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Adaptive Input and Parameter Estimation with Application to Engine Torque Estimation

Jing Na, Guido Herrmann, Richard Burke and Chris Brace

Abstract—This paper presents two estimation methods for systems with unknown time-varying input dynamics. By defining auxiliary filtered variables, an invariant manifold is derived and used to drive the input estimator with only one tuning parameter. Exponential error convergence to a small compact set around the origin can be proved. Robustness against noise is studied and compared with two well-known schemes. Moreover, when the input dynamics to be estimated are parameterized in a quasilinear form with unknown parameters, the proposed idea is further investigated to estimate the associated unknown time-varying parameters. The algorithms are tested by considering the torque estimation of internal combustion engines (ICEs). Comparative simulation results based on a benchmark engine simulation model show satisfactory transient and robustness performance.

I. INTRODUCTION

Engine torque is crucial in automotive powertrain control [1] and other applications, e.g. online estimation of in-car parameters such as mass [2], which has been studied for small-sized passenger cars in Bristol. In a laboratory, the engine torque can be calculated based on the in-cylinder pressure. However, accurate measurements of the in-cylinder pressure via transducers may not be technically feasible in production engines. This motivates the work on the real-time torque estimation using alternative available information from the stock engine sensors [3-5]. This is possible because the engine torque can be considered either as an unknown input variable in the engine rotation dynamics or as a function of other measurable variables, e.g. air/fuel mass flow, engine speed [4].

For the torque estimation, Kalman filters [3] and sliding-mode observers [5] are widely used. The authors of [4] proposed a PI-like torque observer for the sake of a simple implementation. In [6], the estimation was transformed to a tracking control problem and then solved via optimal control methods. Moreover, the principle of unknown input observers has also been used to address this issue [7-9]. In particular, a high-gain observer was presented in [8] and its adaptive

version was studied in [9], where an observer structure with several tuning parameters was used. On the other hand, the engine torque estimation can also be studied via parameter estimation methodologies when one parameterizes the engine rotation dynamics in a quasilinear form. However, classical parameter estimation schemes (e.g. gradient and least squares method) may fail to obtain satisfactory performance, because of the fast time-varying parameters in the torque generation dynamics [10]. Nevertheless, the set-membership estimation algorithm for time-varying parameters [7] depends on the knowledge for precise bounds on the unknown parameters. This assumption may be stringent in practical applications.

With the wish to develop simple, fast and robust online estimation of the engine torque, we will present two new estimation schemes for generic systems with unknown input dynamics, i.e. the input estimator and the time-varying parameter estimator. Filter operations are at first introduced for the measurable variables, and an ideal invariant manifold [11] is constructed to design the input estimator, which exponentially converges to a small compact set around this manifold. Only one tuning parameter needs to be selected. Comparisons to two other input observers in [7] are studied with respect to their performance and robustness. In the second part, the unknown input is parameterized in a quasilinear form with some time-varying parameters (this is particularly true for the engine torque [4]). Thus, our recent idea for parameter estimation [12-14] is further exploited and modified to estimate such time-varying dynamics. In this case, the structural identifiable condition can be presented as a generic persistent excitation (PE) condition [15] and thus can be online verified in this paper. Finally, the proposed algorithms are used to address the indicated engine torque estimation and numerically tested based on a benchmark engine model [16] of a four-cylinder ICE. Simulation results show very encouraging results with small estimation errors even in the presence of measurement noise.

II. UNKNOWN INPUT ESTIMATION

We study the unknown input estimation of a generic system

$$\dot{x} = y + u \quad (1)$$

where $x \in \mathbb{R}^n$ is the system output, $y \in \mathbb{R}^n$ is the known dynamic, and $u \in \mathbb{R}^n$ is the unknown time-varying input, which will be estimated.

System (1) is a generic representation for the automotive engines, which has been widely used to describe various engine dynamics [9], e.g. the cylinder injection dynamics and the torque generation as shown in [8]. In this paper, the

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estimation of the indicated torque for ICEs will be considered as a concrete example in Section IV.

This section first presents theoretical developments of a new input estimator to estimate the unknown input u in (1). The variables x and y are measurable, and the derivative of u is bounded, i.e. $\sup_{t \geq 0} \|\dot{u}\| \leq \bar{h}$ holds for a constant $\bar{h} > 0$.

A. Input Estimation

We define the filtered variables x_f, y_f of x, y as

$$\begin{cases} k\dot{x}_f + x_f = x, & x_f(0) = 0 \\ k\dot{y}_f + y_f = y, & y_f(0) = 0 \end{cases} \quad (2)$$

where $k > 0$ is a filter parameter.

An ideal invariant manifold [11] is then presented, which will be used to design the input estimator.

Lemma 1: Consider system (1) and filter operation (2), the manifold $(x - x_f)/k - y_f - u = 0$ and the associated variable

$$\beta = (x - x_f)/k - y_f - u, \quad (3)$$

then the variable β is ultimately bounded for any finite $k > 0$, and decreases in an exponential sense. Moreover,

$$\lim_{k \rightarrow 0} \lim_{t \rightarrow \infty} \{\beta\} = 0,$$

i.e. $(x - x_f)/k - y_f - u = 0$ is an invariant manifold for $k \rightarrow 0$.

Proof: From (2)-(3), the time derivative of β is given by

$$\dot{\beta} = \frac{\dot{x} - \dot{x}_f}{k} - \dot{y}_f - \dot{u} = -\frac{1}{k}(\beta + k\dot{u}) \quad (4)$$

Select a Lyapunov function as $V_\beta = \beta^T \beta / 2$ such that

$$\dot{V}_\beta = -\frac{1}{k} \beta^T \beta + \beta^T \dot{u} \leq -\frac{1}{k} V_\beta + \frac{k}{2} \bar{h}^2 \quad (5)$$

This implies that $V_\beta(t) \leq e^{-t/k} V_\beta(0) + k^2 \bar{h}^2 / 2$ holds and thus $\beta(t)$ will exponentially converge to a small compact set defined by $\|\beta(t)\| = \sqrt{2V_\beta(t)} \leq \sqrt{\beta^2(0)e^{-t/k} + k^2 \bar{h}^2}$. Hence, $\lim_{t \rightarrow \infty} \beta(t) = k\bar{h}$ which vanishes for small k and/or constant input (i.e. $\bar{h} = 0$). Moreover, for $k \rightarrow 0$, the fact $\lim_{k \rightarrow 0} \lim_{t \rightarrow \infty} \beta(t) = 0$ holds, i.e. $\beta = 0$ is an invariant manifold for $k \rightarrow 0$. \diamond

The ideal invariant manifold provides a mapping from the available variables (x, x_f, y_f) to the unknown input u . Thus, it can be used to design the estimator for u without knowing any information of \dot{x} . Based on the coordinate manifold (3), a feasible input estimator of u is

$$\hat{u} = \frac{x - x_f}{k} - y_f \quad (6)$$

Before showing the convergence of input estimator (6), we give an alternative insight for this estimator by applying the low-pass filter $(\bullet)_f = [\bullet]/(ks + 1)$ of (2) on both sides of (1) as

$$\frac{s}{ks + 1}[x] = \frac{1}{ks + 1}[y] + \frac{1}{ks + 1}[u] \quad (7)$$

This together with the first equation of (2) gives

$$\dot{x}_f = \frac{x - x_f}{k} = y_f + u_f \quad (8)$$

where u_f is the filtered version of the unknown input, which is given by $k\dot{u}_f + u_f = u$. Then it follows from (6) and (8) that $\hat{u} = u_f$. Consequently, the estimation error $v = u - \hat{u}$ is

$$v = u - \hat{u} = \left(1 - \frac{1}{ks + 1}\right)u = \frac{ks}{ks + 1}[u] \quad (9)$$

Clearly, the estimation error can be minimized by setting the filter parameter k sufficiently small and it can even vanish for constant input, i.e. $\dot{u} = 0$. This can be summarized as

Proposition 1: For system (1) with input estimator (6), the estimation error v is bounded by $\|v(t)\| \leq \sqrt{v^2(0)e^{-t/k} + k^2 \bar{h}^2}$, and thus $u \rightarrow \hat{u}$ holds for $k \rightarrow 0$ or $\bar{h} \rightarrow 0$.

Proof: The error dynamics (9) are given in the time-domain as

$$\dot{v} = -\frac{1}{k}v + \dot{u} \quad (10)$$

Select a Lyapunov function as $V = v^T v / 2$, then it follows that $\|v(t)\| \leq \sqrt{v^2(0)e^{-t/k} + k^2 \bar{h}^2}$, and $v(t) \rightarrow 0$ for $k \rightarrow 0, \bar{h} \rightarrow 0$. \diamond

B. Comparison to Other Methods

The performance of estimator (6) is compared with two other input estimators proposed in [8].

1) Sliding mode estimator

The following sliding mode observer [8, 17] is given by

$$\dot{\hat{x}} = y + \lambda \text{sign}(v_x) \quad (11)$$

where $v_x = x - \hat{x}$ is the output error and $\lambda > \bar{h}$ is a positive constant. Then the output error is given as $\dot{v}_x = u - \lambda \text{sign}(v_x)$, thus v_x will reach the sliding mode surface $v_x = 0$ in finite time. Using the notion of the equivalent control, one can obtain that $u = \lambda \text{sign}(v_x)$. To reduce the chattering, a low pass filter is adopted to give the following input estimator

$$\hat{u} = \frac{1}{ks + 1}[\lambda \text{sign}(v_x)] \quad (12)$$

In this case, the estimator error of (12) can be obtained as $v = u - \hat{u} = u - \frac{1}{ks + 1}[u] = \frac{ks}{ks + 1}[u]$, which is the same as (9).

Proposition 2: For system (1) with input estimator (12), the estimation error v is bounded by $\|v(t)\| \leq \sqrt{v^2(0)e^{-t/k} + k^2 \bar{h}^2}$, and thus $u \rightarrow \hat{u}$ holds for $k \rightarrow 0$ or $\bar{h} \rightarrow 0$.

Proof: The proof is similar to Proposition 1 and is omitted.

Remark 1: From (11), the sliding mode observer reaches an invariant manifold $v_x = 0$ by high-frequency switching action in the term $\lambda \text{sign}(v_x)$. This leads to chattering phenomenon [8]. In this case, the bandwidth of filter (12) should be set to compromise between the error performance v and the smoothness of \hat{u} , i.e. k cannot be set arbitrarily small.

2) Dirty differentiation estimator

The so-called ‘dirty derivative’ of x [8] is given as

$$\dot{x} \approx \frac{s}{ks + 1}[x] = \frac{x}{k} - \frac{1}{k(ks + 1)}[x] \quad (13)$$

Hence, the input estimator can be designed as

$$\hat{u} = \frac{x}{k} - \frac{1}{k(ks+1)}[x] - y \quad (14)$$

One essential difference between the proposed estimator (6) and (14) is that the known variable y is filtered by a low pass filter in (2), which is not used in (14).

Proposition 3: For system (1) with input estimator (14), the estimation error v is bounded by $\|v(t)\| \leq \sqrt{v^2(0)e^{-t/k} + k^2\tilde{\lambda}^2}$ with $\sup_{t \geq 0} \|\ddot{x}\| \leq \tilde{\lambda}$, so that $u \rightarrow \hat{u}$ holds for $k \rightarrow 0$ or $\tilde{\lambda} \rightarrow 0$.

Proof: From (1) and (14), we get the estimation error as

$$v = u - \hat{u} = \left(s - \frac{s}{ks+1} \right) [x] = \frac{ks^2}{ks+1} [x] \quad (15)$$

This can be presented in the time-domain as $\dot{v} = -v/k + \ddot{x}$.

Then one may obtain $\|v(t)\| = \sqrt{2V(t)} \leq \sqrt{v^2(0)e^{-t/k} + k^2\tilde{\lambda}^2}$. \diamond

Remark 2: It should be noted that the estimation error of (14) depends on the upper bound of the second order derivative of system output \ddot{x} , while the estimation errors of (6) and (12) are determined by the upper bound of the first order derivative of the unknown input \dot{u} only. Thus, the estimator (14) may be sensitive to output measurement noise, which will be studied.

C. Robustness Analysis

This subsection will address the robustness of the above estimators against measurement noise. Denote w_1, w_2 as the noise signals perturbing x and y , respectively, then the measured variables that are used for the input estimators are

$$\bar{x} = x + w_1, \bar{y} = y + w_2 \quad (16)$$

It is assumed that the noise signals are bounded by $\|w_1\| \leq \eta_1$, $\|w_2\| \leq \eta_2$, $\|\dot{w}_1\| \leq \eta_3$, $\|\dot{w}_2\| \leq \eta_4$ for constants $\eta_1, \eta_2, \eta_3, \eta_4 > 0$.

1) Proposed estimator

The proposed estimator (6) with (2) is modified as

$$\begin{cases} k\dot{\bar{x}}_f + \bar{x}_f = \bar{x}, & \bar{x}_f(0) = 0 \\ k\dot{\bar{y}}_f + \bar{y}_f = \bar{y}, & \bar{y}_f(0) = 0 \end{cases} \quad (17)$$

$$\hat{u} = \frac{\bar{x} - \bar{x}_f}{k} - \bar{y}_f \quad (18)$$

Consider (6) and (16), then (18) can be further presented as

$$\hat{u} = \dot{\bar{x}}_f - \bar{y}_f = \dot{x}_f + \dot{w}_{1f} - y_f - w_{2f} \quad (19)$$

where \dot{w}_{1f} and w_{2f} are the filtered version of \dot{w}_1, w_2 in terms of filter $1/(ks+1)$. Then the estimation error of (17)~(18) is

$$v = \left(1 - \frac{1}{ks+1} \right) u + \frac{1}{ks+1} ([w_2] - s[w_1]) = \frac{ks}{ks+1} [u] + \frac{1}{ks+1} ([w_2] - s[w_1]) \quad (20)$$

which can be rewritten in the time-domain as

$$\dot{v} = -\frac{1}{k}v + \dot{u} + \frac{1}{k}(w_2 - \dot{w}_1) \quad (21)$$

Proposition 4: For system (1) with bounded measurement noise w_1, w_2 , the estimation error of (17)~(18) is bounded by

$$\|v(t)\| \leq \sqrt{v^2(0)e^{-t/k} + k^2[\tilde{h} + (\eta_3 + \eta_2)/k]^2}.$$

The proof is similar to that of Proposition 1 and is omitted.

2) Sliding mode estimator

From (1) and (16), the measured dynamics are given as

$$\dot{\bar{x}} = \dot{w}_1 + \bar{y} - w_2 + u \quad (22)$$

and the sliding mode observer for (22) is designed by

$$\dot{\hat{x}} = \bar{y} + \lambda \text{sign}(v_x) \quad (23)$$

where $v_x = \bar{x} - \hat{x}$ is the output error, which can be given as $\dot{v}_x = u + \dot{w}_1 - w_2 - \lambda \text{sign}(v_x)$. By choosing $\lambda > \tilde{h} + \eta_3 + \eta_2$, then $v_x = 0$ holds in finite-time, and the equivalent control is $u + \dot{w}_1 - w_2 = \lambda \text{sign}(v_x)$, so that the estimator error of (12) is

$$v = u - \frac{1}{ks+1} ([u - w_2 + s[w_1]]) = \frac{ks}{ks+1} [u] + \frac{1}{ks+1} ([w_2] - s[w_1]) \quad (24)$$

Proposition 5: For system (1) with bounded measurement noise w_1, w_2 , the estimation error of (12) with (23) is bounded

$$\|v(t)\| \leq \sqrt{v^2(0)e^{-t/k} + k^2[\tilde{h} + (\eta_3 + \eta_2)/k]^2}.$$

From Proposition 4 and Proposition 5, it is shown that the robustness of the proposed estimator (6) is comparable to the sliding mode estimator (12), while the potential chattering of (12) can be avoided in (6).

3) Dirty differentiation estimator

The estimator (14) with noise signals w_1, w_2 is modified as

$$\hat{u} = \frac{\bar{x}}{k} - \frac{1}{k(ks+1)} [\bar{x}] - \bar{y} = \frac{s}{ks+1} [x] + \frac{s}{ks+1} [w_1] - y - w_2 \quad (25)$$

then the estimation error is obtained as

$$v = \left(s - \frac{s}{ks+1} \right) [x] - \frac{s}{ks+1} [w_1] + w_2 = \frac{ks^2}{ks+1} [x] - \frac{s}{ks+1} [w_1] + w_2 \quad (26)$$

which can be presented in the time-domain as

$$\dot{v} = -\frac{1}{k}v + \ddot{x} + \dot{w}_2 + \frac{1}{k}(w_2 - \dot{w}_1) \quad (27)$$

Proposition 6: For system (1) with bounded measurement noise w_1, w_2 , the estimation error of (25) is bounded by

$$\|v(t)\| \leq \sqrt{v^2(0)e^{-t/k} + k^2[\tilde{\lambda} + \eta_4 + (\eta_3 + \eta_2)/k]^2}.$$

It is shown that the ultimate error bound of the estimator (25) depends also on the upper bound $\|\dot{w}_2\| \leq \eta_4$.

III. INPUT PARAMETER ESTIMATION

In this section, we will study the unknown input estimation from the point of view of parameter estimation, where the unknown dynamics to be estimated can be parameterized in a quasilinear form with unknown time-varying parameters

$$\dot{x} = \Phi(x, y, z)\Theta(t) + y \quad (28)$$

where $\Phi(\bullet) \in \mathbb{R}^{n \times p}$ is the known regressor matrix, which is a smooth function of x, y and the bounded exogenous variable z ; $\Theta(t) \in \mathbb{R}^p$ is the unknown time-varying parameter vector to be estimated. Then the estimation of the unknown input $u = \Phi(\bullet)\Theta(t)$ can be achieved when Θ is precisely estimated. The estimation of unknown dynamics can be significantly

facilitated when they include some well-understood dynamics of measurable data. This is particularly true for the engine torque estimation, because the engine torque is a time-varying function of online measured intake air mass-flow and engine speed [3-5].

Assumption 1: The time derivative of the unknown time-varying vector Θ is bounded by $\|\dot{\Theta}(t)\| \leq \varpi$ for a constant $\varpi > 0$.

We will propose a new parameter estimation by further exploiting the invariant manifold and our previous results [12-14]. Define the filtered variables x_f, y_f in (2) and Φ_f as

$$k\dot{\Phi}_f + \Phi_f = \Phi, \quad \Phi_f(0) = 0 \quad (29)$$

We apply a filter $1/(ks+1)$ of (2) and (29) on (28), then

$$\frac{s}{ks+1}[x] = \frac{1}{ks+1}[\Phi\Theta] + \frac{1}{ks+1}[y] \quad (30)$$

Consider the first equation of (2) and (29), and the Swapping Lemma [15], one can represent (30) as

$$\frac{x-x_f}{k} = y_f + \Phi_f\Theta - \frac{k}{ks+1}[\Phi_f\dot{\Theta}] \quad (31)$$

Since $\Phi(\bullet)$ is a smooth function of bounded variables x, y, z , then Φ_f is bounded as $\|\Phi_f\| \leq \mu$ for constant $\mu > 0$. Moreover, we assume $\|\dot{\Theta}(t)\| \leq \varpi$. Thus, for any finite $k > 0$, the term

$\zeta = \frac{k}{ks+1}[\Phi_f\dot{\Theta}]$ is bounded (i.e. $\|\zeta\| \leq \gamma$ for constant $\gamma > 0$), which can be considered as a small ‘noise’ perturbing the ideal manifold $[(x-x_f)/k - y_f - \Phi_f\Theta] = 0$.

Now, we define auxiliary variables P and Q as

$$\begin{cases} \dot{P} = -\ell P + \Phi_f^T \Phi_f, & P(0) = 0 \\ \dot{Q} = -\ell Q + \Phi_f^T [(x-x_f)/k - y_f], & Q(0) = 0 \end{cases} \quad (32)$$

where $\ell > 0$ is another design parameter.

Finally, other auxiliary vectors W_1 and W_2 are defined as

$$W_1 = P\hat{\Theta} - Q \quad (33)$$

$$W_2 = \Phi_f^T \Phi_f \hat{\Theta} - \Phi_f^T [(x-x_f)/k - y_f] \quad (34)$$

where $\hat{\Theta}$ is the estimated value of Θ .

From (31)~(34) and the definition $\tilde{\Theta} = \Theta - \hat{\Theta}$, it follows

$$W_1 = -P\tilde{\Theta} + \psi \quad (35)$$

$$W_2 = -\Phi_f^T \Phi_f \tilde{\Theta} + \Phi_f^T \zeta \quad (36)$$

where $\psi = \int_0^t e^{-\ell(t-r)} \Phi_f^T(r) \zeta(r) dr$ is a bounded residual error, i.e. $\|\psi\| \leq \|\Phi_f\| \|\zeta\| / \ell = \mu\gamma / \ell$.

Remark 3: The variables W_1, W_2 contain the information of the estimation error $\tilde{\Theta}$. Moreover, W_1 is a filtered version of W_2 in terms of $1/(s+\ell)$. This filter operation can introduce an ‘averaging’ effect, which can improve the robustness against noise but may reduce the ability to track fast time-varying parameters. On the other hand, W_2 contains the instant error information, which may be sensitive to noise, but

is crucial to estimate fast time-varying dynamics.

A. Constant Learning Gain

We first present the following adaptive law to derive $\hat{\Theta}$ as

$$\dot{\hat{\Theta}} = -\Gamma(W_1 + \kappa W_2) \quad (37)$$

where $\Gamma > 0$ is a constant gain, and $\kappa > 0$ is a constant chosen to tradeoff the performance and robustness.

Lemma 2 [12-14]: If the regressor matrix Φ in (28) is PE, i.e. $\int_t^{t+\tau} \Phi(r)\Phi^T(r)dr \geq \varepsilon I, \forall t \geq 0$ for $\tau > 0, \varepsilon > 0$, then the matrix P in (32) is positive definite (e.g. $\lambda_{\min}(P) > \sigma_1 > 0$).

Proposition 7: For system (28) with the regressor matrix Φ being PE, the estimation error $\tilde{\Theta}$ of (37) converges to a compact set $\|\tilde{\Theta}\| \leq \sqrt{\frac{[m\mu^2\gamma^2(\kappa^2+1/\ell^2)+m\varpi^2/\lambda_{\min}^2(\Gamma)]\lambda_{\max}(\Gamma^{-1})}{2(\sigma_1-3/2m)\lambda_{\min}(\Gamma^{-1})}}$.

Proof: Select a Lyapunov function as $V = \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta} / 2$, then \dot{V} is obtained along (35)~(37) as

$$\begin{aligned} \dot{V} &= -\tilde{\Theta}^T P \tilde{\Theta} + \tilde{\Theta}^T \psi - \kappa \tilde{\Theta}^T \Phi_f^T \Phi_f \tilde{\Theta} + \kappa \tilde{\Theta}^T \Phi_f^T \zeta + \tilde{\Theta}^T \Gamma^{-1} \dot{\Theta} \\ &\leq -(\sigma_1 - 3/2m) \|\tilde{\Theta}\|^2 + \frac{m\mu^2\gamma^2}{2\ell^2} + \frac{m\mu^2\gamma^2\kappa^2}{2} + \frac{m\varpi^2}{2\lambda_{\min}^2(\Gamma)} \quad (38) \\ &\leq -\alpha V + \rho \end{aligned}$$

where $\alpha = 2(\sigma_1 - 3/2m) / \lambda_{\max}(\Gamma^{-1})$, $\rho = m\mu^2\gamma^2(\kappa^2 + 1/\ell^2) / 2 + m\varpi^2 / 2\lambda_{\min}^2(\Gamma)$ are positive constants for $m \geq 3/2\sigma_1$. This further implies

$$\|\tilde{\Theta}(t)\| \leq \sqrt{2V(t) / \lambda_{\min}(\Gamma^{-1})} \leq \sqrt{\left(\|\tilde{\Theta}(0)\|^2 \lambda_{\max}(\Gamma^{-1}) e^{-\alpha t} + 2\rho / \alpha\right) / \lambda_{\min}(\Gamma^{-1})}.$$

Thus the estimation error $\tilde{\Theta}$ exponentially converges to a compact set defined in Proposition 7. \diamond

Remark 4: For any constant parameter vector Θ , i.e. $\dot{\Theta} = 0$, it follows from (31)~(36) that $\zeta = \psi = 0$ is true, then global exponential convergence of (37) can be achieved.

Remark 5: As shown in Lemma 2, the classical PE condition is required to prove the estimation convergence, and can be numerically online validated by calculating the minimum eigenvalue of P and testing for $\lambda_{\min}(P) > \sigma_2 > 0$ in this paper. In this respect, the nontrivial validation of the uniquely identifiable property [7] is avoided.

B. Time-varying Learning Gain

From (35)~(36), it is shown that the amplitudes of perturbing disturbances ψ and $\Phi_f^T \zeta$ in W_1, W_2 depend on the filtered regressor Φ_f . In this case, a constant learning gain Γ may not be sufficient to achieve satisfactory performance. Inspired by the Least-squares method and our previous work [12-14], we provide a time-varying gain to compensate for the effects of P and $\Phi_f^T \Phi_f$ in the adaptation. For this purpose, we define another matrix K as

$$\dot{K} = \ell K - K \Phi_f^T \Phi_f K \quad (39)$$

where $K^{-1}(0) = K_0 > 0$ is the initial condition. By using the matrix equality $\frac{d}{dt} K K^{-1} = \dot{K} K^{-1} + K \frac{d}{dt} K^{-1} = 0$, one can obtain

$$K = [e^{-\ell t} K_0 + \int_0^t e^{-\ell(t-r)} \Phi_f^T(r) \Phi_f(r) dr]^{-1} = [e^{-\ell t} K_0 + P]^{-1} \quad (40)$$

It is shown in (40) that K exponentially converges to the inverse of regressor matrix P . Thus, K can be included in the following adaptive law to compensate for the effect of P

$$\dot{\hat{\Theta}} = -\Gamma K (W_1 + \kappa W_2) \quad (41)$$

where $\Gamma > 0$ is a constant scalar.

From (40) and the fact that Φ is PE, we can obtain the boundedness of the gain matrix K as

$$\gamma_1 I \leq K(t) \leq \gamma_2 I \quad (42)$$

where $\gamma_1 = 1/(\lambda_{\min}(K_0) + \mu^2)$, $\gamma_2 = e^{\ell t} / \varepsilon$ are positive constants.

Proposition 8: For system (28) with the regressor matrix Φ being PE, the estimation error $\tilde{\Theta}$ of (41) converges to a

$$\text{compact set } \|\tilde{\Theta}\| \leq \sqrt{\frac{\gamma_2 [m\mu^2\gamma^2(\kappa^2 + 1/\ell^2) + m\bar{\omega}^2 / \gamma_1^2\Gamma^2]}{\gamma_1(2\sigma_1 + \ell / \gamma_2\Gamma - \mu^2 / \Gamma - 3/m)}}$$

Proof: Select a Lyapunov function as $V = \tilde{\Theta}^T K^{-1} \tilde{\Theta} / 2\Gamma$, then

$$\begin{aligned} \dot{V} &= \frac{1}{\Gamma} \tilde{\Theta}^T K^{-1} \dot{\tilde{\Theta}} + \frac{1}{2\Gamma} \tilde{\Theta}^T \dot{K}^{-1} \tilde{\Theta} = \frac{1}{\Gamma} \tilde{\Theta}^T K^{-1} \dot{\tilde{\Theta}} - \tilde{\Theta}^T P \tilde{\Theta} + \tilde{\Theta}^T \psi - \kappa \tilde{\Theta}^T \Phi_f^T \Phi_f \tilde{\Theta} \\ &\quad + \kappa \tilde{\Theta}^T \Phi_f^T \zeta + \frac{1}{2\Gamma} \tilde{\Theta}^T (-\ell K^{-1} + \Phi_f^T \Phi_f) \tilde{\Theta} \\ &\leq -(\sigma_1 + \ell / 2\gamma_2\Gamma - \mu^2 / 2\Gamma - 3/2m) \|\tilde{\Theta}\|^2 + \frac{m\mu^2\gamma^2}{2\ell^2} + \frac{m\mu^2\gamma^2\kappa^2}{2} + \frac{m\bar{\omega}^2}{2\gamma_1^2\Gamma^2} \end{aligned} \quad (43)$$

Hence, the upper bound of $\tilde{\Theta}$ can be obtained from (43) and $\|\tilde{\Theta}\| \leq \sqrt{2\Gamma V / \gamma_2}$. Compared to (38), it is clear that ℓ and Γ can be set larger to improve the error convergence. \diamond

IV. APPLICATION TO ENGINE TORQUE ESTIMATION

The suggested algorithms are used to study the estimation of the produced torque for internal combustion engines.

A. Engine Model

The engine model is based on a commercially available benchmark simulation model [16] built in MATLAB Simulink for a four-cylinder spark ignition ICE. The model is developed from the physics-based model in [18] and [19] using the thermodynamics, fluid mechanics and rigid body mechanics. The mathematical equations [20] are briefly given as:

$$\text{Intake manifold: } \dot{P}_m = \frac{RT_m}{V_m} (\dot{m}_{ai} - \dot{m}_{ao}) \quad (44)$$

$$\text{Throttle body: } \dot{m}_{ai} = f(\theta) g(P_m) \quad (45)$$

$$f(\theta) = k_{r0} + k_{r1}\theta + k_{r2}\theta^2 + k_{r3}\theta^3$$

$$g(P_m) = \begin{cases} 1 & \text{for } P_m \leq \frac{P_{amb}}{2} \\ \frac{2}{P_{amb}} \sqrt{P_{amb} P_m - P_m^2} & \text{for } P_m > \frac{P_{amb}}{2} \end{cases}$$

$$\text{Injection: } \dot{m}_{ao} = k_{m0} + k_{m1}NP_m + k_{m2}NP_m^2 + k_{m3}N^2P_m \quad (46)$$

$$\text{Engine rotation speed: } J\dot{N} = T_{iq} - T_{ld} \quad (47)$$

$$\begin{aligned} T_{iq} &= k_{q0} + k_{q1}m_a + k_{q2}(AFR) + k_{q3}(AFR)^2 + k_{q4}\sigma + k_{q5}\sigma^2 \\ &\quad + k_{q6}N + k_{q7}N^2 + k_{q8}N\sigma + k_{q9}\sigma m_a + k_{q10}\sigma^2 m_a \end{aligned}$$

where R is the gas constant of air, V_m is the manifold volume, T_m is the manifold air temperature, \dot{m}_{ai} is mass flow rate of air into manifold, \dot{m}_{ao} is mass flow rate of air out of manifold; θ is the throttle angle, P_{amb} and P_m are the ambient pressure and manifold pressure; N is the engine angular speed, J is the engine rotational moment of inertia, AFR is the air to fuel ratio and σ is the sparking advance; T_{iq} is the produced engine torque and T_{ld} is the applied load torque. The detail for these parameters of this engine model can be found in [16, 20], which are not presented for conciseness.

The problem to be addressed is to estimate the torque T_{iq} based on the engine rotation dynamics in (47). To guarantee stable operation of the engine and to ensure the required excitation for the estimators (e.g. PE condition), a PI control is used to regulate the throttle angle such that the engine speed can track a given square reference between 2000-3000rpm. Moreover, several other control loops are well configured to maintain the spark advance and the air-fuel ratio at their optimal values. Moreover, the load torque is measurable and assumed to be a function of engine speed $T_{ld} = k_{l1} + k_{l2}N + k_{l3}N^2$, where k_{l1}, k_{l2}, k_{l3} are all constants in this study as [20]. Fig. 1 provides the engine operating profiles (e.g. indicated torque, cylinder mass flow and engine speed).

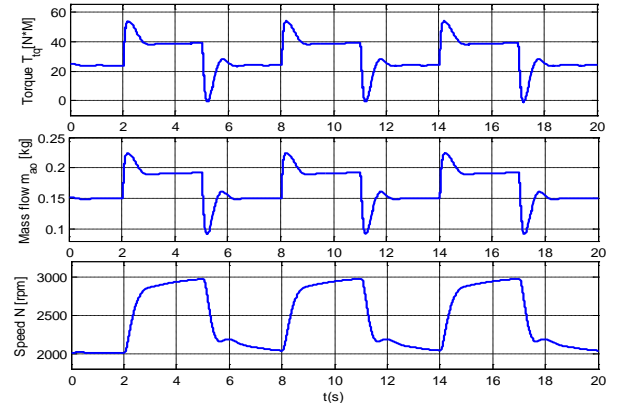


Fig.1 Dynamics of engine system.

B. Unknown Input Estimator

The input estimator (6) presented in Section II is first tested and compared with (12) and (14) with $k = 0.0001$ and $\lambda = 64$. Fig. 2 gives the torque estimation performance with different approaches. It is shown that the proposed estimator (6) achieves better profiles than the sliding-mode estimator (12) and the dirty differentiation based estimator (14). The estimator (14) performs poorly because its error bound depends on the engine acceleration \ddot{N} as shown in Proposition 3. Nevertheless, the chattering (middle subplot of Fig.2) limits the applicability of the sliding mode observer (12). Moreover, the robustness against random noise $w_1 = N(0, 0.2)$ and $w_2 = N(0, 0.1)$ are also tested, and a similar conclusion can be obtained (the results are not plotted due to the limited space). However, in this case, the filter parameter k is increased to $k = 0.05$ to eliminate the effect of high-frequency noise.

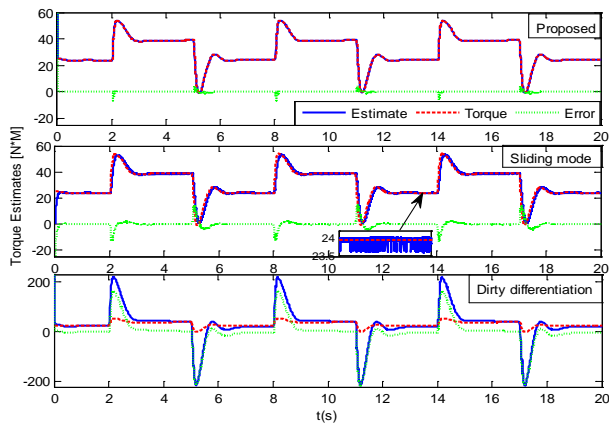


Fig.2 Torque estimation with different Input observers.

C. Torque Parameter Estimator

The parameter estimators proposed in Section III are tested for the torque estimation. From (44)~(47), the indicated torque can be taken as a function of mass flow m_a and engine speed N as $T_{iq} = \theta_1(t)m_a + \theta_2(t)N$, where $\theta_1(t)$ and $\theta_2(t)$ are the lumped unknown time-varying parameters to be estimated. The parameter in (32) is set as $\ell = 5$, and the adaptive gain is $\Gamma = \text{diag}([10,10])$. Then the estimators (37) and (41) are simulated for $\kappa = 0.5$ and $\kappa = 0$. Simulation results are illustrated in Fig.3, from where one can find that the estimator (41) with a time-varying learning gain K can obtain better performance than estimator (37) with a constant learning gain. In particular, the estimator (41) with $\kappa = 0.5$ (i.e. W_2 is activated) can achieve fairly good performance. These simulations validate the claims of Proposition 8, i.e. the inclusion of W_2 with instant error information $\Phi_f^T \Phi_f \hat{\Theta}$ can help to track time-varying dynamics, and the time-varying gain K can improve the error convergence.

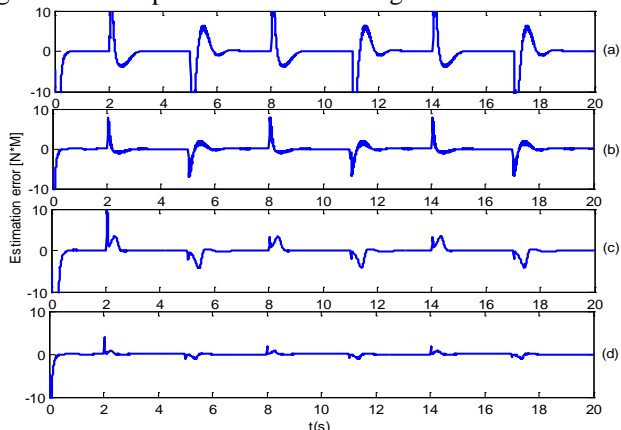


Fig.3 Torque Estimation errors: (a) estimator (37) with $\kappa = 0$; (b) estimator (37) with $\kappa = 0.5$; (c) estimator (41) with $\kappa = 0$; (d) estimator (41) with $\kappa = 0.5$.

V. CONCLUSION

This paper addresses the estimation of unknown system input and time-varying dynamics by employing the invariant manifold method. With appropriate low-pass filters, a simple input estimator with one tuning parameter is first presented,

which has the same robustness as the sliding mode observer but can achieve smoother performance. The problem is further studied from the point of view of time-varying parameter estimation, where a novel adaptation is investigated. In this respect, the required PE condition can be online verified in this paper. The salient feature is that time-varying dynamics can be precisely tracked. The application to the indicated engine torque estimation has been discussed and tested based on a commercially available benchmark engine model of a four cylinder spark ignition ICE. Future work will focus on practical validation of the algorithms via realistic engine tests.

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