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## FOR ONLINE PUBLICATION

### Appendix: Proofs of Results 2 and 3

#### Proof of Result 2

*Result 2: Lottery tickets are not purchased outside the interval*  $x_1 \in \left[ x_2^* - \frac{1-q}{q}, x_2^* + 1 \right]$ .

#### Proof:

The value functions for not buying a lottery ticket is given by:

$$V_1^{l=0} = \begin{cases} u(x_1) & \text{if } x_1 < x_2^* \\ u(x_1 - p) + \eta & \text{if } x_1 \geq x_2^* \end{cases}$$

The expected value function for buying a ticket is given by:

$$E[V_1^{l=1}] = q \left\{ \begin{array}{l} u\left(x_1 + \frac{1-q}{q}\right) \text{ if } x_1 < x_2^* - \frac{1-q}{q} \\ u\left(x_1 + \frac{1-q}{q} - p\right) + \eta \text{ if } x_1 \geq x_2^* - \frac{1-q}{q} \end{array} \right\} + (1-q) \left\{ \begin{array}{l} u(x_1 - 1) \text{ if } x_1 < x_2^* + 1 \\ u(x_1 - 1 - p) + \ln \eta \text{ if } x_1 \geq x_2^* + 1 \end{array} \right\}$$

Now consider separately the incentive to buy a lottery ticket when cash-on-hand is below the interval and above the interval.

1) When

$$x_1 < x_2^* - (1-q)/q,$$

cash-on-hand in period 2 will be sufficiently low that even if the lottery is won,  $x_2 < x_2^*$ , and so the household does not buy the indivisible good, regardless of the lottery outcome.

Thus, the expected value of buying a lottery ticket becomes:

$$E[V_1^{l=1}] = qu\left(x_1 + \frac{1-q}{q}\right) + (1-q)u(x_1 - 1)$$

The value of not buying becomes:  $V_1^{l=0} = u(x_1)$ . Since the gamble is actuarially fair and utility,  $u$ , is concave, the value of not buying a lottery ticket is always greater than the expected value of buying the lottery ticket:

$$\begin{aligned}
V_1^{l=0} &= u(x_1) \\
&\geq qu\left(x_1 + \frac{1-q}{q}\right) + (1-q)u(x_1 - 1) = E[V_1^{l=1}].
\end{aligned}$$

2) ) When

$$x_1 > x_2^* + 1,$$

cash-on-hand in period 2 will be sufficiently high that even if the lottery is lost,  $x_2 > x_2^*$ , and so the household buys the indivisible good regardless of the lottery outcome. Thus, the expected value of buying a lottery ticket becomes:

$$E[V_1^{l=1}] = qu\left(x_1 + \frac{1-q}{q} - p\right) + (1-q)u(x_1 - 1 - p) + \eta$$

And the value of not buying becomes:

$$V_1^{l=0} = u(x_1 - p) + \eta$$

Since the gamble is actuarially fair and utility,  $u$ , is concave, the value of not buying the lottery ticket is always greater than the value of buying the ticket.

$$\begin{aligned}
V_1^{l=0} &= u(x_1 - p) + \eta \\
&\geq qu\left(x_1 - p + \frac{1-q}{q}\right) + (1-q)u(x_1 - p - 1) + \eta = E[V_1^{l=1}],
\end{aligned}$$

### Proof of Result 3

*Result 3: There exists a region,  $x_1 \in [\underline{x}_1, \bar{x}_1]$ , which contains  $x_2^*$  ( $\underline{x}_1 < x_2^* < \bar{x}_1$ ), in which the agent will purchase a lottery ticket.*

#### Proof:

We consider the incentive to buy a lottery ticket in the region of  $x_2^*$ . Define the difference in utility from purchasing the indivisible good and not purchasing it as

$$\delta = u(x_2 - p) + \eta - u(x_2)$$

We consider separately the incentive  $\varepsilon$  above and  $\varepsilon$  below  $x_2^*$ .

1) **Below**  $x_2^*$ : When

$$x_2 = x_2^* - \varepsilon$$

and so  $\delta < 0$

we can write the expected value of buying a lottery ticket as:

$$E[V_1^{l=1}] = q \left( u \left( x_2^* - \varepsilon + \frac{1-q}{q} - p \right) + \eta \right) + (1-q)u(x_2^* - \varepsilon - 1)$$

And the value of not buying a ticket as:

$$\begin{aligned} V_1^{l=0} &= u(x_2^* - \varepsilon) \\ &= q(u(x_2^* - \varepsilon - p) + \eta) - q\delta + (1-q)u(x_2^* - \varepsilon) \end{aligned}$$

$$\begin{aligned} E[V_1^{l=1} - V_1^{l=0}] &= q \left( u \left( x_2^* - \varepsilon + \frac{1-q}{q} - p \right) + \eta - u(x_2^* - \varepsilon - p) - \eta \right) \\ &\quad + q\delta \\ &\quad + (1-q)(u(x_2^* - \varepsilon - 1) - u(x_2^* - \varepsilon)) \end{aligned}$$

This is approximately equal to:

$$\begin{aligned} E[V_1^{l=1} - V_1^{l=0}] &= q \left( u'(x_2^* - \varepsilon - p) \left( \frac{1-q}{q} \right) \right) \\ &\quad + q\delta \\ &\quad + (1-q)(-u'(x_2^* - \varepsilon)) \end{aligned}$$

$$\begin{aligned} E[V_1^{l=1} - V_1^{l=0}] &= (1-q)(u'(x_2 - \varepsilon - p) - u'(x_2 - \varepsilon)) + q\delta \\ &= -p(1-q)u''(x_2 - \varepsilon) + q\delta \end{aligned}$$

As

$$\varepsilon \rightarrow 0, x_2 \uparrow x_2^*, \delta \uparrow 0$$

and

$$E[V_1^{l=1} - V_1^{l=0}] > 0$$

2) **Above  $x_2^*$ :** When

$$\begin{aligned} x_2 &= x_2^* + \varepsilon \\ \text{and so } \delta &> 0 \end{aligned}$$

$$E[V_1^{l=1}] = q \left( u \left( x_2^* + \varepsilon + \frac{1-q}{q} - p \right) + \eta \right) + (1-q)u(x_2^* + \varepsilon - 1)$$

The value of not buying a ticket is:

$$\begin{aligned} V_1^{l=0} &= u(x_2^* + \varepsilon - p) + \eta \\ &= q(u(x_2^* + \varepsilon - p) + \eta) + (1-q)(u(x_2^* + \varepsilon - p) + \eta) \\ &= q(u(x_2^* + \varepsilon - p) + \eta) + (1-q)u(x_2^* + \varepsilon) + (1-q)\delta \end{aligned}$$

$$\begin{aligned} E[V_1^{l=1} - V_1^{l=0}] &= q \left( u \left( x_2^* + \varepsilon + \frac{1-q}{q} - p \right) + \eta - u(x_2^* + \varepsilon - p) - \eta \right) \\ &\quad + (1-q)(u(x_2^* + \varepsilon - 1) - u(x_2^* + \varepsilon)) \\ &\quad - (1-q)\delta \end{aligned}$$

$$\begin{aligned}
E[V_1^{l=1} - V_1^{l=0}] &= q \left( u'(x_2 + \varepsilon - p) \left( \frac{1-q}{q} \right) \right) \\
&\quad + (1-q)(-u'(x_2 + \varepsilon)) \\
&\quad - (1-q)\delta
\end{aligned}$$

$$\begin{aligned}
E[V_1^{l=1} - V_1^{l=0}] &= (1-q)(u'(x_2 + \varepsilon - p) - u'(x_2 + \varepsilon)) - (1-q)\delta \\
&= -p(1-q)u''(x_2 + \varepsilon) - (1-q)\delta
\end{aligned}$$

As

$$\varepsilon \rightarrow 0, x_2 \downarrow x_2^*, \delta \downarrow 0$$

and

$$E[V_1^{l=1} - V_1^{l=0}] > 0$$