Spatial Scale and Measuring Segregation: Illustrated by the Formation of Chicago's Ghetto

Abstract. Few studies of residential segregation in cities have directly addressed the issue of spatial scale, apart from noting that the traditional indices of segregation tend to be larger when calculated for small rather than large spatial units. That observation however ignores Duncan et al.'s (1961) explication that any measure of segregation at a fine-grained scale necessarily incorporates, to an unknown extent, segregation at a larger scale within which the finer-grained units are nested. To avoid that problem, a multi-level modelling perspective is introduced that identifies the intensity of segregation at each scale net of its intensity at any larger scale included in the analysis. It is applied to an analysis of the emergence of Chicago's Black ghetto over the twentieth century's first three decades, using data at the ward and ED scales. It shows that across Chicago as a whole segregation was equally as intense at the two scales, with statistically significant increases in that intensity at both scales across the three decades. At the finer scale, however, segregation was much more intense across the EDs within those wards that formed the core of the emerging ghetto than it was in the reminder of the city.

Keywords: segregation, scale, measurement, Chicago, ghetto

Introduction

Most studies of residential segregation in urban areas analyse the pattern at a single spatial scale only, using descriptive statistics – such as the indices of dissimilarity and isolation – to evaluate the intensity of segregation. Fowler (2015) has however recently argued that segregation patterns result from the operation of location-decision processes at a number of spatial scales, and that analyses should take this into account. (This point was first made some sixty years before – Kish, 1954 – but largely ignored since.) Although there are some examples of this being done (e.g. Voas and Williamson, 2000; Reardon et al., 2008; Clark et al., 2015; Östh et al., 2015), they do not fully explore the analytical problems involved.

Among the small number of studies that have explored segregation within an urban area at several scales, their analyses have look at each scale independently (e.g. Woods, 1976; Peach, 1996), reaching the same conclusion as that stated explicitly by Logan et al. (2015a, 1077): their purpose was 'not to demonstrate that segregation is higher at a finer spatial scale, *which is already well known*' (our emphasis). The analyses reported here challenge that statement and assumption.

In one of the first pieces that looked explicitly at scale issues in the analysis of segregation, Duncan et al., (1961, 84) wrote that:

> … if one system of areal units is derived by subdivision of the units of another system, the index computed for the former can be no smaller than the index for the latter, and usually will be larger. Thus the index of concentration on a county basis will exceed the index on a State basis, because the county index takes into account interstate concentration.

A similar argument – though without reference to Duncan et al. or to segregation per se – was developed by Tranmer and Steel (2001, 942), who showed that if a multi-scalar situation is being analysed, but one of the relevant scales is omitted, then 'variation at the ignored level is redistributed to the levels that are assumed in the model'.

The method introduced here explicitly addresses that issue clarified by Duncan et al. and by Tranmer and Steel, but very rarely noticed, let alone addressed, in the very large literature on segregation. (The scale issue was formally addressed by Haggett, 1964, 1965 – following Krumbein and Slack, 1956 – but not in the context of residential segregation.) It argues that if segregation is studied at one scale in the context of any segregation at a larger scale, then it may not be the case that the finer the spatial scale analysed the greater the observed intensity of segregation; it may be that – reflecting the location-decision-making processes discussed by Fowler (2015) – members of a group are strongly concentrated within a few segments of a metropolitan area but that, within those segments, they are fairly evenly distributed. Whether that – or some other situation – is the case is exemplified first here by a set of ideal type exemplars, and then by an empirical analysis of Black segregation in Chicago at the turn of the nineteenth and twentieth centuries.

This alternative approach to the study of segregation, based in the well-established multi-level modelling procedure, offers new insights to the patterns and processes of segregation. Further developments are feasible, however, and these are identified in a final section.

Multi-scale segregation

The case regarding the importance of assessing segregation at one scale net of that at a coarser scale within which the smaller-scale units are nested is illustrated in Figure 1 by a synthetic city comprising six wards each divided into four EDs. Each ED has a total population of 100; the numbers in each ED in Figure 1 indicate the number of Blacks resident there (the remaining residents are categories as White). In the first situation (the left-hand diagram), the 480 Blacks are all concentrated in two of the six wards, where they form 60 per cent of the population of each of the constituent EDs. There is segregation at both scales, but if that at the ward scale is 'held constant' there is no segregation at the ED scale as well because across the two wards with Black residents there is no difference in the Black's percentage contribution to each ED's population (nor, clearly, in any of the other four wards where Blacks are totally absent from all EDs). In the second situation (the central diagram) there are 600 Blacks, all concentrated into three of the six wards, but they do not form the same percentage of the population in each ED within those wards. There is segregation at both scales: Blacks are concentrated in three of the wards and, within each of them, more concentrated in two of the EDs than the others. Finally, the third situation (the right-hand diagram) also has 600 Blacks, again concentrated in just half of the EDs. But there are 100 Blacks in each of the six wards – and so there is no segregation at the ward scale but there is at the ED scale.

The differences between these three exemplars are only partly revealed by the Indices of Isolation (IIs) for the two groups at the two scales, and the Indices of Dissimilarity (IDs) between the two groups, shown in Table 1.[1] For the first exemplar, the II values are exactly the same at each scale – showing that the index for the finer scale is not always larger than that for the coarser, but also not revealing that there is no segregation within the wards where the Blacks are concentrated; there is, however, a larger ID value at the ED than at the ward scale. For the second and third exemplars, the indices of isolation are the same at the ED

[1] With only two categories being analysed the Index of Dissimilarity and the Index of Segregation are the same.

scale; the number of Blacks and Whites is the same in each case, and they are similarly relatively concentrated in the same number of EDs – but their spatial distribution across the six wards is different. (Technically, there is spatial autocorrelation in the distribution of Blacks in the second exemplar but not the third.) That final difference is reflected in the relative size of the II values at the ward scale in those two exemplars, but once again the large II values at both ward and ED scales for exemplar 2 are misleading. With exemplar 2, the ID values are the same at both scales also; in exemplar 3 there is no apparent segregation at the ward scale – each group is similarly distributed across the six wards (i.e. 100 of the 600 Blacks are to be found in each ward, as are 300 of the 1800 Whites), but segregation at the ED scale is the same as in exemplar 2. As others have argued, these indices show that the value at the finer scale cannot be smaller than that at the larger scale, but as the differences between exemplars 2 and 3 illustrate, major differences in the degree of segregation at the ward scale are not taken into account in calculations at the ED scale.

An alternative approach that separates out the intensity of segregation at a number of scales has recently been developed and applied to studies of contemporary ethnic patterns in London (Jones et al., 2015; Johnston et al., 2016c), Auckland (Manley et al., 2015) and Sydney (Johnston et al., 2016a), as well as in growing spatial polarisation in the partisanship of the US electorate (Johnston et al., 2016b). It is a modification of the well-developed multi-level modelling framework, and produces estimates of the level of segregation at each scale, net of its level at any higher scale within which the areal units deployed are nested – in this case EDs within wards. It begins with a null model of no segregation, and assumes that the members of the ethnic group being analysed are distributed across the areal units forming the finest spatial level (the EDs in this case) in the same proportions as the total population. If the segregation of the Black population is being considered, therefore, this gives an expected number of Blacks in each ED (and also, by imputation, each ward) from which an observed:expected rate can be derived for each spatial unit in the *n*-level hierarchy being deployed. The variances in those rates are then modelled to derive a segregation estimate for each level, net of that at any higher levels. (Full details of the modelling are given in Leckie et al., 2012; as they observed, virtually all of the standard segregation measures – such as the index of dissimilarity – are based on the variances in the observed distributions; the larger the variance, the greater the segregation.)

In the application to be discussed here, the focus is on a binary division of the population of each area – Black:White. The measure of segregation used is termed the Median Odds Ratio (MOR: Larsen and Merlo 2005), whose value can be interpreted in the same way as odds ratios derived from logistic regressions. The derivation of the MOR can be appreciated in the following way. Take a city divided into *n* wards, for each of which the modelling has produced an estimated observed:expected rate for the distribution of the Black population. Select any two of those wards at random and calculate the ratio of the larger to the smaller of those two rates: that is the odds ratio. Repeat that procedure many times, to give a frequency distribution of odds ratios. The median (MOR) is thus the average difference in those ratios.

The modelling procedure generates the MOR values from the variances analysed, and also the associated Bayesian Credible Intervals (CIs) which allow tests of the robustness of the estimated MOR values. (The model specification for estimating the variance is in Appendix 1.) Unlike virtually all other measures of segregation, therefore (though see Cortese et al., 1976; Lee at al., 2015), the equivalent of statistical significance testing can be used in the comparison of MOR values – between scales in one city, for example; between dates for the same scale and city; or between the same scale in different cities. This provides a marked improvement on almost all previous studies of segregation.

The other major advantage of this modelling approach is that it takes the non-independence of scales issue raised by Duncan et al. (1961) and addressed by Tranmer and Steel (2001) into account. In a two-scale analysis, therefore, at the coarser of the two scales the MOR values are interpreted as discussed above – the average difference in the ratios across all pairs of wards. At a finer scale, however, segregation at the coarser scale is 'held constant'. If, as in the exemplars above, we are examining EDs within wards, the MOR values are calculated as the ratios between randomly selected pairs of EDs within the same ward; they are thus estimates of segregation at the ED scale net of any segregation at the ward scale.

An example: the emergence of Chicago's ghetto

Much of the extensive literature on the residential segregation of ethnic groups in urban areas focuses – because of data availability – on the situation some time, often decades, after the initial establishment of a pattern. It is generally assumed, but rarely tested, that segregation builds up over time, as the group involved increases in size, then stabilises and finally declines as group members (almost invariably the descendants of its original members) become economically, socially and culturally integrated into the wider urban society. Although data are available in censuses to explore the second and third stages of that idealised sequence, it is rare to find comparable data with which to map and analyse the first stage – the creation of urban enclaves and/or ghettos.

A major programme aimed at filling that lacuna is the recent work of Logan and his colleagues (Logan et al. 2015a, 2015b) on the emergence of Black Ghettos in American cities, building on earlier studies there (Duncan and Duncan, 1957; Taeuber and Taeuber, 1972; Philpott, 1978: see also Kusmer, 1978, and Logan and Shin, 2016). Their research asks 'When did northern Blacks become highly segregated …?' (Logan et al. 2015b, 21) and uses data on the ethnic composition of both Enumeration Districts (EDs) and Wards – compiled from the original, individual census returns – to measure the degree of segregation at a sequence of dates between 1880 and 1940. Using the well-known indices of dissimilarity (ID) and isolation (II), they showed that as the Black population of a sample of ten cities grew so it became more segregated; each index increased with both the larger the city's ratio of Blacks to Whites and, holding that constant, every succeeding census.

One section of Logan et al.'s (2015b) paper is entitled 'The problem of geographic scale'. This – and the parallel discussion in the other (2015a) paper – is largely concerned with whether the ward scale, used in most studies of early segregation, is the most appropriate for studying the ghetto's emergence, noting that other authors (e.g. Philpott 1978) have argued for the use of a finer spatial scale – hence their use of the Enumeration District (ED) as well as the ward. (Enumeration Districts are the administrative areas to which individual enumerators were assigned for collecting the census data, and are identified in the returns.) Where the population under consideration is small, measurement of its spatial concentration into relatively large units could well under-estimate the degree of segregation. Chicago, for example, had only 29,583 Blacks in 1880 (out of a total population of 1,692,793), and its 18 wards averaged nearly 30,000 residents, whereas the EDs averaged fewer than 1,500; at that date the index of dissimilarity at the ward scale was 0.648; at the ED scale it was 0.694 (Logan et al., 2015a, 1070). A further problem with a small population is that there will almost certainly be few of its members in many of the spatial units, in which random allocation processes may well lead to over-inflation of the degree of segregation – as clearly demonstrated by Carrington and Troske (1997).

To illustrate use of our alternative procedure, we have re-analysed Logan et al.'s data for Chicago across the four censuses 1900-1930; the number of Blacks in Chicago was too small for analysing earlier censuses and the 1940 ED data are not yet available. Given that our focus is on the emergence of the Black ghetto, a binary division of the city's population into Black and White is justified, especially as the Black population differed significantly from Chicago's other major migration streams at the time and experienced discrimination in the labour and housing markets accordingly. Where appropriate, the method can readily be deployed to study multi-group segregation (Jones et al., 2015).

Descriptive data for the four censuses are in Table 2. Over the 30-year period Chicago's total population virtually doubled, whilst its Black population increased by nearly 700 per cent: Blacks formed just 1.75 per cent of the city's population in 1900, and 7.01 per cent three decades later. At the ward scale, there was no evidence of any substantial concentration of Blacks until 1920, and even then only one of the city's 35 wards had more than half of its population Black. A decade later – when the Black population had more than doubled – one ward was more than 90 per cent Black, whereas half of the wards had virtually no Black residents.

At the much finer, ED, scale, on the other hand, there is much clearer descriptive evidence of an emerging ghetto, and by 1910 four of the 1,474 EDs were at least 90 per cent Black. The number of such EDs multiplied many-fold over the next two decades, and by 1930 over 10 per cent of the city's EDs were at least 75 per cent Black. But that localised concentration was spread across several of the city's wards, as the sequence of four maps in Figure 2 indicates. (In those maps, the EDs are divided into groups according to the distribution of quartiles of the Black population. Thus, for example, the 23 EDs shown as Q1 in the map for 1930 comprise the smallest number of EDs containing one-quarter of the total Black population – 58,805 individuals – whereas the 35 shown as Q2 comprises the smallest number containing the next quarter. See Johnston et al. 2016b.)

The intensity of segregation

How segregated was Chicago's Black population at those four dates, at the ward and ED-net-of-ward scales? Table 3 gives the MOR values along with their CIs, and they are graphed in Figure 3. At the ward scale, the MOR quadrupled in size over the four censuses: as the Black population grew, so it became more concentrated in particular wards. The CI spread at each census overlapped that at the previous count, however, so we cannot conclude with any certainty that each year's MOR was significantly larger than that for the previous census (in part because we are dealing with a relatively small number of observations – 35 wards at the 1900-1920 censuses and 50 in 1930). Nevertheless, there is a clear significant difference between the intensity of segregation at the two end-dates: at the ward scale Blacks were significantly more segregated in 1930 than they had been three decades earlier.

At the ED scale, the degree of segregation increased even more rapidly, with the MOR more than quintupling between the 1910 and 1930 censuses. There was little increase between 1910 and 1920, and the two MOR values then were not significantly different from each other. But at each succeeding census the MOR values both doubled and were significantly larger than the value ten years earlier. Thus within-ward segregation at the ED scale – which is measured net of the ward-level segregation – increased at approximately the same rate, as suggested by the trend lines in Figure 3. As Chicago's Black population increased it became more concentrated into particular wards and, within those wards, it also became more concentrated into certain EDs.

One major advantage of this modelling approach to multi-scalar segregation is that – unlike methods that decompose a segregation measure into different scale components (e.g. Fischer et al., 2004; Voas and Williamson, 2000) – it provides estimates of the intensity of segregation at each scale examined, independent of its intensity at the other scale(s). The MOR values in Table 3 show that segregation was slightly more intense at the ward than at the ED scale at the first two censuses (though not significantly so) but of the same intensity at the last two.

Segregation within and outwith the ghetto

At each of the four censuses analysed, Chicago's Blacks were concentrated into a small number of wards – the core of the emerging ghetto. They were also concentrated within certain EDs, most of them within that ghetto. But many of Chicago's Black population lived outside the emerging ghetto. We define the ghetto as, at each date, the three wards with the largest proportion of the city's total Black population, so that in 1900 49 per cent of them lived elsewhere in the city, and at the subsequent three censuses that percentage was 52, 22 and 38. This raises the question: was segregation as great at the ED scale outside the ghetto wards as within them?[2] To answer that we have separately modelled segregation levels in the two sections of the city (using the specification in Appendix 2). Outwith the ghetto, we have modelled segregation across the relevant wards (32 in 1900-1920; 47 in 1930) and across the EDs within those wards: within the ghetto we have just modelled segregation across the EDs, there being too few wards (3) for any reliable estimate of an MOR value at that scale.

The results of those analyses are in Table 4. The first set of MOR values, when compared with those in Table 3 calculated for all wards, indicate that at that scale segregation was less outside the ghetto than it was across the city as a whole, but the MOR values are still substantial – in 1930, for example, it was 33.29 for Chicago as a whole (Table 3) but outwith the ghetto it was about one-third of that value only, at 11.62. Over the first three censuses it hardly changed in non-ghetto Chicago, but between 1920 and 1930 it almost doubled – although, not surprisingly given the relatively small number of wards, the difference was not statistically significant.

Turning to segregation at the ED scale, the next set of MOR values indicates that outside the ghetto there was a very substantial and statistically significant increase in the degree to which Blacks were spatially concentrated, notably from 1910 on. Between then and 1920 the MOR value more than doubled, and over the next decade it almost doubled again. Blacks were concentrated in a small number of Chicago wards, but outside that emerging ghetto area as the Black population expanded so its members were increasingly concentrated into a small number of ED-scale neighbourhoods.

That growth in the degree of segregation outwith the ghetto was not only paralleled but also exceeded within the ghetto – the three wards where half or more of Chicago's Blacks were

[2] Lower spatial segregation outwith the ghetto might be anticipated because a proportion of the Black residents there may have been 'live-in' domestic servants – although they could have been concentrated in those districts whose residents were wealthy enough to afford such servants.

concentrated. The MOR values for EDs at this scale exceeded those for the extra-ghetto wards, though significantly so only in 1930, when the MOR was 77.19 for the three ghetto wards compared to 26.94 for those EDs in the remainder of the city. Comparison of the MOR values for 1920 and 1930 within the ghetto also shows a major increase in the degree of segregation – the MOR was 18.36 at the first date and 77.19 at the second, with no overlap of the CIs.

As Chicago's Black population grew, therefore, it became spatially concentrated into a small number of the city's wards – the emerging ghetto – and, within them, increasingly spatially concentrated into some of those wards' constituent EDs. Growth and spatial concentration at both scales followed a common trajectory. And the same was the case outside the ghetto: many Blacks lived elsewhere in the city, but in those districts too there was not only growing segregation at the ward scale but also a parallel trend for increased clustering into particular neighbourhoods within those wards – though at less intensity than was the case within the ghetto.

Extending the analysis: an agenda

The results of these analyses – which show that segregation is *not* always higher at the finer spatial scales, once its intensity at the coarser of two scales is taken into account – raise a number of other questions, some of which can be addressed here but others await further research.

Comparability of the MOR values

The first concerns the interpretation of the MOR values, in the context of traditional segregation measures. Experimentation has shown that, because derivation of both the MOR and the indices of dissimilarity and segregation is based on the mean and variance of the proportional distribution of the population under consideration (in this case, Chicago's Blacks), it is possible to convert the MOR value into an index-of-dissimilarity-equivalent (IDE), which varies between 0 and 1.0 with, unlike the standard Index of Dissimilarity, associated CIs. (The formulae for these simulations are in Leckie et al. 2012, 14.) This allows us to compare the two measures – Table 5 and Figure 4.[3]

The trends shown in Table 5 are, of course, the same as those for the MORs shown in Table 3. Although segregation increased over time at the ward scale, the only significant difference in its intensity is between the two end dates. At the ED scale, on the other hand, segregation was significantly greater in 1920 than 1910, and again in 1930 than in 1920. More informative is the difference in the intensity of the segregation shown at the two scales by the 'raw' ID and the IDE measures – the former are calculated as in Logan et al.'s papers. Figure 4 shows the ID and IDE values at the ward scale trending in the same direction and not far apart – recalling that the IDE values are modelled estimates and should not be expected to be exactly the same as the ID values. (The differences between the two are 0.03, 0.02, 0.10 and 0.05 for the four dates respectively.) The differences at the ED scale are much more substantial, however, although again both are trending in the same direction: in 1900 the difference between the two values was 0.20 in 1900 and 0.22, 0.23 and 0.22 at the succeeding dates. This is because the IDE values are net of those calculated for the ward scale.

[3] Further exploration is being undertaken to see whether these IDE values should be standardised to take into account the increase in the relative size of Chicago's Black population. We do not believe that, even if this is necessary, it will substantially impact the results but large simulation studies are needed to test that claim.

Segregation increased at both scales over the three decades, but within wards at the ED scale it was never greater than at the ward scale itself. Over time, Blacks became more segregated into particular wards within Chicago and equally as segregated within those wards.

The number of units

The second issue concerns the different number of spatial units analysed at each of the four dates. Although the number of Chicago wards was unchanged across the first three censuses, their boundaries were altered between 1900 and 1910, reflecting the city's expansion (as the maps in Figure 2 show). The number of EDs increased between each pair of censuses, reflecting Chicago's growth and the administrative desire to keep the size of EDs manageable for the enumerators collecting the data. This issue could be overcome by creating a unified set of EDs and wards covering all four censuses, which will make comparisons over time more exact – though that resolution will not obviate the next two issues raised here.

The Modifiable Areal Unit Problem

The third issue concerns the well-known Modifiable Areal Unit Problem (MAUP) that applies to so many spatial statistical analyses (Manley 2014). In much research using areal data, the number and boundaries of the areal units deployed are arbitrary, probably representing administrative decisions and convenience rather than having any direct link to the processes underpinning the observed spatial patterns; Chicago's ward and ED boundaries at the four censuses analysed were not designed to reflect the spatial structuring of the city's housing market. Whereas analysts usually have to accept the smallest scale areal units for which the data are made available – in this case the EDs – that is not the case with regard to those at the larger scale(s). Combinatorially, for example, there is a very large number of ways in which Chicago's 2,459 EDs in 1930 could be combined into 50, spatially-cohesive, larger units (with the same mean and standard deviation for population numbers). Each set of 50 pseudo-wards would have its unique MOR for Black segregation and, because each set would involve a different allocation of EDs across the 50 pseudo-wards, a unique MOR at that scale as well. The analyses here, like almost all other analyses of segregation, assume that the particular configuration of areas is typical of all possibilities, but this is very rarely tested. (Recent examples of methods that could be used for such tests are those developed by Östh et al. 2015.) In the context of the Chicago data analysed here a simulation exercise whereby an algorithm (such as AZP: Martin 2003; Cockings et al. 2011) is used to generate a sufficient number of different aggregations of, for example, Chicago's 2,459 EDs in 1930 to produce a frequency distribution of MORs – at both scales – could be used to determine whether the observed values for the given configuration is either an outlier or typical of all possible ways of grouping the EDs into 50 larger units. Such an exercise is outwith the consideration of the current paper, and the subject for further research.

The number of levels

The fourth issue concerns the number of levels that should be deployed in the analyses. It may be, for example, that Chicago's 50 wards in 1930 should be aggregated into a smaller number of larger units (~20, perhaps) to inquire whether there is concentration at that scale within which segregation at the ward and ED scales should be nested; further it may be that the EDs should also be clustered into a number of sub-ward units (~100, for example). Analyses of other cities (Jones et al. 2015; Manley et al. 2015) have suggested that two scales are generally sufficient, but that may not be the case in every situation. Similarly, introduction of an intermediate scale, between the ward and the ED, could throw greater light on the detailed patterning. Again, answers to these questions – which call for further simulations – will be sought in future research.

Spatial autocorrelation

A common criticism of many segregation analyses using the standard single-number indices is that they take no account of spatial autocorrelation – of the degree to which the areas in which a group is concentrated are spatially contiguous – and ways of measuring that aspect of segregation have been explored (Johnston et al. 2011). To some extent that criticism does not apply to the method discussed here, since by looking at segregation by ED within wards it takes spatial autocorrelation into account; each MOR results from averaging a comparison of rates between EDs within a particular ward, averaged across all wards. Nevertheless, there is potential for exploring the degree to which – in the current context – the wards where Blacks are concentrated are spatially contiguous and similarly the spatial contiguity of EDs within those wards.

Conclusions

A main goal of this paper have been technical, seeking to advance the study and measurement of segregation. It has directly addressed an issue raised explicitly by Fowler (2015), though implicit in a number of other studies, that segregation is a multi-scalar concept, not only in the spatial patterns that it displays but also in its underpinning processes. Using both an idealised set of cities and then data on the distribution of Black and White Chicago residents at four early-twentieth-century censuses, it has deployed a newly-developed, multi-level modelling strategy that identifies the intensity of segregation in the distribution of a population group at any spatial scale independent of its intensity at any larger scale within which the smaller areal units analysed are nested. These illustrate the nature of a multi-scalar reality in the context of arguments that any index of segregation calculated for a set of areal units into which a city has been divided must take into account the intensity of segregation at larger spatial scales, within which those areal units are nested. Without doing that, the identified level of segregation at the finer spatial scale is likely to be over-stated; doing it reveals that the 'conventional wisdom' that segregation is always greater at the finer spatial scale has been empirically challenged.

The paper's substantive analyses contribute further to our appreciation of the segregation of Chicago's Blacks in the twentieth century's first three decades. As the city's Black population multiplied, so its members became more concentrated in particular wards and, within those wards, into some EDs more than others. The evolution of the city's Black ghetto was a clear multi-scalar phenomenon, with clustering occurring at two, nested scales, and by 1930 that clustering was equally as intense at both scales when studied across the entire city. Within the emerging ghetto, however, segregation was much more intense than it was in the remainder of the city, where a substantial proportion of the Black population lived at lower intensities of residential segregation. That separate investigation of different parts of the city (as presaged by Kish, 1954) extends our appreciation of segregation patterns: they may not be as intense across the neighborhoods in some districts as they are in others.

The methodology introduced here provides insights into patterns of segregation from which, as is usual in such studies, the processes underpinning those patterns can be inferred; such insights are not available in either the standard procedures using descriptive indices or others that have recently been advanced. It explicitly acknowledges Duncan et al.'s (1961) argument regarding scale inter-dependence and provides assessments of the intensity with which a group is both concentrated in certain large-area districts within a city and also concentrated within neighborhoods within those districts. Its potential for extending segregation analyses is substantial.

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Appendix 1. The model specification for three-level binomial model

The form of the multilevel model used in the analysis EDs and Wards (derived from Browne et al. 2005) is as follows, with each census year analysed separately:

$$PropBlack_{jk} \sim Binomial(Population_{jk}, \pi_{jk})$$
$$logit(\pi_{jk}) = \beta_0 x_0 + v_{0k} + u_{0jk}$$
$$v_{0k} \sim N(0, \sigma^2_{vo})$$
$$u_{0jk} \sim N(0, \sigma^2_{uo})$$
$$Var\left(PropBlack_{jk} | \pi_{jk}\right) = \frac{\pi_{jk}(1 - \pi_{ik})}{Population_{jk}}$$

Where $PropBlack_{ijk}$ is the observed response variable, the proportion of Black people in ED j in *Ward k* that are Black and $Population$ is the denominator of the Black plus White population. The log of the odds of being Black($logit\ (\pi_{jk})$), is modelled as a function of a fixed effect where x_0 is a constant (a set of 1s for each and every observation) so that β_0 is the overall city-wide average log-odds of being Black. The random part of the model consists of a differential logit for each ward (v_{0k}) and a differential logit for each ED within each ward (u_{0jk}). These logit differentials are assumed to come from a Normal distribution and are summarised by a variance term at each level so that σ^2_{vo} summarises the between Ward differences while σ^2_{uo}summarises the within-Ward between-ED variation. These two variances are our measures of segregation. At lowest level there is a single variance term and this is assumed to follow a Binomial distribution. In practice this is fitted as a three-level model, the Wards at level 3, and exactly the same set of units – the EDs – at level 1 and level 2; that is, each level 2 unit has exactly one level 1 unit. This views the aggregate proportions at level 2 as consisting of replicated binary responses for 'individuals' at level 1. This use of a pseudo-level is fully explained in Brown et al. (2005) and allows the separation of the variance into exact Binomial at level 1 and over-dispersion at higher level so that the higher-level variances summarize differences between areas over and above those expected from a random variation generated by a varying denominator. The estimated higher-level variances are transformed to MORs by using the formula given in Larsen and Merlo (2005).

Appendix 2. Model specification allowing for additional within-ghetto segregation (in three wards) in the model's fixed part

The specification of the model is as follows:

$$PropBlack_{jk} \sim Binomial(Population_{jk}, \pi_{jk})$$
$$logit(\pi_{jk}) = \beta_0 x_0 + \beta_1 x_{1jk} + \beta_2 x_{2jk} + \beta_3 x_{3jk} + v_{0k} x_0 + u_{0jk} x_{4jk} + u_{1jk} x_{5jk}$$
$$v_{0k} \sim N(0, \sigma^2_{vo})$$
$$\begin{bmatrix} u_{0jk} \\ u_{1jk} \end{bmatrix} \sim N(0, \begin{bmatrix} \sigma^2_{uo} & \\ & \sigma^2_{u1} \end{bmatrix})$$
$$Var\left(PropBlack_{jk} | \pi_{jk}\right) = \frac{\pi_{jk}(1 - \pi_{jk})}{Population_{jk}}$$

The underlying model is similar to before with the response being the logit of being Black and the lowest level variance being an exact Binomial distribution. However in the fixed part of the model three dummies (x_{1jk}; x_{2jk} and x_{3jk}) identify the three wards that constitute the ghetto area. The β_0 then gives the overall log-odds of an individual being Black outside the ghetto while $\beta_1$is the differential logit of being Black in ward 1, and so on. In the random part v_{0k} gives the differential ward effect which will be zero for wards 1, 2 and 3 (which have their own mean in the fixed part), so that the variance σ_{vo}^2summarizes the between ward variation in the logit of being Black outside the defined ghetto area. Two separately –coded (Bullen et al. 1997) dummy variables (x_{4jk} and x_{5jk}) identify EDs that are in and outside the ghetto and the two variance terms ($\sigma_{uo}^2$and σ_{u1}^2) summarise the variations within wards between EDs inside and outside the ghetto area. There is no covariance in the level 2 random part as an ED cannot be simultaneously in and outside the ghetto. The MLwiN (Rasbash et al. 2009) software is particularly flexible in estimating such models with complex variance functions.

Table 1. Indices of Isolation (II) and of Dissimilarity (ID) for the three synthetic exemplars in Figure 1.

	II(Black)	II(White)	ID(Black:White)
Exemplar 1			
Ward	0.60	0.90	0.52
ED	0.60	0.90	0.80*Exemplar 2*
Ward	0.50	0.83	0.67
ED	0.52	0.84	0.67
Exemplar 3			
Ward	0.25	0.25	0.00
ED	0.52	0.84	0.67

Table 2. Characteristics of the spatial distribution of Blacks in Chicago, 1900-1930.

	1900	1910	1920	1930
Total Black + White Population	1,692,793	2,182,404	2,675,250	3,356,289
Total Black Population	29,583	45,458	106,846	235,218
Percentage Black	1.75	2.08	3.99	7.01
Number of Wards	35	35	35	50
Number of EDs	1,132	1,474	2,250	2,459
Number of Wards with				
Blacks <1%	24	20	23	25
Blacks 25%<	0	2	2	4
Blacks 50%<	0	0	1	3
Blacks 75%<	0	0	0	1
Blacks 90%<	0	0	0	1
Number of EDs with				
Blacks <1%	840	1,129	1,702	1,970
Blacks 25%<	24	46	141	205
Blacks 50%<	9	24	94	176
Blacks 75%<	4	9	51	155
Blacks 90%<	0	4	23	119

Table 3. The MOR values, with their Low and High CI values containing 95 per cent of the modelled rates, for the distribution of Blacks in Chicago at the ward and ED-net-of-ward scales.

	Ward			ED		
	LowCI	MOR	HighCI	LowCI	MOR	HighCI
1900	4.97	7.95	14.28	5.51	6.15	6.90
1910	5.62	9.30	17.45	6.46	7.19	8.05
1920	8.21	15.17	32.78	13.27	15.22	17.56
1930	16.80	33.29	75.92	28.00	33.12	39.42

Table 4. The MOR values, with their Low and High CI values containing 95 per cent of the modelled rates, calculated separately within and outwith the emerging ghetto.

	Between wards outwith the ghetto			Between EDs outwith the ghetto			Between EDs within the ghetto		
	LowCI	MOR	HighCI	LowCI	MOR	HighCI	LowCI	MOR	HighCI
1900	3.89	6.08	11.03	5.32	5.99	6.78	5.37	7.08	9.74
1910	3.92	6.04	10.40	6.03	6.74	7.58	7.82	11.13	16.69
1920	3.97	6.24	11.01	12.18	14.23	16.80	13.78	18.36	25.06
1930	6.86	11.62	21.87	22.55	26.94	32.46	47.21	77.19	134.55

Table 5. The estimated values of the Index of Dissimilarity (IDE), with their Low and High CI values containing 95 per cent of the modelled rates, for the distribution of Blacks in Chicago at the ward and ED-net-of-ward scales.

	Ward			ED		
	LowCI	IDE	HighCI	LowCI	IDE	HighCI
1900	0.515	0.592	0.662	0.537	0.553	0.572
1910	0.540	0.616	0.682	0.560	0.580	0.596
1920	0.599	0.667	0.729	0.654	0.668	0.682
1930	0.677	0.730	0.775	0.718	0.729	0.740

Figure 1. Ideal–typical segregation patterns: a city with six wards each containing four EDS. (There are 100 persons resident in each ED; the numbers show the number of Blacks in each ED.)

60	60	0	0
60	60	0	0
60	60	0	0
60	60	0	0
0	0	0	0
0	0	0	0

40	40	0	0
60	60	0	0
40	60	0	0
40	60	0	0
60	40	0	0
40	60	0	0

60	0	0	40
0	40	60	0
40	0	40	0
60	0	0	60
0	0	0	0
40	60	60	40

Figure 2: The Development of the Chicago Ghetto, 1900-1930.

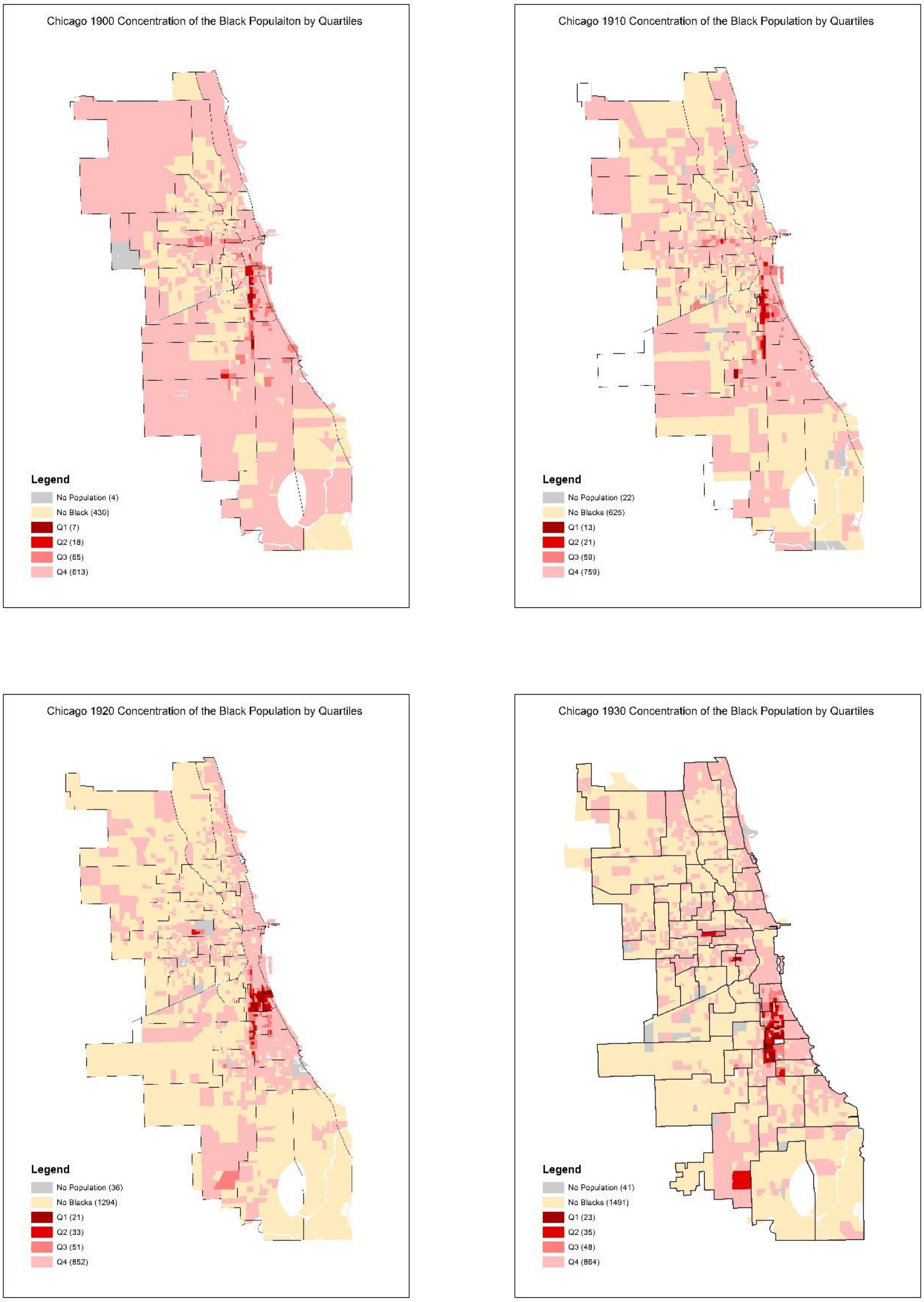

Figure 3. The MOR values for Black segregation at Ward and ED scales, Chicago 1900-1930.

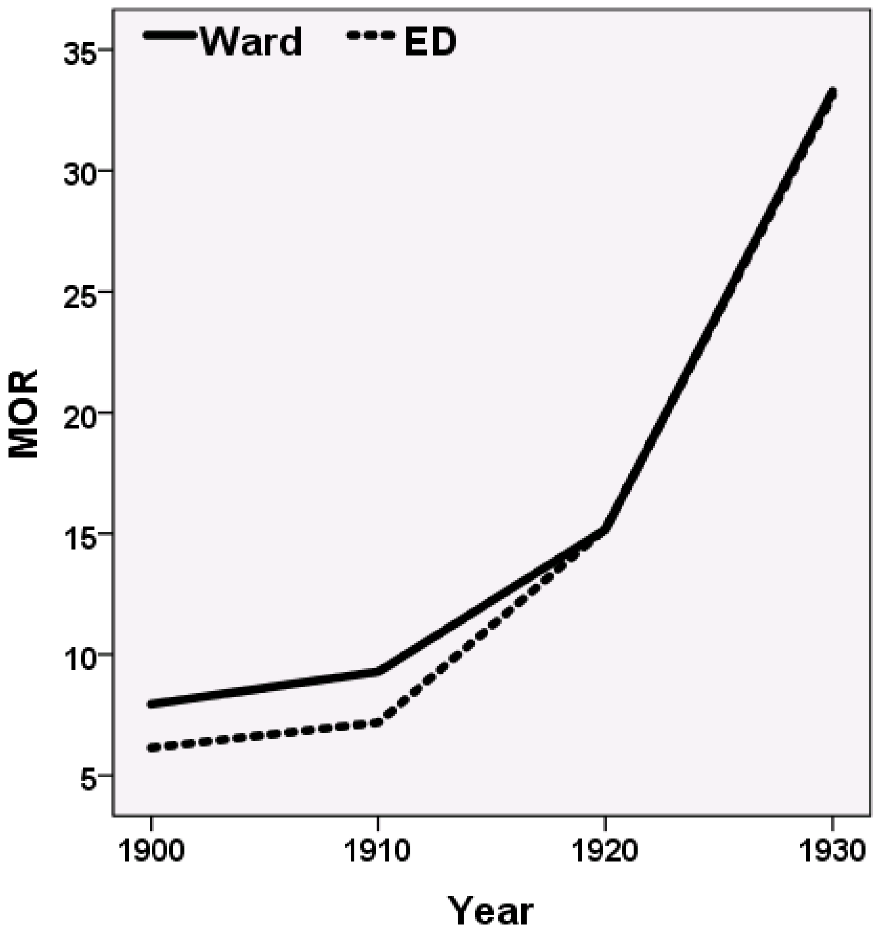

Figure 4. The indices of dissimilarity (ID) and the estimated indices (IDE) for Black segregation at Ward and ED scales, Chicago 1900-1930.