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Discussion of "Undrained Young's Modulus of Fine-Grained Soils" by B. Casey, J. T. Germaine, N. O. Abdulhadim, N. S. Kontopoulos and C. A. Jones

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## Introduction

The discussers read the recent paper by Casey et al. (2015) with great interest. The authors' large body of work, developing geotechnical correlations (e.g., Casey and Germaine, 2013 and Casey et al. 2013) and $CK_0UC$ testing (e.g., Sheahan et al. 1996) is acknowledged. The authors' aims of clarifying the reported ranges of $E_u/s_u$ values according to the effects of overconsolidation ratio, (OCR) and shear stress ratio (Duncan and Buchignani, 1976; Ladd et al. 1977) is very useful for traditional settlement analysis approaches (e.g., D'Appolonia, et al. 1971). The discussers have also had an interest in normalising the stress-strain curves of fine-grained materials and are particularly interested in the $K_0$ effect and the *applied shear stress ratio* as defined in Casey et al. (2015).

## Simple models for $K_0$-data

In Vardanega and Bolton (2011, 2012) equation (1) was presented for shifting upwards the stress-strain curves obtained from an initial state $K_0$ = 1 (note that while equation 1 uses an average value of the exponent, i.e., $b$ = 0.6, which is based on the analysis of a large database, the value of the exponent will vary for individual soil tests)

$$\frac{\tau_{mob}-\tau_0}{s_u} = 0.5\left(\frac{\gamma}{\gamma_{M=2}}\right)^{0.6} \quad (1)$$

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where $\tau_{mob}$ = current maximum shear stress, $\tau_0$ = maximum shear stress in the specimen before undrained shearing commences, $s_u$ = undrained shear strength, $\gamma$ = shear strain and $\gamma_{M=2}$ is the shear strain required to mobilise $0.5s_u$. A similar approach (equation 2) was also successfully used by Li and Bolton (2014) to shift any shear modulus reduction curve (with variable shear modulus, $G$ being less than or equal to the maximum shear modulus, $G_{max}$)

$$\tau_{mob} = \tau_0 + G\gamma \tag{2}$$

A form of equation (3), shown in this discussion with a generalised exponent $b$, was used by Vardanega (2012) to study the applicability of the power-law function to describe the stress-strain behaviour of $K_0$-consolidated materials. Figure 1 shows the digitised data of a reconstituted low plasticity "North Sea Clay" originally reported in Jardine et al. (1984, 1986). Figure 2 shows power curves of the form given as equation (3), fitted to the data between the range $0.2 < B < 0.8$ (analogous to the approach used in Vardanega and Bolton, 2011). Figure 3 shows that equation 3 can reasonably predict the data shown on Figure 2, utilising the computed average exponent of $b = 0.52$ (see Table 1)

$$\frac{\tau_{mob}-\tau_0}{s_u-\tau_0} = B = 0.5\left(\frac{\gamma}{\gamma_{ref,K0}}\right)^b \tag{3}$$

where $\gamma_{ref,K0}$ is the $K_0$-modified reference strain.

Vardanega et al. (2012, 2013) presented data for a kaolin clay that suggested that $\gamma_{M=2}$ is positively correlated with $OCR$ (as is the exponent $b$): accepting some scatter. The regression relations developed are shown as equation 4 and equation 5 with accompanying statistical measures

$$log_{10}(\gamma_{M=2}) = 0.680log_{10}(OCR) - 2.395$$

$$R^2 = 0.81,\ n = 18,\ SE = 0.151,\ p < 0.001 \tag{4}$$

$$b = 0.011(OCR) + 0.371$$

$$R^2 = 0.59,\ n = 18,\ SE = 0.064,\ p < 0.001 \tag{5}$$

Figures 4 and 5 also suggest a relationship of $b$ varying with $OCR$ for the data studied in this discussion but, interestingly, $b$ seems to reduce with increasing $OCR$. The discussers would be interested to know whether any of the authors' datasets can also be successfully described using equation (3).

**Notation**

*The following symbols are used in this discussion:*

$B$ = applied shear stress ratio;

$b$ = an exponent;

$E_u/s_u$ = ratio of undrained modulus to undrained shear strength;

$G$ = shear modulus;

$G_{max}$ = maximum shear modulus;

$K_0$ = earth pressure coefficient at the start of a test;

$n$ = number of data-points used to generate a regression line;

$OCR$ = overconsolidation ratio;

$p$ = smallest level of significance that would lead to the rejection of the null hypothesis;

$R^2$ = coefficient of determination;

$SE$ = standard error of a regression;

$s_u$ = undrained shear strength;

$\gamma$ = shear strain;

$\gamma_{M=2}$ = shear strain required to mobilise $0.5s_u$;

$\gamma_{ref,K0}$ = $K_0$-modified reference strain;

$\varepsilon$ = axial strain;

$\tau_{mob}$ = current maximum shear stress (denoted $\tau$ in the paper under discussion); and

$\tau_0$ = maximum shear stress in the specimen before the undrained shearing commences.

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**Table 1: Test and curve fitting parameters shown on Figure 2 (*OCR* values quoted from Jardine et al. 1984)**

| Test identifier used in Jardine et al. (1984, 1986) | ***b*** | $\gamma_{ref,K0}$ | ***OCR*** |
|---|---|---|---|
| R1 | 0.661 | 0.000155 | 1 |
| R1.4 | 0.587 | 0.000369 | 1.4 |
| R2 | 0.522 | 0.000664 | 2.05 |
| R4 | 0.420 | 0.00195 | 3.73 |
| R8 | 0.425 | 0.00593 | 7.4 |

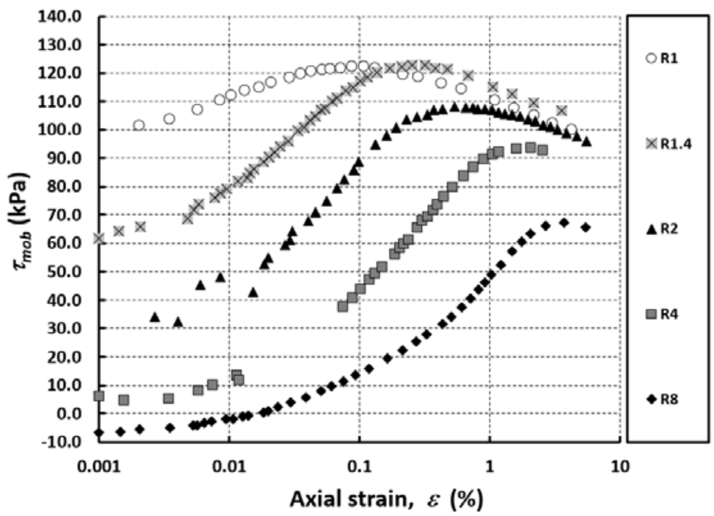


**Figure 1: Digitised triaxial data (data from Jardine et al. 1984, 1986)**

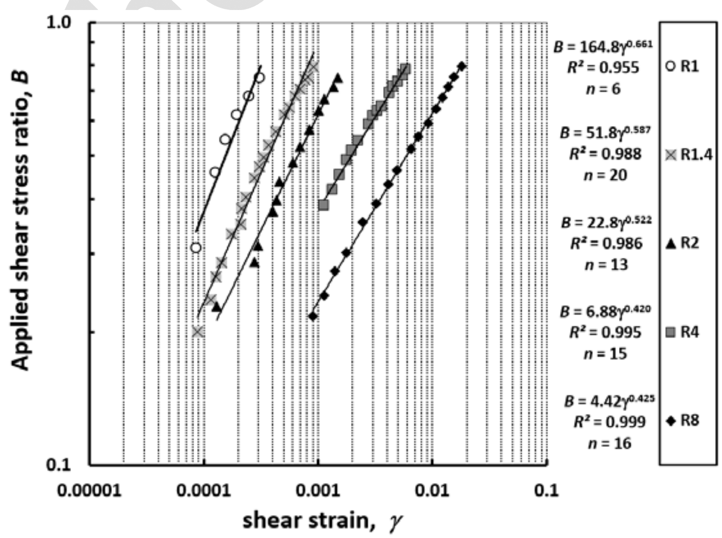


**Figure 2: Power-curve fits to the data from Figure 1 (curves fitted in the range $0.2 < B < 0.8$) (note that $\gamma$ is taken as 1.5 times the axial strain)**

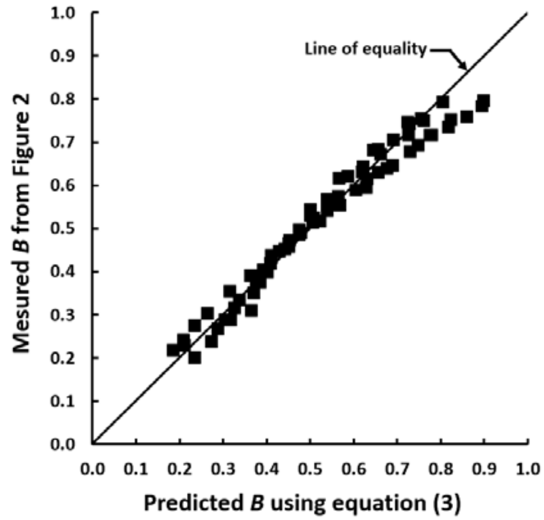


**Figure 3: Comparison of predicted values of *B* using equation 3 (with *b* = 0.52) with the measured values of *B* shown on Figure 2.**

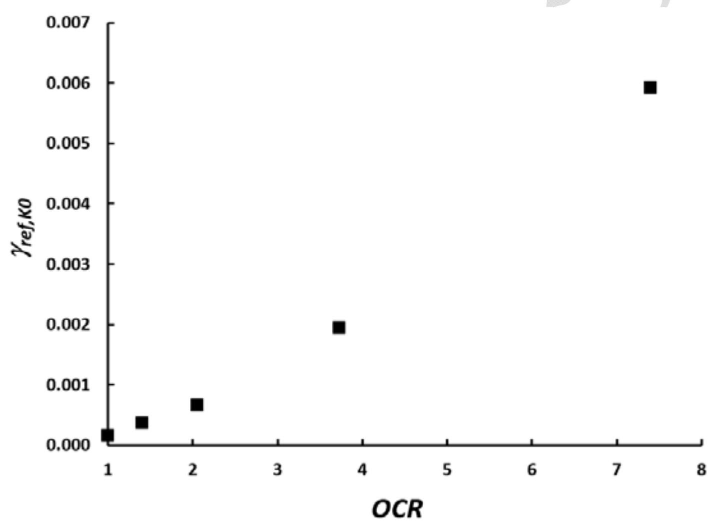


**Figure 4:** $\gamma_{ref,\ K0}$ **plotted against *OCR***

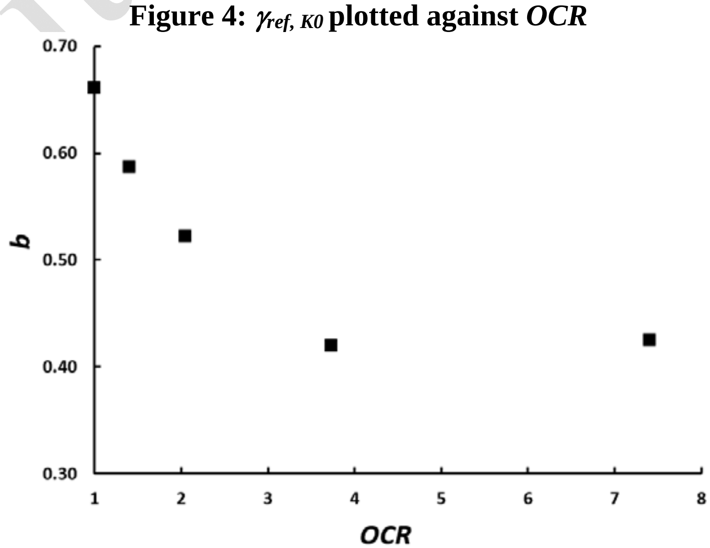


**Figure 5: *b* plotted against *OCR***