
Publisher's PDF, also known as Version of record
License (if available): CC BY
Link to published version (if available): 10.1016/j.jebo.2016.07.023

Link to publication record in Explore Bristol Research
PDF-document

This is the final published version of the article (version of record). It first appeared online via Elsevier at http://www.sciencedirect.com/science/article/pii/S0167268116301585. Please refer to any applicable terms of use of the publisher.

University of Bristol - Explore Bristol Research
General rights
This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available: http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/
Appendix: Proofs of Results 2 and 3

Proof of Result 2

Result 2: Lottery tickets are not purchased outside the interval \( x_i \in \left[ x_2^*, \frac{1-q}{q} x_2^* + 1 \right] \).

Proof:

The value functions for not buying a lottery ticket is given by:

\[
V_{1}^{t=0} = \begin{cases} 
    u(x_i) & \text{if } x_i < x_2^* \\
    u(x_i - p) + \eta & \text{if } x_i \geq x_2^*
\end{cases}
\]

The expected value function for buying a ticket is given by:

\[
E[V_{1}^{t=1}] = q \left\{ \begin{array}{ll}
    u \left( x_i + \frac{1-q}{q} \right) & \text{if } x_i < x_2^* - \frac{1-q}{q} \\
    u(x_i + \frac{1-q}{q} - p) + \eta & \text{if } x_i \geq x_2^* - \frac{1-q}{q}
\end{array} \right\} + (1-q) \left\{ \begin{array}{ll}
    u \left( x_i - 1 \right) & \text{if } x_i < x_2^* + 1 \\
    u(x_i - 1 - p) + \ln \eta & \text{if } x_i \geq x_2^* + 1
\end{array} \right\}
\]

Now consider separately the incentive to buy a lottery ticket when cash-on-hand is below the interval and above the interval.

1) When \( x_i < x_2^* - (1-q)/q \), cash-on-hand in period 2 will be sufficiently low that even if the lottery is won, \( x_2 < x_2^* \), and so the household does not buy the indivisible good, regardless of the lottery outcome. Thus, the expected value of buying a lottery ticket becomes:

\[
E[V_{1}^{t=1}] = qu \left( x_i + \frac{1-q}{q} \right) + (1-q)u \left( x_i - 1 \right)
\]

The value of not buying becomes: \( V_{1}^{t=0} = u \left( x_i \right) \). Since the gamble is actuarially fair and utility, u, is concave, the value of not buying a lottery ticket is always greater than the expected value of buying the lottery ticket:
\[ V_{i=0}^{i} = u(x_i) \]
\[ \geq qu \left( x_i + \frac{1-q}{q} \right) + (1-q)u(x_i-1) = E[V_{i=1}^{i}] \]

2) When \( x_i > x_2^* + 1 \), cash-on-hand in period 2 will be sufficiently high that even if the lottery is lost, \( x_2 > x_2^* \), and so the household buys the indivisible good regardless of the lottery outcome. Thus, the expected value of buying a lottery ticket becomes:
\[ E[V_{i=1}^{i}] = qu \left( x_i + \frac{1-q}{q} - p \right) + (1-q)u(x_i-1-p) + \eta \]
And the value of not buying becomes:
\[ V_{i=0}^{i} = u(x_i - p) + \eta \]
Since the gamble is actuarially fair and utility, \( u \), is concave, the value of not buying the lottery ticket is always greater than the value of buying the ticket.

\[ V_{i=0}^{i} = u(x_i - p) + \eta \]
\[ \geq qu \left( x_i - p + \frac{1-q}{q} \right) + (1-q)u(x_i - p - 1) + \eta = E[V_{i=1}^{i}] \]

Proof of Result 3

**Result 3**: There exists a region, \( x_i \in [x_2, x_1] \), which contains \( x_2^* \left( x_1 < x_2^* < x_1 \right) \), in which the agent will purchase a lottery ticket.

**Proof**: We consider the incentive to buy a lottery ticket in the region of \( x_2^* \). Define the difference in utility from purchasing the indivisible good and not purchasing it as
\[
\delta = u(x_2 - p) + \eta - u(x_2)
\]

We consider separately the incentive \(\varepsilon\) above and \(\varepsilon\) below \(x_2^*\).

1) **Below** \(x_2^*\): When
\[
x_2 = x_2^* - \varepsilon
\]
\[\text{and so } \delta < 0\]

we can write the expected value of buying a lottery ticket as:
\[
E[V_1^{i=1}] = q \left( u \left( x_2^* - \varepsilon + \frac{1-q}{q} - p \right) + \eta \right) + (1-q)u(x_2^* - \varepsilon - 1)
\]

And the value of not buying a ticket as:
\[
V_1^{i=0} = u(x_2^* - \varepsilon)
\]
\[= q \left( u \left( x_2^* - \varepsilon - p \right) + \eta \right) - q\delta + (1-q)u(x_2^* - \varepsilon)
\]

\[
E[V_1^{i=1} - V_1^{i=0}] = q \left( u \left( x_2^* - \varepsilon + \frac{1-q}{q} - p \right) + \eta - u \left( x_2^* - \varepsilon - p \right) - \eta \right)
\]
\[+ q\delta
\]
\[+ (1-q) \left( u \left( x_2^* - \varepsilon - 1 \right) - u \left( x_2^* - \varepsilon \right) \right)
\]

This is approximately equal to:
\[
E[V_1^{i=1} - V_1^{i=0}] = q \left( u'(x_2 - \varepsilon - p) \left( \frac{1-q}{q} \right) \right)
\]
\[+ q\delta
\]
\[+ (1-q) \left( -u'(x_2 - \varepsilon) \right)
\]
$$E[V_1^{i=1} - V_1^{i=0}] = (1-q)(u'(x_2 - \varepsilon - p) - u'(x_2 - \varepsilon)) + q\delta$$
$$= -p(1-q)u^*(x_2 - \varepsilon) + q\delta$$

As
$$\varepsilon \rightarrow 0, \ x_2 \uparrow x_2^*, \delta \uparrow 0$$

and
$$E[V_1^{i=1} - V_1^{i=0}] > 0$$

2) **Above** $x_2^*$: When

$$x_2 = x_2^* + \varepsilon$$

*and so* $\delta > 0$

$$E[V_1^{i=1}] = q\left(u\left(x_2^* + \varepsilon + \frac{1-q}{q} - p\right) + \eta\right) + (1-q)u(x_2^* + \varepsilon - 1)$$

The value of not buying a ticket is:

$$V_1^{i=0} = u\left(x_2^* + \varepsilon - p\right) + \eta$$
$$= q\left(u\left(x_2^* + \varepsilon - p\right) + \eta\right) + (1-q)\left(u\left(x_2^* + \varepsilon - p\right) + \eta\right)$$
$$= q\left(u\left(x_2^* + \varepsilon - p\right) + \eta\right) + (1-q)u(x_2^* + \varepsilon) + (1-q)\delta$$

$$E[V_1^{i=1} - V_1^{i=0}] = q\left(u\left(x_2^* + \varepsilon + \frac{1-q}{q} - p\right) + \eta - u\left(x_2^* + \varepsilon - p\right) - \eta\right)$$
$$+(1-q)\left(u\left(x_2^* + \varepsilon - 1\right) - u\left(x_2^* + \varepsilon\right)\right)$$
$$-(1-q)\delta$$
\[ E[V_{1}^{l=1} - V_{1}^{l=0}] = q \left( u'(x_2 + \varepsilon - p) \left( \frac{1-q}{q} \right) \right) + (1-q) \left( -u'(x_2 + \varepsilon) \right) - (1-q)\delta \]

\[ E[V_{1}^{l=1} - V_{1}^{l=0}] = (1-q) \left( u'(x_2 + \varepsilon - p) - u'(x_2 + \varepsilon) \right) - (1-q)\delta \]

\[ = -p(1-q)u''(x_2 + \varepsilon) - (1-q)\delta \]

As
\[ \varepsilon \to 0, \quad x_2 \downarrow x_2^*, \quad \delta \downarrow 0 \]

and
\[ E[V_{1}^{l=1} - V_{1}^{l=0}] > 0 \]