exact distance distribution, denoted as $\mathbb{E}_{L\sim f_L}\{L^m\}$:

$$\mathbb{E}_{L\sim f_L}\{L^m\} = \int_0^{2R} f_L(L)L^m \mathrm{d}L = \frac{2R^m\Gamma(2+m)}{\Gamma\left(2+\frac{m}{2}\right)\Gamma\left(3+\frac{m}{2}\right)}, \tag{3}$$

where $\Gamma(x) = \int_0^\infty z^{x-1}\exp(-z)\mathrm{d}z$ is the gamma function, representing the Euler's integral of the second kind.

Proof: By (1), the mth moment of the exact distance distribution L can be expanded as

$$\begin{aligned} \mathbb{E}_{L\sim f_L}\{L^m\} = &\underbrace{\int_0^{2R} L^m\left(\frac{4L}{\pi R^2}\right)\arccos\left(\frac{L}{2R}\right)\mathrm{d}L}_{=I_1} \\ &\underbrace{-\int_0^{2R} L^m\left(\frac{2L^2}{\pi R^3}\right)\sqrt{1-\left(\frac{L}{2R}\right)^2}\mathrm{d}L}_{=I_2}. \end{aligned} \tag{4}$$

By expanding the integral expression, integrals I_1 and I_2 can be evaluated separately in order to obtain the closed form expression of (4). Through the method of integration by substitution for definite integrals, we can substitute $\phi = L/(2R)$ into I_1 to obtain the following expression in closed form:

$$I_1 = \frac{2^{m+4}R^m}{\pi}\int_0^1 \phi^{m+1}\arccos(\phi)\mathrm{d}\phi \overset{\text{(a)}}{=} \frac{2^{m+4}R^m\Gamma\left(\frac{3+m}{2}\right)}{\sqrt{\pi}(m+2)^2\Gamma\left(1+\frac{m}{2}\right)}, \tag{5}$$

where the integral solved through (a) is obtained by combining Eqs. (4.523.3) and (4.523.4) in [12]. In a similar way, we can substitute $\cos(\varpi) = L/(2R)$ into I_2 to obtain the closed form of I_2 as

$$\begin{aligned} I_2 &= -\frac{2^{m+4}R^m}{\pi}\int_0^{\frac{\pi}{2}}(\sin(\varpi))^2(\cos(\varpi))^{m+2}\mathrm{d}\varpi \\ &\overset{\text{(a)}}{=} -\frac{2^{m+3}R^m}{\pi}\mathsf{B}\left(\frac{3}{2}, \frac{3+m}{2}\right) \overset{\text{(b)}}{=} -\frac{2^{m+2}R^m\Gamma\left(\frac{3+m}{2}\right)}{\sqrt{\pi}\Gamma\left(3+\frac{m}{2}\right)}, \end{aligned} \tag{6}$$

where the integral solved through (a) is in accordance with Eq. (3.623.1) in [12], and $\mathsf{B}(\cdot,\cdot)$ is the beta function, representing the Euler integral of the first kind; (b) can be deduced by the relation between the beta function and the gamma function, i.e., $\mathsf{B}(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$, and identity $\Gamma(3/2) = \sqrt{\pi}/2$.

Adding (5) and (6) together and performing algebraic manipulations give the closed-form expression of $\mathbb{E}_{L\sim f_L}\{L^m\} = I_1 + I_2$ as in (3). ∎

Similarly, the mth moment of L abiding by the polynomial distributional substitute, denoted as $\mathbb{E}_{L\sim \hat{f}_L}\{L^m\}$ can be derived as

$$\mathbb{E}_{L\sim \hat{f}_L}\{L^m\} = \int_0^{2R} \hat{f}_L(L)L^m\mathrm{d}L = (2R)^m\left(\sum_{t=1}^{T}\frac{t\varphi_t}{m+t}\right). \tag{7}$$

Remark 1: As shown in (7), the mth moment of L abiding by the polynomial approximate distance distribution can be derived in a closed and polynomial form without any special function, $\forall\ m \geq 1$, which is a computing advantage brought by the proposed polynomial distributional substitution technique.

Solving the moment matching equation set given as

$$\left\{ \underset{L\sim f_L}{\mathbb{E}} \{L^m\} = \underset{L\sim \hat{f}_L}{\mathbb{E}} \{L^m\} \right\}_{m=1}^{T-1} \quad \text{and} \quad \sum_{t=1}^{T} \varphi_t = 1 \tag{8}$$

directly yields the solution of $\{\varphi_t\}_{t=1}^{T}$:

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_T \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} & \cdots & \frac{T}{1+T} \\ \frac{1}{3} & \frac{2}{4} & \cdots & \frac{T}{2+T} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{(T-1)+1} & \frac{2}{(T-1)+2} & \cdots & \frac{T}{2T-1} \\ 1 & 1 & \cdots & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{\Gamma(3)}{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{7}{2}\right)} \\ \frac{\Gamma(4)}{2\Gamma(3)\Gamma(4)} \\ \vdots \\ \frac{\Gamma(2+(T-1))}{2^{(T-1)-1}\Gamma\left(2+\frac{T-1}{2}\right)\Gamma\left(3+\frac{T-1}{2}\right)} \\ 1 \end{bmatrix}. \tag{9}$$

Remark 2: As shown in (9), taking advantage of the explicit form of the higher moments, the computational complexity predominantly comes from inverting a $T \times T$ square matrix for solving the linear equation system. It also becomes clear from (9) that the moment matching method is suited for being used by computers for application scenarios where the explicit expressions of higher moments exist, and the only challenge is to invert a $T \times T$ matrix. The $T \times T$ matrix contains the coefficients as the constraints for matching the moments of the original and approximate distributions and is non-singular. There exist developed algorithms to invert matrices in this form. Assuming the classical Gauss-Jordan elimination technique is used to invert the matrix, the computational complexity is $O\left(T^3\right)$.

B. Empirical Evidence Based Method

The moment matching method relies on the prior knowledge of the original distance distribution, i.e., (1). However, this prior knowledge might not always be accessible in real-world WSNs [14]. In order to generalize the proposed polynomial distributional substitution technique and enhance its practicality, we propose the generic resort termed the empirical evidence based method in this subsection, which is suited to be applied for cases where the original distance distribution is unknown or is too complex to process. Specifically, the prerequisite of using the empirical evidence based method is the availability of plenty of samples drawn from the original distance distribution, which can be collected by inspecting a stochastic WSN when the number of wireless sensors is large. Given a stochastic WSN consisting of N sensors, there exist $(N-1)^2$ inter-sensor distances $\{L_{n,k}\}_{n,k\in N, n\neq k}$, and $L_{n,k} = L_{k,n}$. Subsequently, we can randomly take ϵ independent samples from these $(N-1)^2$ distances without replacement to form a sampling vector $\mathbf{s} = [S_1, S_2, \ldots, S_\epsilon]^T$, where[1] $\epsilon \leq (N-1)^2$. With $\mathbf{s} = [S_1, S_2, \ldots, S_\epsilon]^T$, we can construct the mth sample moment by $A_m = (1/\epsilon)\sum_{i=1}^{\epsilon} S_i^m$, which can be used to estimate $\underset{L\sim f_L}{\mathbb{E}} \{L^m\}$ without bias, $\forall\ m = 1, 2, \ldots, T-1$ and imitate the moment matching method when the exact expressions of moments are not obtainable.

[1]On the other hand, if N is small in very special cases or sampling is too difficult to conduct, we can rely on the parametric bootstrapping technique to mimic the sampling process. As the formalism of the polynomial substitution model is known, the parametric bootstrapping technique can generate $\epsilon \geq (N-1)^2$ dependent samples from the $(N-1)^2$ independent samples and form a re-sampling vector with ϵ elements.

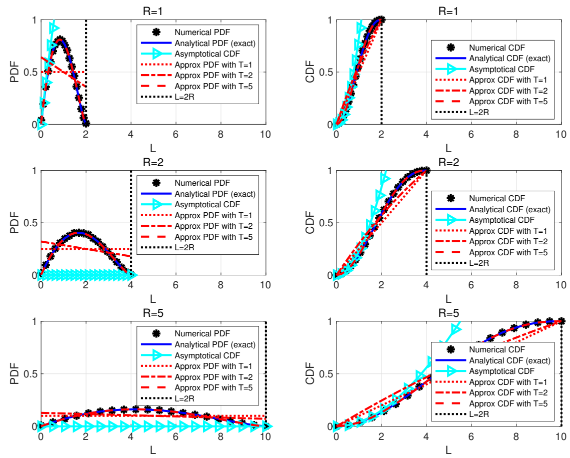

Fig. 1. PDF and CDF yielded by the distributional parameter sets via the moment matching method, given different setups of R and L.

Consequently, solving the equation set for matching the approximate expressions of moments and the sample moments yields the empirical solution of $\{\varphi_t\}_{t=1}^{T}$ through the ϵ collected samples:

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_T \end{bmatrix} = \begin{bmatrix} R & \frac{4R}{3} & \cdots & \frac{2RT}{1+T} \\ \frac{4R^2}{3} & 2R^2 & \cdots & \frac{4R^2T}{2+T} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{(2R)^{T-1}}{T} & \frac{2(2R)^{T-1}}{T+1} & \cdots & \frac{(2R)^{T-1}T}{2T-1} \\ 1 & 1 & \cdots & 1 \end{bmatrix}^{-1} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{T-1} \\ 1 \end{bmatrix}. \tag{10}$$

Remark 3: The computational complexity is comprised mainly of the computation of sample moments and the inverse of the $T \times T$ square matrix. Again, assuming the schoolbook elementary operations are applied, the former renders $O\left(T\epsilon\varsigma^2 \cdot 2^{\lceil \log(T-1) \rceil}\right)$, where ς is the maximum number of digits of precision, at which the exponentiation is to be computed. The latter results in $O\left(T^3\right)$, same as the moment matching method. Consequently, the overall computational complexity is $O\left(T^3 + T\epsilon\varsigma^2 \cdot 2^{\lceil \log(T-1) \rceil}\right)$.

Remark 4: Note that the empirical evidence based method does not rely on the formalism of the original inter-sensor distance distribution, i.e., $F_L(L)$ and $f_L(L)$, and is thereby a generic method suited for any spatial distribution for wireless sensors in random fields as long as a set of observed samples are available. This method can thus generalize the proposed polynomial distributional substitution technique to other complex circumstances. Without loss of generality, when the distribution space is not a disk, distance distribution limits of 0 and $2R$ can be replaced by $L_{\min}$ and $L_{\max}$ that are the minimum and maximum distances between two sensors in the random field.

IV. SIMULATION VERIFICATION AND DISCUSSION

First, to assess the efficacy of the moment matching method, we present visualizations of the PDF and CDF generated by different distributional parameter sets using this method in Fig 1. Additionally, we conduct an asymptotic analysis for the distribution of inter-sensor distance as $R \to \infty$, applying the approximations $\arcsin(x) \approx x$ and $\arccos(x) \approx \frac{\pi}{2}$ as $x \to 0$. This yields the asymptotic CDF $F_L^A(L) \approx \frac{L^2}{R^2} - \frac{L^3}{2\pi R^3}$ and the corresponding asymptotic PDF $f_L^A(L) \approx \frac{2L}{R^2} - \frac{3L^2}{2\pi R^3}$. We vary the distribution radius R and the number of polynomial

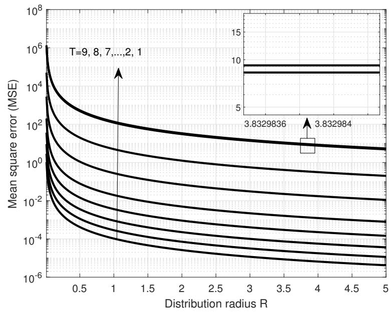

Fig. 2. MSE versus distribution radius yielded by the distributional parameter sets via the moment matching method, given different T.

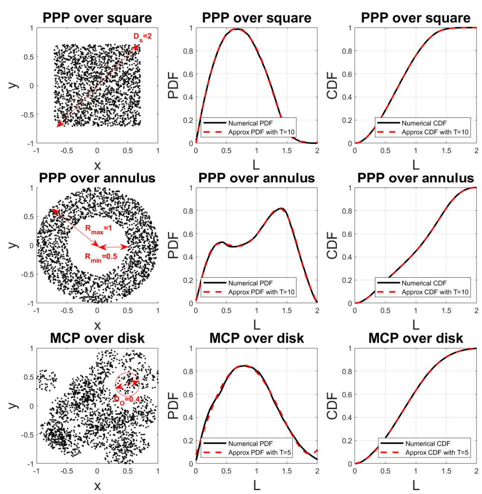

Fig. 3. PDF and CDF yielded by the distributional parameter sets via the empirical evidence based method for various WSNs.

terms T to examine their impacts on substitution accuracy. The results presented in Fig 1 provide a direct and insightful depiction of the moment matching method's accuracy for various combinations of $R = 1, 2, 5$ and $T = 1, 2, 5$. Notably, the proposed approach consistently outperforms the asymptotic method, even in scenarios with large coverage areas and a small number of polynomial terms.

Then, to quantitatively assess the efficiency of the moment matching method, we depict the mean square error (MSE) as a function of the distribution radius R for varying polynomial terms T in Fig. 2. The results showcased in this figure distinctly reveal the impacts of both R and T on the efficiency of the moment matching method. Increasing the distribution radius R leads to a reduction in MSE, indicating more accurate substitutions, albeit with a marginal effect. This trend can be attributed to the sharper changes in the PDF curve within a smaller range of L for smaller R, making it more challenging to capture its statistical features using the polynomial form. Conversely, the flattened PDF curves associated with larger values of R are more regular and easier to mimic. Hence, elevating the number of polynomial terms T is a beneficial strategy for achieving accurate substitutions. With an increase in the number of terms, the MSE experiences a substantial reduction by several orders of magnitude. However, a conspicuous saturation phenomenon emerges, hindering further improvement in substitution accuracy by indefinitely increasing T.

Finally, to study the generality of the polynomial distributional substitution technique for inter-sensor distances in complex WSNs, we select three representative cases with the specifications as follows to simulate the use of empirical evidence based parameter mapping:

- Homogeneous PPP over a square with diagonal length $D_S = 2$ (an inscribed square of a disk with radius $R = 1$);
- Homogeneous PPP over an annulus with internal and external radii $R_{\min} = 0.5$ and $R_{\max} = 1$;
- Matérn cluster process over a disk with radius $R = 1$, and the diameter of circular cluster $D_O = 0.4$.

The realizations of the above cases plus the corresponding PDFs and CDFs yielded by fitting using the polynomial forms through the empirical evidence based method are presented in Fig. 3. The fitted PDFs and CDFs are also compared to the numerical benchmarks. From the simulation results presented in this figure, we have demonstrated the good generality of the empirical evidence based method for complex application scenarios in practice, as the analytical and numerical PDFs and CDFs closely match each other over large ranges of L. Even the bimodal characteristic of the PDF for the annulus distribution space can be accurately retained through the proposed polynomial fitting using empirical samples.

Through the simulation results presented above, the effectiveness and efficiency of the distributional substitution technique for inter-sensor distances enabled by both parameter mapping strategies have been verified, making it suitable for diverse WSN and IoT scenarios, including environment monitoring, disaster response, and various smart city applications. By applying this verified distributional substitution technique, the mathematical tractability of performance analysis and optimization for WSNs can be improved.

REFERENCES

[1] X. Lu, M. Salehi, M. Haenggi, E. Hossain, and H. Jiang, "Stochastic geometry analysis of spatial-temporal performance in wireless networks: A tutorial," *IEEE Communications Surveys & Tutorials*, vol. 23, no. 4, pp. 2753–2801, 2021.

[2] ——, "Stochastic geometry analysis of spatial-temporal performance in wireless networks: A tutorial," *IEEE Commun. Surv. Tutor.*, vol. 23, no. 4, pp. 2753–2801, 2021.

[3] L. Valentini, A. Giorgetti, and M. Chiani, "Density estimation in randomly distributed wireless networks," *IEEE Trans. Wirel. Commun.*, vol. 21, no. 8, pp. 6687–6697, 2022.

[4] M. Haenggi, "On distances in uniformly random networks," *IEEE Trans. Inf. Theory.*, vol. 51, no. 10, pp. 3584–3586, 2005.

[5] S. Srinivasa and M. Haenggi, "Distance distributions in finite uniformly random networks: Theory and applications," *IEEE Trans. Veh. Technol.*, vol. 59, no. 2, pp. 940–949, 2010.

[6] A. Behnad and N. C. Beaulieu, "Best neighbor communication in a Poisson field of nodes," *IEEE Trans. Veh. Technol.*, vol. 64, no. 2, pp. 818–823, 2015.

[7] D. Moltchanov, "Distance distributions in random networks," *Ad Hoc Networks*, vol. 10, no. 6, pp. 1146–1166, 2012.

[8] S. Lellouche and M. Souris, "Distribution of distances between elements in a compact set," *Stats*, vol. 3, no. 1, pp. 1–15, 2019.

[9] S.-J. Tu and E. Fischbach, "Random distance distribution for spherical objects: General theory and applications to physics," *J. Phys. A Math. Gen.*, vol. 35, no. 31, p. 6557, 2002.

[10] R. Arshad and L. Lampe, "Stochastic geometry analysis of user mobility in RF/VLC hybrid networks," *IEEE Trans. Wirel. Commun.*, vol. 20, no. 11, pp. 7404–7419, 2021.

[11] J. Tang, G. Chen, J. P. Coon, and D. E. Simmons, "Distance distributions for Matérn cluster processes with application to network performance analysis," in *Proc. IEEE ICC*, Paris, France, 2017, pp. 1–6.

[12] D. Zwillinger and A. Jeffrey, *Table of Integrals, Series, and Products*. Elsevier Science, 2007.

[13] S. Atapattu, C. Tellambura, and H. Jiang, "A mixture gamma distribution to model the SNR of wireless channels," *IEEE Trans. Wirel. Commun.*, vol. 10, no. 12, pp. 4193–4203, 2011.

[14] M. Zhao, L. Mason, and W. Wang, "Empirical study on human mobility for mobile wireless networks," in *Proc. IEEE MILCOM*, San Diego, USA, 2008, pp. 1–7.