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## Appendix: Proofs of Results 2 and 3

### Proof of Result 2

*Result 2: Lottery tickets are not purchased outside the interval* $x_1 \in \left[ x_2^* - \frac{1-q}{q}, x_2^* + 1 \right]$.

**Proof:**

The value functions for not buying a lottery ticket is given by:

$$V_1^{l=0} = \begin{cases} u(x_1) \text{ if } x_1 < x_2^* \\ u(x_1 - p) + \eta \text{ if } x_1 \geq x_2^* \end{cases}$$

The expected value function for buying a ticket is given by:

$$E\left[V_1^{l=1}\right] = q \begin{Bmatrix} u\left(x_1 + \frac{1-q}{q}\right) \text{ if } x_1 < x_2^* - \frac{1-q}{q} \\ u(x_1 + \frac{1-q}{q} - p) + \eta \text{ if } x_1 \geq x_2^* - \frac{1-q}{q} \end{Bmatrix} + (1-q) \begin{Bmatrix} u\left(x_1 - 1\right) \text{ if } x_1 < x_2^* + 1 \\ u(x_1 - 1 - p) + \ln \eta \text{ if } x_1 \geq x_2^* + 1 \end{Bmatrix}$$

Now consider separately the incentive to buy a lottery ticket when cash-on-hand is below the interval and above the interval.

1) When

$$x_{1`} < x_2^* - (1-q)/q,$$

cash-on-hand in period 2 will be sufficiently low that even if the lottery is won, $x_2 < x_2^*$, and so the household does not buy the indivisible good, regardless of the lottery outcome. Thus, the expected value of buying a lottery ticket becomes:

$$E\left[V_1^{l=1}\right] = qu\left( x_1 + \frac{1-q}{q} \right) + (1-q)u\left(x_1 - 1\right)$$

The value of not buying becomes: $V_1^{l=0} = u\left(x_1\right)$. Since the gamble is actuarially fair and utility, u, is concave, the value of not buying a lottery ticket is always greater than the expected value of buying the lottery ticket:

$$V_1^{l=0} = u(x_1)$$
$$\geq qu\left(x_1 + \frac{1-q}{q}\right) + (1-q)u(x_1 - 1) = E\left[V_1^{l=1}\right].$$

2) ) When

$$x_1 > x_2^* + 1,$$

cash-on-hand in period 2 will be sufficiently high that even if the lottery is lost, $x_2 > x_2^*$, and so the household buys the indivisible good regardless of the lottery outcome. Thus, the expected value of buying a lottery ticket becomes:

$$E\left[V_1^{l=1}\right] = qu\left(x_1 + \frac{1-q}{q} - p\right) + (1-q)u(x_1 - 1 - p) + \eta$$

And the value of not buying becomes:

$$V_1^{l=0} = u(x_1 - p) + \eta$$

Since the gamble is actuarially fair and utility, u, is concave, the value of not buying the lottery ticket is always greater than the value of buying the ticket.

$$V_1^{l=0} = u(x_1 - p) + \eta$$
$$\geq qu\left(x_1 - p + \frac{1-q}{q}\right) + (1-q)u(x_1 - p - 1) + \eta = E\left[V_1^{l=1}\right],$$

**Proof of Result 3**

*Result 3: There exists a region,* $x_1 \in \left[\underline{x_1}, \overline{x_1}\right]$, *which contains* $x_2^*$ $\left(\underline{x_1} < x_2^* < \overline{x_1}\right)$*, in which the agent will purchase a lottery ticket.*

**Proof:**

We consider the incentive to buy a lottery ticket in the region of $x_2^*$. Define the difference in utility from purchasing the indivisible good and not purchasing it as

$\delta = u(x_2 - p) + \eta - u(x_2)$

We consider separately the incentive ε above and ε below $x_2^*$.

1) **Below** $x_2^*$: When

$$\begin{aligned} &x_2 = x_2^* - \varepsilon \\ &\textit{and so } \delta < 0 \end{aligned}$$

we can write the expected value of buying a lottery ticket as:

$$E\left[V_1^{l=1}\right] = q\left(u\left(x_2^* - \varepsilon + \frac{1-q}{q} - p\right) + \eta\right) + (1-q)u\left(x_2^* - \varepsilon - 1\right)$$

And the value of not buying a ticket as:

$$\begin{aligned} V_1^{l=0} &= u\left(x_2^* - \varepsilon\right) \\ &= q\left(u\left(x_2^* - \varepsilon - p\right) + \eta\right) - q\delta + (1-q)u\left(x_2^* - \varepsilon\right) \end{aligned}$$

$$\begin{aligned} E\left[V_1^{l=1} - V_1^{l=0}\right] = q&\left(u\left(x_2^* - \varepsilon + \frac{1-q}{q} - p\right) + \eta - u\left(x_2^* - \varepsilon - p\right) - \eta\right) \\ &+q\delta \\ &+(1-q)\left(u\left(x_2^* - \varepsilon - 1\right) - u\left(x_2^* - \varepsilon\right)\right) \end{aligned}$$

This is approximately equal to:

$$\begin{aligned} E\left[V_1^{l=1} - V_1^{l=0}\right] = q&\left(u'\left(x_2 - \varepsilon - p\right)\left(\frac{1-q}{q}\right)\right) \\ &+q\delta \\ &+(1-q)\left(-u'\left(x_2 - \varepsilon\right)\right) \end{aligned}$$

$$E\left[V_1^{l=1} - V_1^{l=0}\right] = (1-q)\left(u'(x_2-\varepsilon-p) - u'(x_2-\varepsilon)\right) + q\delta$$
$$= -p(1-q)u''(x_2-\varepsilon) + q\delta$$

As

$$\varepsilon \to 0,\ x_2 \uparrow x_2^*, \delta \uparrow 0$$

and

$$E\left[V_1^{l=1} - V_1^{l=0}\right] > 0$$

2) **Above $x_2^*$:** When

$$x_2 = x_2^* + \varepsilon$$
$$and\ \ so\ \ \delta > 0$$

$$E\left[V_1^{l=1}\right] = q\left(u\left(x_2^* + \varepsilon + \frac{1-q}{q} - p\right) + \eta\right) + (1-q)u\left(x_2^* + \varepsilon - 1\right)$$

The value of not buying a ticket is:

$$V_1^{l=0} = u\left(x_2^* + \varepsilon - p\right) + \eta$$
$$= q\left(u\left(x_2^* + \varepsilon - p\right) + \eta\right) + (1-q)\left(u\left(x_2^* + \varepsilon - p\right) + \eta\right)$$
$$= q\left(u\left(x_2^* + \varepsilon - p\right) + \eta\right) + (1-q)u\left(x_2^* + \varepsilon\right) + (1-q)\delta$$

$$E\left[V_1^{l=1} - V_1^{l=0}\right] = q\left(u\left(x_2^* + \varepsilon + \frac{1-q}{q} - p\right) + \eta - u\left(x_2^* + \varepsilon - p\right) - \eta\right)$$
$$+(1-q)\left(u\left(x_2^* + \varepsilon - 1\right) - u\left(x_2^* + \varepsilon\right)\right)$$
$$-(1-q)\delta$$

$$E\left[V_1^{l=1}-V_1^{l=0}\right]=q\left(u'\left(x_2+\varepsilon-p\right)\left(\frac{1-q}{q}\right)\right)$$
$$+\left(1-q\right)\left(-u'\left(x_2+\varepsilon\right)\right)$$
$$-\left(1-q\right)\delta$$

$$E\left[V_1^{l=1}-V_1^{l=0}\right]=\left(1-q\right)\left(u'\left(x_2+\varepsilon-p\right)-u'\left(x_2+\varepsilon\right)\right)-\left(1-q\right)\delta$$

$$=-p\left(1-q\right)u''\left(x_2+\varepsilon\right)-\left(1-q\right)\delta$$

As

$$\varepsilon\to 0,\ x_2\downarrow x_2^*,\delta\downarrow 0$$

and

$$E\left[V_1^{l=1}-V_1^{l=0}\right]>0$$