The Disturbing Interaction between Countercyclical Capital Requirements and Systemic Risk*

Bálint L. Horváth† Wolf Wagner‡

June 3, 2016

Abstract

We present a model in which flat (state-independent) capital requirements are undesirable because of shocks to bank capital. There is a rationale for countercyclical capital requirements that impose lower capital demands when aggregate bank capital is low. However, such capital requirements also have a cost as they increase systemic risk taking: by insulating banks against aggregate shocks (but not bank-specific ones), they create incentives to invest in correlated activities. As a result, the economy's sensitivity to shocks increases and systemic crises can become more

*We thank participants at the FIRS conference in Dubrovnik, the workshop on "Understanding Macroprudential Regulation" at Norges Bank, the LSE workshop on financial models, the joint conference by the Central Bank of the Republic of Turkey, Bank for International Settlements and the International Monetary Fund on "Macroprudential Policy" in Istanbul, the joint conference by the Bank of England, the Basel Committee Research Task Force and the Centre for Economic Policy Research on "The interaction of regulatory instruments" in London, the workshop on systemic risk at Nottingham University, the workshop on "Liquidity, Banking and Financial Markets" at the University of Bologna, the conference on financial stability in Luxembourg, the summer workshop of the Hungarian Science Academy, the EFMA conference in Reading and seminars at Tilburg University, Vienna University, Aarhus University, Federal Reserve Board, Koc University, Ozyegin University, Federal Reserve Bank of Kansas City, Banque de France, Bank of Lithuania, Luxembourg School of Finance and the University of Bristol as well as Mathias Drehmann, Harry Huizinga, Peter Kondor, Enrico Perotti, Frédéric Malherbe, Xavier Freixas, John Theal, Javier Suarez and two anonymous referees for valuable comments.

†University of Bristol. Email: balint.horvath@bristol.ac.uk

‡Erasmus University Rotterdam - Rotterdam School of Management; Centre for Economic Policy Research. Email: wagner@rsm.nl

likely. Capital requirements that directly incentivize banks to become less correlated dominate countercyclical policies as they reduce both systemic risk-taking and cyclicality.

1 Introduction

Following the experience of the crisis of 2007-2008, there is broad interest in policies that mitigate *procyclicality*, that is, the tendency of the financial system to amplify shocks. The new Basel Accord, for instance, incorporates capital buffers that are built up in good times and can be run down when economic conditions deteriorate. In addition, the liquidity coverage ratio of Basel III – which aims at safeguarding banks against short-term outflows – contains a countercyclical element to the extent that such liquidity buffers are released in bad times. On the accounting side, there is a discussion about whether mark-to-market accounting – which has the potential to amplify the impact of asset price changes – should be suspended when prices are depressed. There is also a growing debate about whether monetary policy should "lean against the wind" with respect to the financial cycle, that is, raise interest rates when the economy experiences excessive credit expansion and asset price inflation, but lower interest rates in times of significant contraction in lending or general stress in the financial system.

In this paper we argue that procyclicality cannot be separated from the second dimension of systemic risk: the extent to which institutions in the financial system are correlated with each other.[1] Focusing on capital requirements as the policy tool, we show that there is a two-way interaction between these two dimensions of systemic risk: mitigating procyclicality affects the correlation of risks in the financial system and correlation (and policies that mitigate it) influences procyclicality. It is thus not possible to address the two dimensions of systemic risk in isolation, which has profound implications for the design of macroprudential regulation.

We consider an economy in which banks invest excessively[2] and face shocks to their capital. Flat capital requirements – even if optimally set – lead to excessive cyclicality in

[1]It is common in the literature to see procyclicality and common risk exposures as the two key – but separate – dimensions of systemic risk (e.g. Borio, 2003).

[2]This is because of deposit insurance and because banks do not internalize the cost of joint failures.

this economy. Following a positive shock to capital, banks will expand beyond the efficient level, while a negative shock forces them to contract investment to an inefficiently low level. We show that welfare-maximizing capital requirements – for given correlation of risks in the financial system – are countercyclical: in order to stabilize investment at its optimal level, capital requirements have to be tightened when banks have plenty of capital, and loosened if capital is scarce. Effectively, countercyclical capital requirements improve welfare by mitigating the impact of aggregate shocks on investment.

This result no longer necessarily holds when the correlation of risks is endogenous. We allow banks to choose between a common and a bank-specific project. Since a bank's capital is determined by prior returns on its activities, capital conditions become more correlated when banks invest in the same project. At the same time, correlation makes it also more likely that banks fail jointly. In this case output in the economy is very low and risk-averse consumers suffer. Banks do not fully internalize this cost, and hence may choose more correlation than is socially optimal.

We show that countercyclical capital requirements worsen the problem of excessive correlation. The reason is the following: they insulate banks against common shocks, but not against bank-specific ones. The expected cost from being exposed to aggregate risk hence falls relative to bank-specific exposures, which increases banks' incentives to invest in the common project. A bank that continues to focus on bank-specific activities would run the risk of receiving a negative shock when aggregate capital is plenty, in which case it would be subject to high capital requirements precisely when it is most costly. Countercyclical capital requirements thus trade off benefits from reducing the impact of a shock for given exposures in the financial system with higher correlation of risks in the financial system. Their overall welfare implications are hence ambiguous. Perversely, countercyclical policies may even increase the economy's sensitivity to aggregate conditions. The reason is that by inducing banks to become more correlated, they make the financial system more exposed to aggregate shocks, which may result in a greater likelihood of joint bank failures.

While correlation has negative effects in our model, we show that having a correlated economy can also be socially optimal. In such an economy shocks to bank capital will be identical across banks and aggregate conditions match individual bank conditions. Capital requirements based on the aggregate state are then able to perfectly control investment at individual banks, increasing welfare by lowering investment volatility.

There is an alternative macroprudential policy in our model: a regulator could directly incentivize banks to become less correlated (for example, by charging higher capital requirements for correlated banks). We show that such a policy (if feasible) dominates countercyclical policies. This is because it addresses the two dimensions of systemic risk at the same time: it discourages correlation but also makes the system less procyclical as more heterogenous institutions will respond less strongly to aggregate shocks. In contrast – as discussed before – countercyclical policies improve systemic risk along one dimension at the cost of worsening it along another one.

The key message of our paper is that the two dimensions of systemic risk (common exposures and procyclicality) are inherently linked. The consequence is that policies addressing one risk dimension will also affect the other – and possibly in undesired ways. Our paper also derives several policy implications for countercyclical regulation, such as whether discretionary policies are desirable and whether countercyclicality should vary with the charateristics of a country's financial system.

This paper connects two strands of literature. The first investigates whether capital regulation should respond to the economic cycle.[3] Kashyap and Stein (2004) argue that capital requirements that do not depend on economic conditions are suboptimal and suggest that capital charges for a given unit of risk should vary with the scarcity of capital in the economy. Repullo and Suarez (2013) demonstrate that fixed risk-based capital re-

[3] See Galati and Moessner (2011) for a general overview of macroprudential policies.

quirements (such as in Basel II) result in procyclical lending. They also show that banks have an incentive to hold precautionary buffers in anticipation of capital shortages – but that these buffers are not effective in containing procyclicality. As a result, introducing a countercyclical element into regulation can be desirable. Malherbe (2013) considers a macroeconomic model where a regulator trades off growth and financial stability and finds that optimal capital requirements depend on business cycle characteristics. Martínez-Miera and Suarez (2012) consider a dynamic model where capital requirements reduce banks' incentives to take on aggregate risk (relative to investment in a diversified riskless portfolio). The reason is that capital requirements increase the value of capital to surviving banks in a crisis. This in turn provides banks with incentives to invest in safer activities in order to increase the chance of surviving when other banks are failing (the "last bank standing" effect).

A second strand of the literature analyzes the incentives of banks to correlate with each other. In particular, it has been shown that inefficient correlation may arise from investment choices (e.g. Acharya and Yorulmazer, 2007), diversification (Wagner, 2011; Allen et al., 2012), interbank insurance (Kahn and Santos, 2010) or through herding on the liability side (Segura and Suarez, 2011; Stein, 2012; Farhi and Tirole, 2012). In Acharya and Yorulmazer (2007), regulators cannot commit not to bail out banks if they fail jointly. Anticipating this, banks have an incentive to (inefficiently) invest in the same asset in order to increase the likelihood of receiving the bailout. Farhi and Tirole (2012) consider herding in funding choices. They show that when the regulator lacks commitment, bailout expectations provide banks with strategic incentives to increase their sensitivity to market conditions. The mechanism in our paper is not based on commitment problems (nor on strategic incentives). Banks do not expose themselves to the aggregate shock in order to get more lenient regulatory treatment. Rather, the incentives for correlation arise because being exposed to the aggregate state means that banks can benefit from countercyclical capital requirements that tend to offset shocks to bank capital, resulting in lower volatility of investment (average capital requirements are the same regardless of whether a bank correlates or not). A key difference of our paper relative to Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) (and other papers in the literature) is also that in our model it can in fact be socially optimal for the economy to be correlated (because capital requirements that vary with the aggregate state can then better reflect the individual conditions of banks).

The remainder of the paper is organized as follows. Section 2 contains the model. Section 3 provides several extensions. Section 4 concludes.

2 Model

2.1 Preview

We present a model that has a role for state-dependent regulation and in which there is endogenous systemic risk. We focus the analysis on capital requirements, the latter being probably the most prominent form of bank regulation. The rationale for imposing *cyclical* requirements comes from shocks to bank capital. In particular, bank capital is affected by the returns on existing projects.[4] Under flat capital requirements, variations in bank capital result in inefficient changes in investment and procyclical economic activity. In order to keep investment at its efficient level, a regulator hence has to tighten capital requirements in response to positive shocks to bank capital, and loosen requirements when there are negative shocks. Systemic costs arise because when banks fail at the same time, there are not enough funds in the economy to satisfy subsistence needs in the economy.[5] Systemic

[4]Our view of bank capital is based on Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Martínez-Miera and Suarez (2012) in that bank capital derives from accumulated bank profits.

[5]In our model joint failure costs are thus created by risk aversion arising from a minimum consumption requirement. While novel in the context of systemic risk, such preferences are widely used in other disciplines, such as development economics ("subsistence needs"), labor economics (e.g. Barzel and McDonald,

risk and capital requirements interact because banks can affect the correlation of their projects. In particular, anticipation of cyclical capital requirements determines whether banks want to invest in the same project or not. This in turn affects the likelihood of systemic crises where banks are failing jointly.

2.2 Setup

The economy consists of two bankers and a consumer. There are three dates: $0, 1, 2$.

Bankers (denoted with A and B) have no endowment and derive utility from consumption at date 1 and date 2:

$$u_i(c_{i,1}, c_{i,2}) = c_{i,1} + c_{i,2}, \tag{1}$$

where $i \in \{A, B\}$. The consumer is endowed with ω units of funds at date 1. She also consumes at dates 1 and 2. Her utility is linear as for the banker, except that she has a requirement for positive consumption at date 2. If this requirement is not met, the consumer suffers a utility loss of $\gamma > 0$:

$$u_C(c_{C,1}, c_{C,2}) = \begin{cases} c_{C,1} + c_{C,2} \text{ if } c_{C,2} > 0 \\ c_{C,1} + c_{C,2} - \gamma \text{ if } c_{C,2} = 0. \end{cases} \tag{2}$$

At date 0 banker A has access to two projects: an economy-wide project (the "common" project) and a project that is only available to him (the "alternative" project). The project choice is not observable. Banker B only has access to the common project.[6] The returns on the common and the alternative project (to be specified later) are independently and identically distributed. Each banker can undertake only one project; we can hence summarize the project choices in the economy by C (correlated projects) and U (uncorrelated projects). There is no storage technology in the economy.

1973), trade (Bergstrand, 1990) and taxation (Rosen, 1978). The idea of a utility loss if consumption falls below a threshold also features in habit formation, with applications to asset pricing (e.g. Abel, 1990) and RBC models (Lettau and Uhlig, 2000).

[6]This is without loss of generality since there is no benefit to having two alternative assets in our economy.

At date 1, a project returns an amount x_H with probability $\pi \in (0,1)$ and x_L otherwise. We denote with $\widehat{x}$ $(= \pi x_H + (1-\pi)x_L)$ the expected interim pay-off and with Δx $(= x_H - x_L)$ the difference between the pay-off in the high and the low state. At date 1, an investment in the project can be made to affect returns at date 2. In particular, an investment of z $(z \geq 0)$ returns zR $(R > 1)$ with probability $p(z) = a - \frac{z}{2}$ $(a \in (\frac{1}{2}, 1))$ at date 2, with probability $1-p$ that the project returns zero. Note that there is hence a trade-off between bank safety (low z) and return (high z), as for example in Allen and Gale (2000). The choice of z can be interpreted as a (short-term) scaling up of lending activities in response to shocks and the assumed technology captures that doing so may lead to lower lending quality (consistent with evidence that lending growth increases bank risk, e.g. Sinkey and Greenawalt (1991) and Foos et al. (2010)). The banker also has to decide how much of z to finance externally, using deposits d. Deposits are raised from the consumer and are fully insured. The deposit insurance fund is financed through lump sum taxation from the consumer at date 2.

There is a regulator who maximizes utilitarian welfare. The regulator sets a rule that has to be obeyed by bankers at date 1. Specifically, the regulator sets an upper limit $\overline{d}$ $(\overline{d} \geq 0)$ on the amount of debt each bank can raise. Alternatively, this can be interpreted as a requirement that capital at the bank has to exceed $z - \overline{d}$. The primary purpose of the capital constraint is to induce efficient investment as bankers would otherwise choose an excessive level of z due to deposit insurance and because of the cost of joint failure (Section 3.5 discusses the two sources of inefficiency in more detail). We assume that the return on the common (economy-wide) project is observable to the regulator, but not on the bank-specific one. The regulator can hence condition capital requirements only on the return of the common project.[7]

We make the following assumptions about parameter values:

[7] This captures that a regulator may be able to set capital requirements based on the state of the economy, but not on conditions at an individual bank.

Assumptions

1. $x_H < a - \frac{1+\frac{\gamma}{4}}{R}$,

2. $\omega + 2x_L > 2(a - \frac{1+\frac{\gamma}{4}}{R})$,

3. $\Delta x < \frac{-aR^2+R+(a-\frac{1+\frac{\gamma}{4}}{R})(R+\frac{\gamma}{4})}{2(R+\frac{\gamma}{4})(\frac{1}{2}-R)(1-\frac{\pi}{2})}$.

Assumption 1) is an upper bound on the interim return, which ensures that external finance is always needed to implement efficient investment. This assumption also guarantees that efficient investment is strictly positive. Assumption 2) states that the consumer's endowment, together with the interim returns, are always sufficient to undertake efficient investment in the economy. Assumption 3) constrains the differences in the interim returns across states. This assumption ensures that optimal capital requirements are also binding in the case where the return on the common project is low (and hence optimal capital requirements lax) but a bank operating the alternative project has a high return (and hence plenty of capital).

Timing

The sequence of actions is as follows. At date 0, the regulator announces how date-1 capital requirements will be set depending on whether the interim return on the economy-wide project is high (H) or low (L). These capital requirements can be summarized by $\overline{d}_H$ and $\overline{d}_L$ (a higher $\overline{d}$ translates into a looser constraint); the special case of flat capital requirements arises when $\overline{d}_H = \overline{d}_L$. Following this, bank A makes its project choice. At date 1, the interim returns materialize at each bank. In the correlated economy, these returns are $\widetilde{x}^C$ at both banks, in the uncorrelated bank one bank has $\widetilde{x}^C$ and the other $\widetilde{x}^U$. Each banker decides how much to invest in the project, z_i, and how much to finance this with capital, k_i. The shortfall, $z_i - k_i$, is financed through deposits and has to respect capital requirements (that is, it cannot exceed $\overline{d}_s$, $s \in \{H, L\}$). Following this, the consumer and the bankers consume.

Figure 1: Timing of actions

$t = 0$	$t = 1$	$t = 2$
The regulator announces state-dependent capital requirements $\bar{d}_H$ and $\bar{d}_L$.	Interim returns x_i realize. Bankers decide on scale of project z_i, the amount of equity finance (k_i) and debt finance ($z_i - k_i \leq \bar{d}_s$). Agents consume.	Successful projects return Rz_i. Depositors are paid. Agents consume.

At date 2, the projects mature and pay off in the case of success. Each banker repays the consumer – if there are sufficient funds. Any shortfall is covered by the deposit insurance fund. Finally, all agents consume.

Figure 1 summarizes the timing.

2.3 Benchmark: Project choice is observable

To establish a benchmark, we first analyze an economy in which the project choice is observable and hence can be directly controlled by the regulator. The regulator's actions at the beginning of date 0 then consist of setting capital requirements $(\bar{d}_H, \bar{d}_L)$ and the project type for bank A.

We first solve for the bankers' decisions at date 1. At this date, each banker has to decide on the level of investment (z_i) as well as on the funding, that is, how much capital (k_i) to put into the bank and how much debt $(z_i - k_i)$ to raise. A banker's expected utility at this date is

$$E_1[u(c_{i,1}, c_{i,2})] = c_{i,1} + E_1[c_{i,2}] = x_i - k_i + p(z_i)(z_i R - (z_i - k_i)). \tag{3}$$

Defining with $f(z_i) := p(z_i)z_i R - z_i$ the (expected) net return of the project, this can be written as

$$E_1[u(c_{i,1}, c_{i,2})] = f(z_i) + (1 - p(z_i))(z_i - k_i) + x_i. \tag{4}$$

Equation (4) shows that a banker maximizes the project's surplus, $f(z_i)$, plus the value of the put-option arising from limited liability, $(1 - p(z_i))(z_i - k_i)$.

We can think of the banker's maximization problem as choosing the amount of equity, k_i, and debt, $d_i = z_i - k_i$ (which then jointly determines the scale of the project). The maximization problem for bank i can hence be stated as

$$\max_{k_i, d_i} \; f(k_i + d_i) + (1 - p(k_i + d_i))d_i + x_i, \text{ s.t. } k_i \leq x_i \text{ and } d_i \leq \overline{d}_s \; (s \in \{H, L\}) \tag{5}$$

The first constraint ($k_i \leq x_i$) states that capital cannot exceed the banker's endowment, the second constraint ($d_i \leq \overline{d}_s$) reflects the capital requirements.

From differentiating (4) with respect to k_i and d_i, we obtain the first order conditions for equity and debt:

$$f'(z_i) - p'(z_i)(z_i - k_i) = 0, \tag{6}$$

$$f'(z_i) - p'(z_i)(z_i - k_i) + 1 - p(z_i) = 0. \tag{7}$$

Comparing the LHS of (6) and (7) we can see that the marginal benefit from equity financing is strictly lower than financing through debt. This is because of the put-option value of debt financing.

We assume in the following that capital requirements $\overline{d}$ are binding in that a banker that operates the bank at the maximum permissible scale, that is, he invests his entire endowment at date 1 ($k_i = x_i$) and scales the bank up to the capital constraint ($z_i = x_i + \overline{d}_s$). Since a banker has a lower benefit from financing with equity, the condition for this is that at $k_i = x_i$ and $z_i - k_i = \overline{d}_s$, the banker would benefit from further scaling up the bank using equity:

$$f'(z_i) - p'(z_i)(z_i - x_i) > 0. \tag{8}$$

We verify later that this inequality is indeed fulfilled for optimally set capital requirements.

Note that – for given capital requirements – higher availability of capital at date 1 leads to more risk at date 2 since a banker will use the spare capacity to invest in more risky activities. This captures the idea that "a boom lies the seeds for a crisis" (Schularick and Taylor (2012) provide evidence that crises are preceded by expansionary periods; Gorton and Ordoñez (2016) provide a theoretical justification for this where after an initially desirable expansion, firms with lower credit quality are admitted, eventually resulting in a crisis).

2.3.1 The regulator's problem

The regulator maximizes welfare W, consisting of the utilities of bank owners and the consumer. Since capital requirements are binding at date 1, they directly determine investment levels.[8] For a capital requirement of $\overline{d}_s$, the resulting scale of the common project and the alternative project are $z^C = x^C + \overline{d}_s$ and $z^U = x^U + \overline{d}_s$. Note that since capital requirements can be conditional on the return of the common project, the regulator can fully control the scale of the common project. Instead of the regulator choosing capital requirements, we can hence alternatively view the regulator's problem as one of choosing the scale of the common project in the high and the low state. However, the regulator cannot control the scale of the alternative project. In particular, we have for the scale of the alternative project that $z^U = x^U + \overline{d}_s = z^C + (x^U - x^C)$. The scale of the alternative project will hence differ from that of the common project whenever $x^A \neq x^C$.

Since the utility of agents is linear except for the consumer's subsistence need, welfare consists of total expected consumption in the economy, minus the expected utility loss from not reaching the subsistence level. Total expected consumption equals the endowment plus (expected) interim returns, $\omega + \widehat{x}$, plus the surplus from undertaking two projects, $f(z_A)+f(z_B)$. The losses arising for the consumer when date-2 consumption is not (strictly) positive can be determined as follows. The consumer's budget at date 2 consists of her

[8]This is because bank equity is given at date 1. If banks could raise external funds (as emphasized by recent literature, see, for example, Martin et al. (2013)), it will become more difficult for the regulator to control investment. In this case, capital requirements may only indirectly influence investment levels by affecting bank risk-taking incentives (as in, e.g. Furlong and Keeley (1989)).

combined payments from both banks (the payments to and from the deposit insurance fund cancel out). These payments will be positive when at least one bank's project succeeds. Thus, the utility loss γ only arises when both banks do not succeed. We can hence also interpret this as a systemic cost arising from joint bank failures.

Consider first the correlated economy. In this economy, two common projects are undertaken. With probability π they will both reach the high state and investment will be z_H. The expected surplus per project is then $f(z_H)$. The likelihood of both projects failing at date 2 is $1 - p(z_H)$, in which case the cost γ is incurred. Similarly, with probability $1 - \pi$, the low state is reached with investment level z_L. In the low state, the expected project surplus is $f(z_L)$ and projects will jointly fail with probability $1 - p(z_L)$.

We can hence write welfare as

$$E[W^C] = \pi \left(2f(z_H) - (1 - p(z_H))\gamma\right) + (1 - \pi) \left(2f(z_L) - (1 - p(z_L))\gamma\right) + \omega + \widehat{x}. \tag{9}$$

In the uncorrelated economy, both projects reach the high state with probability π^2 only, as their returns are now independent. In this case, the pay-off is the same as in the correlated economy, except that the likelihood of joint failure at date 2 is $(1 - p(z_H))^2$ and not $1 - p(z_H)$. The low state is reached for both projects with probability $(1 - \pi)^2$. Again, the same pay-off as in the correlated economy arises, except for the lower likelihood of joint failure, $(1 - p(z_L))^2$.

In the other cases, the common and alternative projects reach different states at date 1. Recall that the scale of the alternative project is $z^U = z^C + (x^U - x^C)$. Consider first the case of the common project being in the high state, and the alternative project being in the low state. This case arises with probability $\pi(1 - \pi)$. The joint surplus from both projects is then $f(z_H) + f(z_H - \Delta x)$ and the likelihood of joint failure is $(1 - p(z_H))(1 - p(z_H - \Delta x))$. Similarly, we have the case when the common project reaches the low state and the alternative project the high state, again occurring with probability $\pi(1 - \pi)$. In this case the joint surplus is $f(z_L) + f(z_L + \Delta x)$ and the likelihood of joint failure is

$(1-p(z_L))(1-p(z_L+\Delta x))$. Combining we obtain welfare in the uncorrelated economy:

$$E[W^U] = \pi^2\left(2f(z_H) - (1-p(z_H))^2\gamma\right) + (1-\pi)^2\left(2f(z_L) - (1-p(z_L))^2\gamma\right) \tag{10}$$
$$+\pi(1-\pi)(f(z_H) + f(z_H - \Delta x) - (1-p(z_H))(1-p(z_H-\Delta x))\gamma)$$
$$+\pi(1-\pi)(f(z_L) + f(z_L + \Delta x) - (1-p(z_L))(1-p(z_L+\Delta x))\gamma) + \omega + \widehat{x}.$$

We next solve for optimal investment (that is, optimal capital requirements) for given project choices. Consider first the correlated economy (that is, bank A has chosen the common project). Differentiating welfare (equation 9) with respect to z_H and z_L, we obtain the following first-order conditions:

$$f'(z_H) + \frac{p'(z_H)\gamma}{2} = 0, \tag{11}$$
$$f'(z_L) + \frac{p'(z_L)\gamma}{2} = 0 \tag{12}$$

Optimal investment hence requires equating the marginal increase in output $(f'(z_H))$ with higher expected costs due to systemic failures $(\frac{p'(z_H)\gamma}{2} < 0)$. The latter effect arises because larger projects have a higher likelihood of failure. Note that the first order conditions do not depend on the interim returns. The optimal scale hence does not depend on the state of the economy.

Optimal investment in the uncorrelated economy differs from that of the correlated economy. This is because capital requirements will then also affect investment for the uncorrelated project, which may result in investment levels that differ from that in the correlated economy. Proposition 1 states the optimal investment levels in the correlated and uncorrelated economies.

Proposition 1 *The optimal investment in the correlated economy is*

$$z^{*C} = z_H^{*C} = z_L^{*C} = a - \frac{1+\frac{\gamma}{4}}{R}, \tag{13}$$

while the optimal investments in the uncorrelated economy are

$$z_H^{*U} = \frac{a(R+\frac{\gamma}{2})-(1+\frac{\gamma}{2})}{R+\frac{\gamma}{4}} + (1-\pi)\frac{\triangle x}{2}, \tag{14}$$

$$z_L^{*U} = \frac{a(R+\frac{\gamma}{2})-(1+\frac{\gamma}{2})}{R+\frac{\gamma}{4}} - \pi\frac{\triangle x}{2}. \tag{15}$$

Proof. *See Appendix.* ■

From the proposition we have that the optimal scales decrease in the cost of joint failure γ. This is because a higher scale makes projects riskier, and increases the likelihood of failure. A larger cost of joint failure thus makes it optimal to have safer projects and hence a lower scale.

While in the correlated economy the optimal scale is state-independent, this is not the case in the uncorrelated economy. Comparing equation (14) and (15) we can see that the optimal scale is larger in the high state. The reason is as follows. When the common project is in the high state, there is a chance of $1-\pi$ that the alternative project will be in the low state. The (marginal) gain from investing is then strictly higher for the alternative project than for the common project ($f'(z-\Delta x) > f'(z)$). This implies that the optimal scale lies above the one that is optimal if there are only common projects. Similarly, when the common project is in the low state, there is a chance that the alternative project will be in the high state. In this case, the alternative project has a lower marginal product, making it optimal to require a lower scale.

What can be said about the relative magnitude of the average scales in the two economies? Let us define with $\widehat{z}^{*U} := \pi z_H^{*U} + (1-\pi)z_L^{*U}$ the *expected* scale in the uncorrelated economy. From (14) and (15) we have

$$\widehat{z}^{*U} = \frac{a(R+\frac{\gamma}{2})-(1+\frac{\gamma}{2})}{R+\frac{\gamma}{4}}, \tag{16}$$

which is strictly larger than z^{*C}. The reason for this is that the cost of scaling up is lower in the uncorrelated economy: a higher level of investment increases the likelihood of project failure; the systemic cost of this is lower in the uncorrelated economy as in this economy it is less likely that projects fail jointly.

We now turn to the cyclical properties of capital requirements. We call capital requirements procyclical if they are loosened when the common project is in the high state. In our setting this implies $\overline{d}_H > \overline{d}_L$, that is, the constraint set by the regulator is less strict in states of the world where bank capital is high. Conversely, capital requirements are countercyclical if $\overline{d}_H < \overline{d}_L$; flat capital requirements arise for $\overline{d}_H = \overline{d}_L$. The degree of cyclicality can be measured by $\left|\overline{d}_H - \overline{d}_L\right|$.

Corollary 1 *Optimal capital requirements are countercyclical:* $\overline{d}_H^* < \overline{d}_L^*$.

Proof. *In the correlated economy we have that the optimal scale is cycle independent:* $z_H^{*C} = z_L^{*C}$. *Using that* $z = x + \overline{d}$, *this implies* $\overline{d}_H - \overline{d}_L = -\Delta x < 0$. *In the uncorrelated economy we have that* $\overline{d}_H^* - \overline{d}_L^* = z_H^* - x_H - (z_L^* - x_L) = -\frac{\triangle x}{2} < 0$. ■

The reason for this result is the following. In the correlated economy, optimal investment is independent of the state. However, banks have more capital in the high state than in the low state. For flat capital requirements they would thus lend more in the high state than in the low state, which would be inefficient. Capital requirements thus have to be stricter in the high state.[9] In the uncorrelated economy, the optimal scale in the high state is higher than in the low state because of the presence of the alternative project. However, this effect cannot outweigh the effect that is present in the correlated economy and capital requirements are still countercyclical.

A further direct corollary is

[9] The reason why capital requirements are countercyclical is that we have shocks to equity availability while optimal investment does not change. If the project's expected productivity at date 2 were higher in the high state, there would also be benefits to *loosening* capital requirements in order to allow the banker to invest more. Productivity shocks have been analyzed by Malherbe (2013), who shows that optimal capital regulation can be procyclical in the presence of such shocks.

Corollary 2 *The optimal degree of countercyclicality is lower in the uncorrelated economy:* $\left|\overline{d}_H^{*C} - \overline{d}_L^{*C}\right| > \left|\overline{d}_H^{*U} - \overline{d}_L^{*U}\right|$.

Proof. *Follows directly from* $\overline{d}_H^{*C} - \overline{d}_L^{*C} = -\Delta x$ *and* $\overline{d}_H^{*U} - \overline{d}_L^{*U} = -\frac{\triangle x}{2}$. ■

The reason is that while in the correlated economy countercyclical capital requirements improve the efficiency of investment at both banks, in the uncorrelated economy they only do so at one bank. The gains from countercyclicality are thus lower in the uncorrelated economy, and hence it is optimal to choose a lower degree of it.

Proposition 1 states the optimal policy rule for given projects. Whether it is optimal to have correlated or uncorrelated projects in the economy can be determined by comparing the welfare levels that obtain in either case, presuming that the regulator implements the optimal policy rules as given in Proposition 1.

In order to obtain an intuition for what determines the optimal project choice, let us for the moment consider that the regulator imposes a capital requirement rule irrespective of the correlation choice. We denote the resulting scales with z_H and z_L for the high and low state, respectively. Taking the difference between welfare in the correlated and uncorrelated economies, we obtain

$$E[W^C] - E[W^U] = \pi(1-\pi)(f(z_H) - f(z_H - \Delta x) + f(z_L) - f(z_L + \Delta x)) \qquad (17)$$
$$-\pi(1-p(z_H))p(z_H)\gamma - (1-\pi)(1-p(z_L))p(z_L)\gamma - (1-\pi)\pi(p(z_L) - p(z_H))\gamma\frac{\triangle x}{2}.$$

Generally, differences in welfare arise because in the correlated economy two common projects are operated, while in the uncorrelated economy there is one common and one alternative project. The expression in the first line is the welfare impact arising because investment levels can differ between the common and alternative project and hence surplus differs. If the common and the alternative project have the same interim return, the scales are the same, and no difference arises. However, this is not the case when they have different interim returns. With probability $\pi(1-\pi)$ the correlated project is in the high state but the alternative project is in the low state. In this case investment in the common project is z_H while in the alternative project it is Δx less $(z_H - \Delta x)$. With probability $\pi(1-\pi)$ we also get the opposite case: the correlated project is in the low state while the alternative project is in the high state. Investment levels in the common and alternative project are then z_L and $z_L + \Delta x$. In sum, the differences in surplus whenever common and alternative projects are in different states are $f(z_H) - f(z_H - \Delta x) + f(z_L) - f(z_L + \Delta x)$.

This expression represents the benefit from correlation. The benefit arises because in a correlated economy both banks profit from countercyclical capital requirements (while in the uncorrelated economy only one bank benefits). To see this, suppose that we have capital requirements that fully smooth out the impact of interim returns on investment. In this case, investment in the high and the low states are identical for the common project: $z_H = z_L = z$. The first line in (17) then simplifies to

$$2\pi(1-\pi)\left(f(z) - \frac{f(z-\Delta x) + f(z+\Delta x)}{2}\right). \tag{18}$$

By an application of Jensen's inequality, this expression is larger than zero since the production function $f(.)$ is concave. More generally, the benefits are strictly positive whenever capital requirements reduce fluctuations in investment levels, $z_H - z_L$, below the level that would prevail under flat capital requirements.

Lemma 1 *For $z_H - z_L \in [0, \Delta x)$ we have that $\pi(1-\pi)(f(z_H) - f(z_H - \Delta x) + f(z_L) - f(z_L + \Delta x)) > 0$.*

Proof. *We can write the expression as $(1-\pi)\pi\left(\int_{z_H-\Delta x}^{z_H} f'(t)dt - \int_{z_L}^{z_L+\Delta x} f'(t)dt\right) = (1-\pi)\pi\int_0^{\Delta x}(f'(t+z_H-\Delta x) - f'(t+z_L))dt$. We have $z_H - \Delta x < z_L$ by assumption. It follows that $f'(t+z_H-\Delta x) > f'(t+z_L)$ by the concavity of $f(x)$. Hence the integral is positive.* ■

By contrast, for flat capital requirements $(z_H - z_L = \Delta x)$ the benefits are zero as in this case $f(z_H) - f(z_H - \Delta x) + f(z_L) - f(z_L + \Delta x) = 0$.

The expressions in the second line in (17) represent the cost of correlation. The first two terms, $-\pi(1-p(z_H))p(z_H)\gamma - (1-\pi)(1-p(z_L))p(z_L)\gamma$, are the welfare impact arising because the likelihood of joint failures is lower in the uncorrelated economy as projects are not correlated at date 2. Finally, the term $-(p(z_L) - p(z_H))\gamma\frac{\triangle x}{2}$ in (17) gives the impact on joint failures arising from the fact that success probabilities at date 2 are also different because of differences in date-1 investment levels. This effect overall reduces the likelihood of joint failures since when investment in the common project is high (and hence failure risk is also high), in the uncorrelated economy with probability $1 - \pi$ a low scale project (and hence a relatively safe project) is undertaken. In this case, alternative investment reduces joint failure risk. It is true that the opposite case also arises (the alternative project is in the high state while the common project is in the low state) – but the impact of this on joint failure risk is lower since in this case the risk of failure of the common project is low. Taken together, systemic costs are thus higher in the correlated economy – as expected.

Correlation thus leads to a trade-off between higher costs from joint failures and higher benefits from countercyclical policies. When the regulator tailors capital requirements to the correlation choice, additional effects to those discussed arise because optimal capital requirements depend on correlation in the economy.

The following proposition shows that it is possible for correlation to be optimal, but also for no correlation to be optimal.

Proposition 2 *When the project choice is observable the uncorrelated economy is optimal if and only if the following inequality holds:*

$$\Delta x < 2\sqrt{\frac{2f(\widehat{z}^{*U}) - 2f(z^{*C}) + (1 - p(z^{*C}))\gamma - (1 - p(\widehat{z}^{*U}))^2\gamma}{\pi(1-\pi)(3R - \frac{\gamma}{4})}}.$$

For a sufficiently small Δx *the uncorrelated economy is optimal, while for a sufficiently small* γ *the correlated economy is.*

Proof. *See Appendix.* ■

A correlated economy is preferred when joint failure costs are small, as then the cost of higher correlation weighs less. In addition, when the difference between the high and the low state becomes small at the interim, there is limited scope for adjusting capital requirements in response to shocks. In this case there is only a small benefit from having more projects in the economy to which capital requirements can be tailored. In such a constellation, it is optimal to implement an uncorrelated economy.

The result that correlation can be optimal is noteworthy as most of the literature has emphasized the cost of correlation in a systemic context. The benefit of correlation arises because, due to the concavity of production, there is a gain from stabilizing "output" in the economy. When the shocks that hit banks are similar (which is the case in the correlated economy), stabilizing policies are more effective as the variability of investment can then be lowered at each individual bank.

2.4 Optimal capital requirements when project choice is unobservable

We now assume that the regulator cannot observe the project type. The consequence is that the correlation choice has to be privately optimal for bank A. Specifically, at date 0 the regulator announces capital rules $\overline{d}_H$ and $\overline{d}_L$, and following this bank A chooses a project.

Consider first the case that the common project is chosen. With probability π the bank's project reaches the high state. The banker then faces a scale restriction z_H $(= x_H + \overline{d}_H)$ and raises $\overline{d}_H$ from depositors. With probability $p(z_H)$ the project succeeds and yields $z_H R - \overline{d}_H$ to the banker. With probability $1 - p(z_H)$ the project fails and the banker defaults and receives nothing. Total expected consumption is hence $p(z_H)(z_H R - \overline{d}_H) = x_H + f(z_H) + (1 - p(z_H))(z_H - x_H)$. Similarly, with probability $1 - \pi$ he reaches the low state with an expected pay-off of $x_L + f(z_L) + (1 - p(z_L))(z_L - x_L)$.

The expected utility of the banker is hence

$$E[W_A^C] = \pi(f(z_H)+(1-p(z_H))(z_H-x_H))+(1-\pi)(f(z_L)+(1-p(z_L))(z_L-x_L))+\widehat{x}. \quad (19)$$

A banker who chooses the alternative project faces more cases. With probability π^2 his project reaches the high state at the same time as the common project. In this case his pay-off is $x_H + f(z_H) + (1 - p(z_H))(z_H - x_H)$ in expectation, as above. With probability $(1-\pi)^2$ he reaches the low state jointly with the common project and his expected pay-off is $x_L + f(z_L) + (1 - p(z_L))(z_L - x_L)$. With probability $\pi(1-\pi)$ he reaches the low state while the common project pay-off is high. The project's scale is then $z_H - \Delta x$ and utility is $x_L + f(z_H - \Delta x) + (1 - p(z_H - \Delta x))(z_H - x_H)$. Equally, with probability $\pi(1-\pi)$ he reaches the high state and the common project pay-off is low. Utility is then $x_H + f(z_L + \Delta x) + (1 - p(z_L + \Delta x))(z_L - x_L)$. Putting these together we obtain for the expected return:

$$
\begin{aligned}
E[W_A^U] = {} & \pi^2(f(z_H) + (1 - p(z_H))(z_H - x_H)) + (1-\pi)^2(f(z_L) + (1 - p(z_L))(z_L - x_L)) \\
& + \pi(1-\pi)(f(z_H - \Delta x) + (1 - p(z_H - \Delta x))(z_H - x_H)) \\
& + \pi(1-\pi)(f(z_L + \Delta x) + (1 - p(z_L + \Delta x))(z_L - x_L)) + \widehat{x}. \qquad (20)
\end{aligned}
$$

From this we can derive the net gain from choosing the common (as opposed to the alternative project):

$$
E[W_A^C] - E[W_A^U] = \pi(1-\pi)(R - \frac{1}{2})\Delta x(\Delta x - (z_H - z_L)). \qquad (21)
$$

We hence have that $E[W_A^C] - E[W_A^U] > 0$ whenever $\Delta x > z_H - z_L$. Since $\Delta x > z_H - z_L$ is equivalent to $\overline{d}_H < \overline{d}_L$, this means that the common project is chosen whenever the capital requirements are countercyclical. Assuming a weak preference for the alternative project, we hence obtain for the correlation choice:

Proposition 3 *Banks choose correlated projects if and only if the policy rule is countercyclical ($\overline{d}_H < \overline{d}_L$).*

Proof. Follows directly from (21). ■

The reason for this is the following. The banker maximizes the expected benefits from operating a (debt-financed) project. Due to the concavity of production, these benefits are higher when there is less fluctuation in investment scales. Since under countercyclical regulation investment scales vary less when the common project is chosen, the banker thus prefers such a project.

The project choice is, however, not necessarily socially efficient. This is, first, because a banker ignores the impact on the consumer – who suffers in the event of joint failure. Since the likelihood of joint failure is higher for correlated projects, choosing the common project is associated with a negative externality. The banker thus has a bias for the common project, relative to what is desirable from the perspective of welfare. There is a second effect, arising because of changes in the value of the put-option, $(1 - p(z))(z - x)$. The latter is a convex function of the investment scale z, implying that the banker will prefer a higher variability. However, Proposition 3 shows that under a countercyclical rule this effect is dominated by the first one.

The consequence is that an inefficient project choice occurs whenever the policy rule is countercyclical (and bank A hence chooses correlation) but no correlation is welfare-optimal:

Corollary 3 *For given capital requirements $\overline{d}_H$ and $\overline{d}_L$, banks may choose correlated projects even though zero correlation leads to higher welfare. This occurs precisely when $\overline{d}_H < \overline{d}_L$ and $E[W^U(\overline{d}_H, \overline{d}_L)] > E[W^C(\overline{d}_H, \overline{d}_L)]$.*

It follows that there are situations where the welfare level of the benchmark case can no longer be obtained. In fact, this happens whenever in the benchmark uncorrelated projects are optimal. Since welfare-maximizing regulation (in the benchmark case) requires countercyclical capital requirements, banks would find it privately optimal to choose correlated projects, necessarily resulting in lower welfare:

Corollary 4 *Whenever $E[W^{*U}] > E[W^{*C}]$, attainable welfare is lower than in the benchmark case.*

The regulator's problem

When correlation is optimal in the benchmark case (that is, when $E[W^{*C}] > E[W^{*U}]$), the regulator can still obtain the same level of welfare as before. For this he simply implements the scales z_H^{*C} and z_L^{*C} and banks (efficiently) choose correlated projects. In the case where the benchmark stipulates no correlation, we know that we can no longer reach the welfare level of the benchmark case as optimal capital requirements are countercyclical and would hence induce banks to choose correlated projects (Corollary 3). This still leaves open what the regulator should do in this case.

Suppose first that the regulator implements correlation in the economy. In this case the regulator is not constrained by banks' private incentives (since banks have a bias towards correlation). The regulator can hence implement the scales of the benchmark case (z_H^{*C} and z_L^{*C}). Consider next that the regulator wants to implement an uncorrelated economy. In this case, the regulator is constrained by the incentive compatibility constraint of bank A. Proposition 3 tells us that he then has to choose a policy that is not countercyclical. Since procyclical policies cannot be optimal, he will hence choose flat (state-independent) capital requirements ($\overline{d}_H = \overline{d}_L$).

Proposition 4 derives conditions under which the correlated and uncorrelated economies are optimal as well as the corresponding investment levels.

Proposition 4 *Flat capital requirements are optimal when project choice is observable if and only if:*

$$\Delta x < \sqrt{\frac{2f(\widehat{z}^{*U}) - 2f(z^{*C}) + (1-p(z^{*C}))\gamma - (1-p(\widehat{z}^{*U}))^2\gamma}{\pi(1-\pi)R}}.$$

For a sufficiently small Δx the inequality holds, the regulator implements investment scales

$$\widehat{z}_H^U = \frac{a(R+\frac{\gamma}{2}) - (1+\frac{\gamma}{2})}{R+\frac{\gamma}{4}} + (1-\pi)\Delta x, \quad (22)$$

$$\widehat{z}_L^U = \frac{a(R+\frac{\gamma}{2}) - (1+\frac{\gamma}{2})}{R+\frac{\gamma}{4}} - \pi\Delta x, \quad (23)$$

and the economy is uncorrelated. For a sufficiently small γ the inequality does not hold, the regulator implements investment scale z^{*C} and the economy is correlated.

Proof. *See Appendix.* ■

Underlying this result is a trade-off between the variability of shocks (Δx) and the cost of joint failures (γ). When the difference between capital in the high and the low states is large, there are higher benefits of correlation (as correlation allows to better smooth shocks) and correlation is desirable. However, when γ is high, correlation brings about significant costs to consumers, making it undesirable. This suggests that countercyclicality is more desirable in volatile economies with low cost of systemic crises.

Figure 2 depicts the trade-off for various parameter constellations. It shows indifference curves, that is, combinations of Δx and γ for which the regulator is indifferent to correlation (for points above the line, no correlation is preferred, below the line the economy is better off with correlation). It can be seen that the trade-off is not linear: for larger values of volatility, increasingly higher levels of systemic costs are needed to offset the gains from correlation. The intuition for this is that volatility induces welfare losses by causing inefficient production, and this cost is increasing because of concavity of the production function. For highly volatile countries (such as emerging economies), there is thus a particularly high cost to giving up the benefit from correlation and countercyclicality is more desirable for them.

Figure 2a and 2b show the comparative statics with respect the productivity of investment (captured by R and a in our model). It shows that an increase in productivity (large R or a) enlarges the area where correlation is optimal. The reason is that when production is more valuable, the cost from inefficient fluctuations due to shocks is higher, and hence smoothing becomes more valuable. Figure 2c considers the impact of uncertainty (moving π away from $\frac{1}{2}$ increases uncertainty). The area where correlation is optimal increases when there is more uncertainty – this is because the gains from smoothing of shocks are higher in the presence of significant uncertainty.

Proposition 4 has an interesting implication for financial regulation: it implies that whenever it is optimal to implement uncorrelated projects, the regulator has to reduce the countercyclically of capital requirements (compared to the benchmark case). The message is thus that in the presence of risk-shifting by banks, regulators should be less aggressive in smoothing financial cycles. In a recent paper, Martínez-Miera and Suarez (2012) demonstrate that in the presence of risk-shifting, optimal capital requirements may even be procyclical. They consider a model where capital requirements affect the value of capital to surviving banks (the "last-bank standing effect") and show for a parameterization that it can be optimal to tighten capital requirements following a negative shock, in order to increase incentives to become a surviving bank. While their effect is derived from the last-bank-standing effect that limits risk-taking, in our paper countercyclicality affects the incentives to correlate by reducing the cost of being exposed to aggregate shocks (another difference to Martínez-Miera and Suarez is that correlation itself can be desirable from a welfare perspective, precisely because of lower costs of aggregate shocks).

2.5 The role of commitment

We have assumed that at the beginning of date 0, the regulator can commit to a policy rule. We now relax this assumption. We assume that the regulator decides on the policy rule at the same time as when projects are chosen. Specifically, the regulator and bank A play Nash at date 0: the regulator maximizes welfare taken as given the project choice of bank A, while banker A maximizes his utility taken as given the policy function.

Consider first a candidate equilibrium where correlated projects are chosen. In such an equilibrium, the best response of the regulator is z_H^{*C}, z_L^{*C} (since z_H^{*C}, z_L^{*C}, by Proposition 1, is the optimal policy given that projects are correlated). Since z_H^{*C}, z_L^{*C} implies counter-cyclical capital requirements, it is also optimal for bank A to choose the common project (Proposition 3). Correlation and scales z_H^{*C}, z_L^{*C} thus form an equilibrium.

Consider next a (candidate) equilibrium with uncorrelated projects. The regulator's best response to an uncorrelated economy is z_H^{*U} and z_L^{*U}. However, since this policy is countercyclical, a bank would want to choose the common project. An equilibrium with uncorrelated projects hence cannot exist.

We summarize:

Proposition 5 *When the regulator lacks commitment, the unique equilibrium is one where correlated projects are chosen and scale limits* z_H^{*C}, z_L^{*C} *are imposed.*

In the case where no correlation was optimal in the absence of commitment problems, welfare is now lower. Lack of commitment thus amplifies the cost of countercyclical policies arising from banks' correlation incentives. Interestingly, this means that endowing regulators with countercyclical tools may reduce welfare in the economy if such tools do not come with commitment.[10]

3 Extensions

3.1 Diversification

In the baseline model, bank A had a choice between a common and an alternative project. One can also consider a third option: a mixed project that diversifies the risk of the common and the alternative project. Such a project has two types of social benefits (the formal analysis of diversification can be found in the internet appendix). First, diversification makes date-1 returns less volatile. This leads to higher welfare as fluctuations in investment levels are lowered. Second, diversification can reduce the likelihood of joint failures at date 2. Notwithstanding these benefits, it turns out that a bank does not necessarily prefer the

[10]Whereas Basel III sets out benchmarks for when buffers should be considered, the actual decision whether to invoke them lies within the respective countries: "The relevant authority in each jurisdiction will ... make *assessments* of whether such growth is excessive ... they will need to use their *judgement* ... to determine whether a countercyclical buffer requirement should be imposed." [emphasis added] (Basel Committee on Banking Supervision (BCBS), 2010).

diversified project. The reason is that the bank wants to maximize the put-option value arising from limited liability (as discussed in Section 2.3). As there are fewer failures under diversification, a banker's benefit is lowered when choosing the diversified project. This creates an additional wedge between the incentives of the bank and the regulator: besides the bias for correlation, the banker now also has a bias for being undiversified. Capital requirements thus also have to address this wedge. It can be shown that optimal capital requirements are still countercyclical and implement either a correlated or a diversified economy (the undiversified economy is always dominated). In addition, bankers may still have a bias for correlation, as in the baseline model.

3.2 Capital reallocation

In the baseline model, the allocation of capital across banks is inefficient: A bank that has a large amount of capital at date 1 can invest more than a bank with low capital – even though their production opportunities are the same.

We now consider the possibility for banks to exchange equity at date 1 (there is no role for *lending* among banks as lending cannot alleviate the regulatory constraint), which allows banks reap gains from trade. The allocation of capital across banks can be given two interpretations. First, there can be a competitive market for bank capital at date 1. Alternatively, we can think of the two banks being part of a conglomerate, where each bank chooses its project (and is subject to own regulatory requirements) but capital is allocated centrally. We focus our exposition on the first interpretation.

We now assume that at date 1 banks can raise capital from each other. Raising equity amounts to obtaining funds from the other bank in return for a promise of a share in its profits at date 2. Let us denote with r_e the price of a unit of equity, expressed in terms of date 2 consumption. Given that a bank's project only delivers a return at date 2 with a probability of p, this translates into a promise of $\frac{r_e}{p}$ in case of project success. We assume that banks behave as price-takers in the market for equity, that is, they take r_e as given.

At date 1, each bank decides to raise (or to provide) equity, in order to maximize its expected profits. The benefit from raising equity (that is, obtaining capital from the other bank by selling a stake of date-2 output) is that it allows a constrained bank to expand its investment. In particular, a bank that raises an amount of equity q_i can expand its investment from $x_i + \overline{d}$ to $x_i + q_i + \overline{d}$. Similarly, a bank that provides funds ($q_i < 0$) has to lower investment.

The maximization problem of bank i at date 1 can be expressed as

$$\max_{q_i} \; x_i + f(x_i + q_i + \overline{d}) + (1 - p(x_i + q_i + \overline{d}))\overline{d} - r_e q_i, \tag{24}$$
$$\text{s.t. } q_i + x_i \geq 0 \text{ and } z_i R - \overline{d} \geq \frac{r_e}{p(x_i + q_i + \overline{d})} q_i.$$

The first inequality states that a bank cannot sell more than its date-1 endowment of capital, while the second inequality states that a bank cannot promise more than its profits at date 2. The market clearing condition takes the form of

$$q_A + q_B = 0. \tag{25}$$

The first order condition for q_i is

$$f'(z_i) + (1 - p'(z_i))\overline{d} = r_e, \tag{26}$$

where $z_i = x_i + q_i + \overline{d}$ denotes the bank's investment post-trade. From (26) we can see that the benefits from a unit of capital, $f'(z_i) + (1 - p'(z_i))\overline{d}$, will be equalized across banks. This in turn implies that investment levels are identical across banks: $z_i = z$. Using the market clearing condition, it follows that bank A raises funds (provides funds if negative) of an amount $\frac{x_B - x_A}{2}$ and bank B provides an amount of $\frac{x_B - x_A}{2}$. The level of investment at either bank is hence $z = \frac{x_A + x_B}{2} + \overline{d}$. Plugging into equation (26) and rearranging, gives us the equilibrium price of equity:

$$r_e(\overline{d}) = f'(\frac{x_A + x_B}{2} + \overline{d}) + (1 - p'(\frac{x_A + x_B}{2} + \overline{d}))\overline{d}. \tag{27}$$

Note that the allocation that arises after trade is the same as the one that obtains in a conglomerate with centralized capital allocation. A parent that maximizes the joint returns of both subsidiaries would also allocate capital such that the marginal product is equalized, and hence investment levels are again identical across banks.

How does trade affect welfare in the two economies? The correlated economy is not affected as the two banks then have always the same returns and there is hence no scope for trade. Welfare is then still given by equation (9). As for the uncorrelated economy, capital will be reallocated whenever the banks have different date-1 returns; in all other situations the outcome is the same as in the baseline model. Consider first the case where the common project has a high return and the alternative project has a low return, occurring with probability $\pi(1-\pi)$. Given that trading equalizes investment levels, we thus have that each bank will invest $\frac{x_H+x_L}{2}+\overline{d}_H = z_H - \frac{\triangle x}{2}$, while in the absence of an trade one bank would have invested $\frac{x_H}{2}+\overline{d}_H = z_H$ and the other $\frac{x_L}{2}+\overline{d}_H = z_H - \Delta x$. Similarly, when the common project returns the low amount, and the alternative the high amount, investment levels will be $z_L + \frac{\triangle x}{2}$, instead of z_L and $z_L + \Delta x$.

Total welfare in the uncorrelated economy is hence

$$E[W^{U,Trade}] = \pi^2\left(2f(z_H) - (1-p(z_H))^2\gamma\right) + (1-\pi)^2\left(2f(z_L) - (1-p(z_L))^2\gamma\right) \quad (28)$$
$$+\pi(1-\pi)(2f(z_H - \frac{\triangle x}{2}) - (1-p(z_H - \frac{\triangle x}{2}))^2\gamma)$$
$$+\pi(1-\pi)(2f(z_L + \frac{\triangle x}{2}) - (1-p(z_L + \frac{\triangle x}{2}))^2\gamma) + \omega + \widehat{x}.$$

Proposition 6 next shows optimal capital requirements are still countercyclical when project choices are observable, and in fact the same as when there is no market for equity

Proposition 6 *Optimal capital requirements in the uncorrelated economy are the same as in the absence of capital reallocation.*

Proof. *See Appendix.* ■

We next analyze how capital reallocation affects the desirability of a correlated economy as opposed to the uncorrelated economy. For the purpose of intuition, let us again consider the welfare effect of moving to a correlated economy for given capital requirements $\overline{d}_H$ and $\overline{d}_L$. Using (9) and (28) we have for the welfare gain from moving to a correlated economy:

$$\begin{aligned} E[W^C] - E[W^{U,Trade}] &= 2\pi(1-\pi)(f(z_H) - f(z_H - \frac{\triangle x}{2}) + f(z_L) - f(z_L + \frac{\triangle x}{2})) \quad (29) \\ &\quad -\pi(1-p(z_H))p(z_H)\gamma + (1-\pi)(1-p(z_L))p(z_L)\gamma \\ &\quad +\pi(1-\pi)((1-p(z_H - \frac{\triangle x}{2}))^2 + (1-p(z_L + \frac{\triangle x}{2}))^2)\gamma. \end{aligned}$$

As in the baseline model, the term in the first line is the effect arising from differences in the volatility of investment levels, while the other terms represent the costs from more joint failures in the correlated economy. Let us have a closer look at the effect arising from the volatility of investment. Suppose that we have fully countercyclical capital requirements: $z_H = z_L = z$. The benefits are then

$$2\pi(1-\pi)(2f(z) - f(z - \frac{\triangle x}{2}) - f(z + \frac{\triangle x}{2}). \quad (30)$$

This expression is positive and is the familiar benefit of correlation in the presence of countercyclicality. Suppose next that capital requirements are flat. In this case we have $z_H - z_L = \Delta x$ and the expression in the first line becomes

$$2\pi(1-\pi)(f(z_H) + f(z_L) - 2f(\frac{z_H + z_L}{2})). \quad (31)$$

This expression is negative because of the concavity of $f(.)$. There are two reasons for this. First, when capital requirements are flat, there is no longer the benefit from having a correlated economy. Second, the correlated economy foregoes the gains from trade that are available when banks choose different assets.

The presence of this second effect means that it is no longer clear whether investment levels are less volatile in the correlated economy. However, as the following proposition shows, it can still be optimal to have a correlated economy (meaning that capital requirements will be countercyclical):

Proposition 7 *There exist parameter values for which it is optimal to have countercyclical capital requirements.*

Proof. *See Appendix.* ■

The reason behind the result is that, although equity trading reduces the potential benefit from a correlated economy, it cannot eliminate it because the correlated economy may still offer a lower investment volatility. For sufficiently low costs of joint failure, this will make the correlated economy preferable to the uncorrelated one.

In this section we have focused on how trade affects the social benefits from correlation. The impact on the private benefits from correlation are straightforward: the presence of trade will also increase a bank's private gain from an uncorrelated economy, as it provides the bank with the chance that it can engage in trade with the other bank. This, however, does not create a wedge between private and social incentives, since the social benefits from a uncorrelated economy are higher as well. Hence, banks maintain a bias towards excessive correlation.

3.3 Strategic interaction among banks

There is no role for strategic interaction among banks in the baseline model. To see this, let us introduce an asset choice for bank B: the bank can either invest in the common asset (the one also accessible to bank A), or in an alternative asset that is unique to the bank (and not correlated with that of bank A). In the baseline model, bank B's choice has no effect on the profits of bank A. This can be appreciated by examining equations (9) and (10), which show that, for given capital requirements, bank A's profits only depend on the return on its own project.

The introduction of trade among banks considered in the previous section, however, introduces the possibility for interdependencies. This is because a bank's ability to use trade to smooth shocks depends on the project choice of the other bank. In the internet appendix we show that this causes banks' choices to become strategic substitutes. The reason is that banks benefit from being different from each other as this allows them to smooth shocks bilaterally. The incentives to be different in turn also alleviate the incentive compatibility constraint of the regulator. This provides the regulator with more freedom to impose welfare-improving countercylical capital requirements, while retaining the uncorrelated economy. A policy implication is thus that it may be beneficial to promote trading among banks, as this lowers banks' incentives to take on systemic risk.

3.4 Monopolistic behavior

We have assumed that banks behave competitively at date 0 and date 1. In the internet appendix we also analyze monopoly power, and in particular its consequences for the inefficiency in the bank's project choice. We show that in the presence of capital reallocation, monopoly power limits the distortions arising from a bank's incentives to correlate with the other bank. The reason is that a monopolistic bank will be able to extract more of the surplus from trade (arising in the case the banks are hit by different shocks). Thus the bank internalizes a larger part of the gains from being uncorrelated. It can however not extract the full benefits as the latter also accrue to the consumer. Correlation hence remains excessive.

3.5 Deposit insurance

We have assumed that deposits are insured – how does this influence the results? Deposit insurance causes banks to underprice risk because they do not internalize the losses that are borne by the deposit insurance fund. However, in our setting there is a second distortion, arising from joint failure risk. When a bank takes on more risk by scaling up, it also increases the likelihood of joint failures in the economy. This effect is not fully internalized by a risk-taking bank because it also affects the value of deposits at the other bank. Thus, even in the absence of deposit insurance risk-taking is not socially optimal and there is still a rationale for capital regulation.

We analyze the economy without deposit insurance in the internet appendix. Banks' incentives to correlate remain excessive. The reason is that – even though consumers have lower utility in the correlated economy – joint failure costs do not show up in the cost of deposits because the *marginal* benefit of a unit of deposits to the consumer (which determines the interest rate) does not differ between the two economies. It follows that the basic trade-off is the same as in the baseline model: countercylical capital requirements cause a tension between efficient investment and inefficient correlation.

3.6 Bank-specific capital requirements

The cost of countercyclical policies (in the form of higher correlation) can be avoided entirely if capital requirements can be made contingent on banks' individual project returns (x^A and x^B) instead of the return on the common project only (as we have assumed). In this case regulators can isolate each bank against shocks to its own capital, and there is no longer an incentive to increase exposure to common risk.

However, there are several reasons why such capital requirements may not be attractive nor feasible. First, there are higher informational requirements for setting bank-specific requirements. The regulator would need to observe (and verify) the return on the assets of each bank in the economy, while in the case of aggregate capital requirements, observing a single asset is sufficient. Even leaving this point aside, the return on an economy-wide asset (for example, a listed firm or a syndicated loan) is more easily observed than the return on an individual bank's portfolio, which will consist of opaque assets. Second, there are issues of inequality and competition as weaker banks would then be subjected to less stringent regulation. Third, bank-specific capital requirements create incentive problems to the extent that banks can influence the return on their projects. In an extension (see the internet appendix) we analyze such incentive problems. We consider a situation where the regulator has to rely on banks reporting the shocks they have been subject to. In this setting, it is no longer optimal to condition on a bank's specific information (as this prevents truth-telling), while it remains optimal to condition on the average reported by all banks.

3.7 Reducing procyclicality versus reducing cross-sectional risk

We have assumed that the date-0 project choice of banks, that is, whether or not projects are correlated, is unobservable. What happens if banks can be regulated directly on the correlation of their projects?[11] It is easy to see that this can eliminate the systemic risk-shifting problem. For example, Acharya (2009) considers capital requirements that increase in bank correlation, and shows that they can reduce systemic risk. Similarly, higher capital requirements for correlated banks in our model will incentivize banks to remain uncorrelated and hence reduce systemic costs. There is, however, also a second benefit to reducing correlation. In a less correlated economy, the sensitivity of bank capital to shocks will be lower (the volatility of aggregate bank capital is lower in the uncorrelated economy), which reduces the need for countercyclical policies.

Countercyclical capital requirements, in contrast, have the cost of increasing correlation risk – as we have shown. Perversely, they can even increase the sensitivity of the economy to aggregate conditions. To see this, consider that starting from flat capital requirements, the regulator (marginally) increases countercyclicality. The economy will then move from an uncorrelated to a correlated equilibrium (Proposition 3). This will increase the likelihood of joint failures but also increase the sensitivity of aggregate bank capital to shocks. The latter is because shocks now affect both banks equally – while the (marginal) increase in countercyclicality only has a second-order effect.

[11] An example of such regulation would be a capital requirement based on measures of banks' systemic importance, such as the CoVar (Adrian and Brunnermeier, 2011) or the Systemic Expected Shortfall (Acharya et al., 2012).

4 Conclusion

We have developed a model in which there is a rationale for regulation in reducing the impact of shocks on the financial system. Aggregate risk is endogenous in this model since banks can influence the extent to which they correlate with each other. We have shown that countercyclical macroprudential capital requirements – while reducing the impact of shocks on the economy ex-post – provide banks with incentives to become more correlated ex-ante. This is because such capital requirements lower a bank's cost from exposure to aggregate risk – but not the cost arising from taking on idiosyncratic risks. The overall welfare implications of countercyclical policies are hence ambiguous.

Our results have important consequences for the design of macroprudential policies. First, policy makers typically view different macroprudential tools in isolation: there are separate policies for dealing with procyclicality (e.g. countercyclical capital buffers) and correlation risk (e.g. higher capital charges for Systemically Important Financial Institutions as under Basel III). Our analysis suggests that there are important interactions among these tools. In particular, policies that mitigate correlation are a substitute for countercyclical policies since lowering correlation also means less procyclicality (while the reverse is not true). This suggests that if regulators prefer to employ a single policy instrument (for example, in order to keep regulatory complexity low), they should focus on reducing cross-sectional risk rather than on implementing countercyclical measures.

Second, Basel III envisages countercyclical capital buffers that are imposed when (national) regulators deem credit expansion in their country excessive. Such discretionary buffers create a new time-inconsistency problem since a regulator will always be tempted to lower capital requirements in bad times, while it will be difficult for regulators to withstand pressure and raise capital requirements in boom times. Our analysis suggests in that context providing domestic regulators with the option to modify capital requirements during the cycle may be counterproductive for the objective of containing systemic risk as it may increase banks' correlation incentives.

Third, optimal "countercyclicality" is likely to vary across countries. In particular, in countries with highly developed banking systems it may be optimal to introduce only a modest element of countercyclicality. In such countries there are more possibilities for banks to correlate with each other, making it worthwhile to limit countercyclicality in order to keep correlation incentives under control. In addition, in such countries it is easier for banks to share risks with each other (for example, through interbank markets). This further reduces the benefit from countercyclical policies as the banking sector can then better insure itself against shocks.

References

Abel, A. B. (1990). Asset prices under habit formation and catching up with the joneses. *The American Economic Review*, 80(2):38–42.

Acharya, V. V. (2009). A theory of systemic risk and design of prudential bank regulation. *Journal of Financial Stability*, 5(3):224–255.

Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. P. (2012). Measuring systemic risk. CEPR Discussion Papers 8824, C.E.P.R. Discussion Papers.

Acharya, V. V., Shin, H. S., and Yorulmazer, T. (2011). Crisis resolution and bank liquidity. *Review of Financial studies*, 24(6):2166–2205.

Acharya, V. V. and Yorulmazer, T. (2007). Too many to fail–an analysis of time-inconsistency in bank closure policies. *Journal of Financial Intermediation*, 16(1):1–31.

Adrian, T. and Brunnermeier, M. K. (2011). Covar. NBER Working Papers 17454, National Bureau of Economic Research, Inc.

Allen, F., Babus, A., and Carletti, E. (2012). Asset commonality, debt maturity and systemic risk. *Journal of Financial Economics*, 104(3):519–534.

Barzel, Y. and McDonald, R. J. (1973). Assets, subsistence, and the supply curve of labor. *The American economic review*, 63(4):621–633.

Basel Committee on Banking Supervision (BCBS) (2010). Guidance for national authorities operating the countercyclical capital buffer. *Bank for International Settlements, Basel*.

Bergstrand, J. H. (1990). The heckscher-ohlin-samuelson model, the linder hypothesis and the determinants of bilateral intra-industry trade. *The Economic Journal*, 100(403):1216–1229.

Borio, C. (2003). Towards a macroprudential framework for financial supervision and regulation? *CESifo Economic Studies*, 49(2):181–215.

Farhi, E. and Tirole, J. (2012). Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review*, 102(1):60–93.

Foos, D., Norden, L., and Weber, M. (2010). Loan growth and riskiness of banks. *Journal of Banking & Finance*, 34(12):2929–2940.

Furlong, F. T. and Keeley, M. C. (1989). Capital regulation and bank risk-taking: A note. *Journal of banking & finance*, 13(6):883–891.

Galati, G. and Moessner, R. (2011). Macroprudential policy - a literature review. BIS Working Papers 337, Bank for International Settlements.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of monetary economics*, 3(3):547–599.

Gorton, G. and Ordoñez, G. (2016). Good booms, bad booms. Technical report, National Bureau of Economic Research.

Kahn, C. M. and Santos, J. A. (2010). Liquidity, payment and endogenous financial fragility. In *EFA 2005 Moscow Meetings Paper*.

Kashyap, A. and Stein, J. C. (2004). Cyclical implications of the Basel II capital standards. *Economic Perspectives*, (Q I):18–31.

Lettau, M. and Uhlig, H. (2000). Can habit formation be reconciled with business cycle facts? *Review of Economic Dynamics*, 3(1):79–99.

Malherbe, F. (2013). Optimal capital requirements over the business and financial cycles. *London Business School, mimeo.*

Martin, A., McAndrews, J., Palida, A., and Skeie, D. R. (2013). Federal reserve tools for managing rates and reserves. *FRB of New York Staff Report*, (642).

Martínez-Miera, D. and Suarez, J. (2012). A macroeconomic model of endogenous systemic risk taking. mimeo, CEMFI and Universidad Carlos III.

Perotti, E. C. and Suarez, J. (2002). Last bank standing: What do I gain if you fail? *European Economic Review*, 46(9):1599–1622.

Repullo, R. and Suarez, J. (2013). The procyclical effects of bank capital regulation. *Review of Financial Studies*, 26(2):452–490.

Rosen, H. S. (1978). The measurement of excess burden with explicit utility functions. *The Journal of Political Economy*, pages S121–S135.

Schularick, M. and Taylor, A. M. (2012). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870–2008. *The American Economic Review*, 102(2):1029–1061.

Segura, A. and Suarez, J. (2011). Liquidity shocks, roll-over risk and debt maturity. CEPR Discussion Papers 8324, C.E.P.R. Discussion Papers.

Sinkey, J. F. and Greenawalt, M. B. (1991). Loan-loss experience and risk-taking behavior at large commercial banks. *Journal of Financial Services Research*, 5(1):43–59.

Stein, J. (2012). Monetary policy as financial stability regulation. *Quarterly Journal of Economics*, 127:57–95.

Wagner, W. (2011). Systemic liquidation risk and the diversity–diversification trade-off. *The Journal of Finance*, 66(4):1141–1175.

Appendix

Proof of Proposition 1. Correlated economy: Equation (13) follows from solving the first-order conditions (11) and (12) with respect to z_H and z_L.

Uncorrelated economy: The first order condition for the high state is given by

$$\pi^2 \left(2f'(z_H) + 2(1-p(z_H))p'(z_H)\gamma\right) + \pi(1-\pi)(f'(z_H) + f'(z_H - \Delta x)) \quad (32)$$
$$+\pi(1-\pi)((1-p'(z_H))(1-p(z_H-\Delta x))\gamma + (1-p(z_H))(1-p'(z_H-\Delta x))\gamma) = 0.$$

Similarly, the first order condition for the low state is given by

$$\pi^2 \left(2f'(z_L) + 2(1-p(z_L))p'(z_L)\gamma\right) + \pi(1-\pi)(f'(z_L) + f'(z_L + \Delta x)) \quad (33)$$
$$+\pi(1-\pi)((1-p'(z_L))(1-p(z_L+\Delta x))\gamma + (1-p(z_L))(1-p'(z_H+\Delta x))\gamma) = 0.$$

Solving the first-order conditions for z_H and z_L we obtain equations (14) and (15).

We still have to show that at the optimal capital requirements, bankers find it optimal to scale up to the maximum permissible amount using equity ($z = x + \overline{d}$). In the correlated economy this is obvious. Comparing the banker's private first-order condition for equity-induced investment with the social conditions for optimal investment, we can see that the private gains from investment exceed the social ones: from (6) and (11) (or (12)) we have that the private gains are higher by an amount of $-p'(z)(\overline{d} + \frac{\gamma}{2}) > 0$. However, in the uncorrelated economy, the state at bank A may differ from the state on which capital requirements are based. In particular, when the common project is in the low state, the regulator sets low capital requirements that may permit bank A to scale beyond its privately optimal amount if the alternative project reaches the high state. The maximum scale a bank with a project in the high state (when the common project is in the low state) is allowed to choose is $z_L^{*U} + \Delta x$. From (6) we have that the private gains from increasing investment beyond $z_L^{*U} + \Delta x$ are $f'(z_L^{*U} + \Delta x) - p'(z_L^{*U} + \Delta x)(z_L^{*U} + \Delta x - x_H)$. Assumption (3) guarantees that this expression is positive for z_L^{*U}. Thus, a bank with a high interim return will scale up to the maximum amount even when the capital requirements are set according to the low state. ■

Proof of Proposition 2. Inserting $z^{*C} = z_H^{*C} = z_L^{*C}$ (Proposition 1) into the expression for welfare in the correlated economy, we obtain:

$$E[W^{*C}] = 2f(z^{*C}) - (1 - p(z^{*C}))\gamma + \omega + \widehat{x}. \tag{34}$$

Inserting z_H^{*U} and z_L^{*U} into the expression for welfare in the uncorrelated economy, we obtain:

$$E[W^{*U}] = 2f(\widehat{z}^{*U}) - (1 - p(\widehat{z}^{*U}))^2\gamma - \frac{1}{8}\pi(1-\pi)(\Delta x)^2(6R - \frac{\gamma}{2}) + \omega + \widehat{x}. \tag{35}$$

Comparing (34) and (35) yields (23).

For γ tending to zero ($\gamma \downarrow 0$) the optimal scales in the correlated and uncorrelated economies converge to one another $\widehat{z}^{*U} - z^{*C} \to 0$ and hence the left hand side of (20) approaches zero. Since $\Delta x > 0$ the inequality does not hold for sufficiently small γ.

For Δx sufficiently close to zero ($\Delta x \downarrow 0$) and z_H^{*C} we have $E[W^C(z^{*C})] - E[W^U(z^{*C})] < 0$ from equation (17). Since $E[W^U(\widehat{z}^{*U})] \geq E[W^U(z^{*C})]$, it follows that we have $E[W^U(\widehat{z}^{*U})] > E[W^C(z^{*C})]$. Hence there are also parameter values for which an uncorrelated economy is preferred. ■

Proof of Proposition 4. Consider first welfare in an uncorrelated economy. In an uncorrelated economy we need to have that $\overline{d}_H = \overline{d}_L$ as otherwise capital requirements are countercyclical and banks would choose correlation. Flat capital requirements ($\overline{d}_H = \overline{d}_L$) imply that $z_H - z_L = \Delta x$. Using this condition to substitute z_H in equation (10) we obtain

$$\begin{aligned} E[W^U]_{z_H = z_L + \Delta x} &= \pi^2 \left(2f(z_L + \Delta x) - (1 - p(z_L + \Delta x))^2\gamma\right) + (1-\pi)^2 \left(2f(z_L) - (1 - p(z_L))^2\gamma\right) \\ &+ \pi(1-\pi)(f(z_L + \Delta x) + f(z_L) - (1 - p(z_L + \Delta x))(1 - p(z_L))\gamma) \\ &+ \pi(1-\pi)(f(z_L) + f(z_L + \Delta x) - (1 - p(z_L))(1 - p(z_L + \Delta x))\gamma) + \omega + \widehat{x}. \end{aligned} \tag{36}$$

Taking derivative with respect to z_L we obtain the first-order condition:

$$-(2R + \gamma)\pi\Delta x + 2(a - z_L)R - 2(1+\gamma) + 2\gamma(a - \frac{z_L}{2}) = 0. \tag{37}$$

Solving for z_L we obtain (23). From $z_H = \Delta x - z_L$ we then obtain (22).

Inserting the optimal scales (22) and (23) into the expression for welfare in the uncorrelated economy, we obtain:

$$E[W^{*U}] = 2f(\widehat{z}^{*U}) - (1 - p(\widehat{z}^{*U}))^2\gamma - \pi(1-\pi)\Delta x^2 R + \omega + \widehat{x}. \tag{38}$$

Comparing (34) and (38) yields (24).

Analogous to Proposition 2 it can be shown that for small γ we have that $E[W^{*U}]_{z_H = z_L + \Delta x} < E[W^{*C}]$, while for small Δx we have $E[W^{*U}]_{z_H = z_L + \Delta x} > E[W^{*C}]$. Hence there are cases where correlation is optimal and there are cases where no correlation is optimal. ■

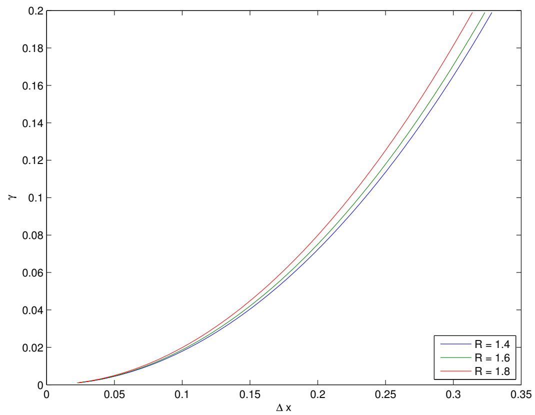
Proof of Proposition 6. Comparing equations (10) and (28) we find that the difference between the welfare levels of the two uncorrelated economies (when project choices are observable) – with and without trade – is

$$E[W^{U,Trade}] - E[W^U] = 2\pi(1-\pi)\frac{\triangle x^2}{4}(R - \frac{\gamma}{4}),$$

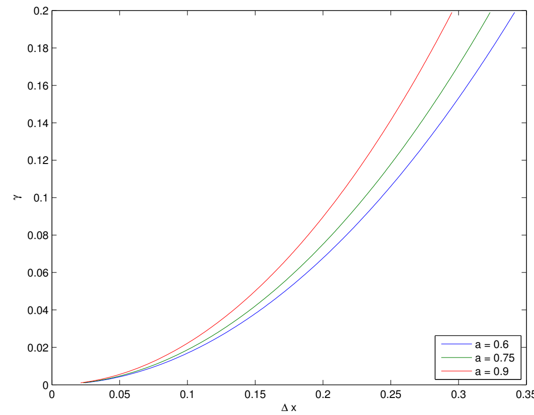
which is positive because $\frac{\gamma}{4} < R - 1$ ($\frac{\gamma}{4} < R - 1$ follows from assumption (i) and $a < 1$). This expression does not depend on capital requirements. It follows that optimal capital requirements have to be the same regardless of whether there is the possibility to trade. ■

Proof of Proposition 7. Assume that systemic costs are sufficiently small ($\gamma \downarrow 0$). For z_H^{*U} and z_L^{*U} (that is, optimal capital requirements in the uncorrelated economy) we have $z_H^{*U} - z_L^{*U} = \frac{\Delta x}{2}$ and hence $E[W^C(z^{*U})] - E[W^{U,Trade}(z^{*U})]$ tends to zero. For optimally chosen capital requirements in the correlated economy, z_H^{*C} and z_L^{*C}, we must have that $E[W^C(z^{*U})] > E[W^C(z^{*U})]$. Thus we have that $E[W^C(z^{*C})] > E[W^U(z^{*U})]$, that is, welfare is higher in the correlated economy. ■

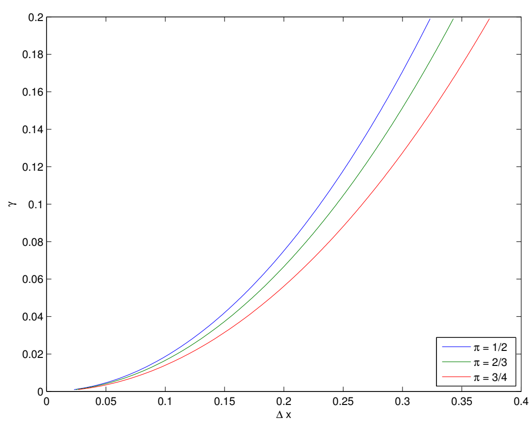
Figure 2: The trade-off between variability of shocks (Δx) and the cost of joint failures (γ)



(a)



(b)



(c)

Note: The lines in a graph shows combinations of the variability of shocks and the cost of joint failure for which welfare is the same in the correlated and in the uncorrelated economy. The baseline parameters are R=1.6, a=0, π=0.5

Internet Appendix for the paper "The disturbing interaction between countercyclical capital requirements and systemic risk"

Diversification

In the baseline model, bank A had a choice between a common and an alternative project. We now consider a third option: investing in a "diversified project" that combines the risk of the common and the alternative project. Specifically, the diversified project delivers interim pay-offs that consist of half the pay-off of the common and the alternative project: $x^D = \frac{x^C + x^U}{2}$. As before, at date 1 the scale, z, of the project is determined. At date 2, the project pays $\frac{zR}{2}$ whenever a common project of size z succeeds, and in addition, $\frac{zR}{2}$, whenever an alternative project of size z succeeds. As each leg of the project succeeds with probability $p(z)$, the total expected pay-off at date 2 is hence $p(z)zR$ as in the baseline model (We model diversification in this way in order to avoid "hard-wiring" diversification benefits that would arise if the banker could simply combine two concave production functions.) Note that diversification brings benefits at date 1 *and* date 2 by reducing the volatility of pay-offs relative to holding either the common or the alternative asset.

The welfare that obtains from choosing the diversified project can be derived as follows. With probability $\pi^2 + (1-\pi)^2$ the alternative and the common projects have the same date-1 return and investment levels are either z_H or z_L at both projects. In these cases, diversification does not matter from the perspective of date 1. However, there are still potential diversification benefits at date 2. Systemic costs arise only when the returns of both banks are zero. Bank A has a zero return when both the common and the alternative projects fail, while bank B has a zero return if the common project fails. Systemic costs thus arise when both the common and the alternative projects fail, as in the case of the uncorrelated economy. Given the investment levels at date 1, the likelihood of joint failure is $(1-p(z_H))^2$ and $(1-p(z_L))^2$, respectively. The total expected pay-off whenever the assets have identical returns at date 1 is hence $2f(z_H)-(1-p(z_H))^2\gamma$ and $2f(z_L)-(1-p(z_L))^2\gamma$.

With probability $\pi(1-\pi)$ the common asset is in the high state and the alternative asset is in the low state. Investment in the diversified asset is then $z=\frac{x_H+x_L}{2}+\overline{d}_H=z_H-\frac{\triangle x}{2}$. Since systemic costs arise only when the returns on both banks are zero, the likelihood of joint failure is $(1-p(z_H))(1-p(z_H-\frac{\triangle x}{2}))$. With probability $\pi(1-\pi)$ the opposite case arises where the common asset reaches the low state and the alternative asset the high state. In this case investment in the diversified asset is $z_L+\frac{\triangle x}{2}$ and the likelihood of joint failure is $(1-p(z_L))(1-p(z_L+\frac{\triangle x}{2}))$.

We thus have that welfare is

$$\begin{aligned}E[W^D] &= \pi^2\left(2f(z_H)-(1-p(z_H))^2\gamma\right)+(1-\pi)^2\left(2f(z_L)-(1-p(z_L))^2\gamma\right)\\ &+\pi(1-\pi)(f(z_H)+f(z_H-\frac{\triangle x}{2})-(1-p(z_H))(1-p(z_H-\frac{\triangle x}{2}))\gamma)\\ &+\pi(1-\pi)(f(z_L)+f(z_L+\frac{\triangle x}{2})-(1-p(z_L))(1-p(z_L+\frac{\triangle x}{2}))\gamma)+\omega+\widehat{x}. \qquad \text{(A1)}\end{aligned}$$

Note that this is identical to the expression for welfare in the uncorrelated economy, $E[W^U]$, when one replaces $\frac{\triangle x}{2}$ with Δx. We can express the difference in welfare levels in the diversified and the uncorrelated economy as

$$\begin{aligned}E[W^D]-E[W^U] &= \pi(1-\pi)(f(z_H-\frac{\triangle x}{2})+f(z_L+\frac{\triangle x}{2})-f(z_H-\Delta x)-f(z_L+\Delta x))\\ &\quad -\pi(1-\pi)(p(z_L)-p(z_H))\gamma\frac{\Delta x}{4}. \qquad \text{(A2)}\end{aligned}$$

The expression in the first line is the impact of diversification at date 1, arising because of a lower variance of returns for bank A. This leads to lower fluctuations in the scale of bank A's project. Similarly to the baseline model (Lemma 1), these gains are positive when the policy is countercyclical. The second line is the effect that arises because different scale combinations at date 1 lead to a different likelihood of joint failures. This effect was already present when we compared the correlated and the uncorrelated economies but is now only half as large. As before, the effect is negative.

Project choice is observable

The following proposition states the optimal scales in the diversified economy.

Proposition 8 *The optimal levels of investment in the diversified economy are*

$$z_H^{*D} = \frac{a(R+\frac{\gamma}{2})-(1+\frac{\gamma}{2})}{R+\frac{\gamma}{4}}+(1-\pi)\frac{\Delta x}{4}, \tag{A3}$$

$$z_L^{*D} = \frac{a(R+\frac{\gamma}{2})-(1+\frac{\gamma}{2})}{R+\frac{\gamma}{4}}-\pi\frac{\Delta x}{4}. \tag{A4}$$

Proof. *From (A1) we can derive the first order condition for z_H:*

$$\pi(1-\pi)f'(z_H)+\pi(1-\pi)f'(z_H-\frac{\triangle x}{2})R-2\pi+4\pi^2p'(z_H)(1-p(z_H))\gamma \tag{A5}$$
$$+2\pi(1-\pi)(p'(z_H)(1-p(z_H-\frac{\triangle x}{2}))\gamma+p'(z_H-\frac{\triangle x}{2})(1-p(z_H))\gamma=0.$$

Solving for z_H yields equation (A3). Next, we have the first-order condition for z_L:

$$\pi f'(z_L+\frac{\triangle x}{2})R+(2-\pi)f'(z_L)R-2-(1-\pi)(1-p(z_L))\gamma \tag{A6}$$
$$-\pi(1-p(z_L+\frac{\triangle x}{2})+(1-p(z_L))\gamma=0.$$

Solving for z_L yields equation (A4). ■

Comparing to Proposition 1 we can see that optimal scales in the diversified economy vary less across states than in the uncorrelated economy ($z_H^{*D}-z_L^{*D}<z_H^{*U}-z_L^{*U}$). It follows that

Corollary 5 *Optimal countercyclicality in the diversified economy is higher than in the uncorrelated economy ($\overline{d}_L^{*D}-\overline{d}_H^{*D}>\overline{d}_L^{*U}-\overline{d}_H^{*U}$).*

The reason is the following. In the diversified economy, bank A's project is correlated (albeit imperfectly) with the common project. Thus, implementing a countercyclical policy (based on the common project) will also lead to lower variability at bank A. As a result, it is optimal to implement a higher amount of countercyclicality.

Proposition 9 provides results about which economy (uncorrelated, diversified, correlated) may be preferred from a welfare perspective.

Proposition 9 *There are no parameter values for which an uncorrelated economy is optimal. However, there are parameter values for which i) a correlated economy is optimal, ii) a diversified economy is optimal.*

Proof. *Inserting the expressions for the optimal scales in the diversified economy (equations (A3) and (A4)) into welfare (equation (A1)), we obtain:*

$$E[W^{*D}] = 2(p(\widehat{z}^*)R - 1)\widehat{z}^* - 2(1 - p(\widehat{z}^*))^2 \frac{\gamma}{2} - \frac{1}{8}\pi(1-\pi)(\frac{\triangle x}{2})^2(6R - \frac{\gamma}{2}) + \omega + \widehat{x}. \quad \text{(A7)}$$

Comparing to $E[W^{*U}]$ we see that $E[W^{*D}] > E[W^{*U}]$ since $\gamma < 6R$ (by assumption (i) and $a < 1$ we have that $\gamma < R - 1$). Thus, diversification always dominates the undiversified economy and hence an undiversified economy is never optimal. For $\gamma \downarrow 0$ we have that $E[W^{*D}] < E[W^{*C}]$ and for $\frac{\triangle x}{2} \downarrow 0$ we have that $E[W^{*D}] > E[W^{*C}]$. Thus there exist parameter values for which a diversified economy is preferred and for which a correlated economy is preferred. ■

An uncorrelated economy is never optimal because it is dominated by a diversified economy. The latter has the benefit that the returns on the diversified project are less volatile and hence the resulting scales are more efficient. There is no cost associated with the diversified economy as its likelihood of joint failure is the same as in the undiversified economy.

Relative to the baseline model, correlation now becomes less attractive since there is a better alternative to it (diversification). Nonetheless, as the proposition shows, it can still be optimal to have correlated projects in the economy.

Project choice is not observable

We now turn to the analysis of a bank's optimal project choice, taking as given capital regulation. Relative to the baseline model, the bank now has the additional opportunity to invest in the diversified asset.

This case is more complex since it is not clear when a diversified bank defaults at date 2. In particular, depending on the level of debt, a bank may or may not default when one of the projects in which it has invested fails while the other does not. We develop the analysis for the case where the bank does not default iff either the alternative or the common asset fails (in this case, diversification thus provides benefits in terms of reducing failures at date 2).

The analysis mirrors that of the baseline model. Choosing a diversified (as opposed to the alternative) asset provides benefits to the banker as his returns then become more correlated to the common asset; he can hence benefit when countercyclical regulation is in place because investment will become less volatile. One can also show that there is a cost from choosing the diversified as opposed to the common asset, because in this case the banker's return becomes less correlated to the asset on whose return countercyclical capital requirements are based. The analysis of diversification, however, highlights a new effect, coming through the put-option value of limited liability. Under diversification, the banker fails less often at date 2. In order to maximize the value of the put, the banker hence has an interest in avoiding the diversified asset. This creates an additional wedge between the private and social benefits of asset choice.

In the following we show that, notwithstanding this new effect, a similar problem as in the baseline model arises: because the banker ignores the impact of his choices on the social cost of joint failures he may choose the correlated instead of the diversified investment, potentially resulting in lower welfare compared to the benchmark.

We first derive the bank's utility resulting from investment in the diversified asset. Different cases arise with respect to default at $t = 2$. Depending on parameter values, the bank may (or may not) default at $t = 2$ if either the common or the alternative asset pays off. We assume in the following that in cases where one of the assets pays off, the bank survives (that is, diversification helps the bank to avoid failure).

A condition that ensures this can be derived as follows. Consider first the case when the common project is in the high state, and hence the limit on external finance is $\overline{d}_H$. When the alternative project is in the low state at $t = 1$, the investment scale of the diversified project is $z_H - \frac{\triangle x}{2}$. When one of the projects succeeds at $t = 2$, the return on the diversified asset at date 2 is hence $\frac{z_H - \frac{\triangle x}{2}}{2} R$. The condition that the bank does not default is hence

$$\frac{z_H - \frac{\triangle x}{2}}{2} R \geq \overline{d}_H. \tag{A8}$$

When the alternative project is in the high state at date 1, investment level is z_H and the condition for bank survival is $\frac{z_H}{2} R > \overline{d}_H$, which is dominated by (A8).

Consider next the case when the common project return is low. External finance is then limited to $\overline{d}_L$. When the alternative project is in the low state, investment in the diversified project is z_L. The no-default condition is then

$$\frac{z_L}{2} R \geq \overline{d}_L. \tag{A9}$$

Again, we can ignore the case of a high return on the alternative project, as this will lead to a less strict condition. Noting that we have $z_H - \frac{\triangle x}{2} \geq \overline{d}_H$ and $z_L \geq \overline{d}_L$ (as only a part of investment is financed through debt), we can see that a sufficient condition that guarantees (A8) and (A9) is $R \geq 2$. We assume in the following that $R \geq 2$ holds.

We next derive the expected utility for a diversified banker who at date 1 faces an external finance limit $\widehat{d}$ and implements a project scale $\widehat{z}$. With probability $p^2(\widehat{z})$ both the alternative and the common projects succeed at date 2. In this case, the return on the diversified portfolio is $\widehat{z}R$, of which the banker can keep $\widehat{z}R - \widehat{d}$. With probability $2p(\widehat{z})(1 - p(\widehat{z}))$ either the alternative or the common project succeeds. In this case the diversified asset returns $\widehat{z}\frac{R}{2}$, of which the banker can keep $\frac{\widehat{z}}{2}R - \widehat{d}$. With probability $(1 - p(\widehat{z}))^2$ both projects fail, in which case the bank defaults, leaving the banker with zero. Summing up, we have for the expected return for the banker (conditional on $\widehat{z}$ and $\widehat{d}$): $E[W_A^D \big| z = \widehat{z}, d = \widehat{d}] = p(\widehat{z})\widehat{z}R - \widehat{d}p(\widehat{z})(2 - p(\widehat{z}))$. This can be simplified to

$$E[W_A^D \big| z = \widehat{z}, d = \widehat{d}] = f(\widehat{z}) + (1 - p(\widehat{z}))^2(\widehat{z} - x) + \omega + x. \tag{A10}$$

Recall that there are four states at date 1 (the common and alternative project can be in states (H, H), (L, L), (H, L) and (L, H)) with associated probabilities π^2, $(1-\pi)^2$, $\pi(1-\pi)$ and $\pi(1-\pi)$. We can hence write the unconditional return for the banker as

$$\begin{aligned} E[W_A^D] &= \pi^2(f(z_H) + (1-p(z_H))^2(z_H - x_H)) \\ &+ (1-\pi)^2(f(z_L) + (1-p(z_L))^2(z_L - x_L)) \\ &+ \pi(1-\pi)(f(z_H - \frac{\triangle x}{2}) + (1 - p(z_H - \frac{\triangle x}{2}))^2(z_H - x_H)) \\ &+ \pi(1-\pi)(f(z_L + \frac{\triangle x}{2}) + (1 - p(z_L + \frac{\triangle x}{2}))^2(z_L - x_L)) + \omega + \widehat{x}. \end{aligned} \tag{A11}$$

From this we can derive the banker's benefit from choosing the diversified as opposed to the alternative asset:

$$\begin{aligned} &E[W_A^D] - E[W_A^U] = \pi(1-\pi)(f(z_H - \tfrac{\triangle x}{2}) + f(z_L + \tfrac{\triangle x}{2}) - f(z_H - \Delta x) - f(z_L + \Delta x)) \\ &-\pi(\pi(1-p(z_H))p(z_H) + (1-\pi)((1-p(z_H - \Delta x)) - (1-p(z_H - \tfrac{\triangle x}{2}))^2))(z_H - x_H) \\ &-(1-\pi)((1-\pi)(1-p(z_L))p(z_L) + \pi((1-p(z_L + \Delta x)) - (1-p(z_L + \tfrac{\triangle x}{2}))^2))(z_L - x_L). \end{aligned} \tag{A12}$$

The first line is the diversification benefit, familiar from the discussion of welfare. The second and third line represents the loss in the value of the put-option arising from limited liability. The reason for this effect is the following. When the banker holds the diversified asset, he is less likely to default. Because of limited liability, this means that in expectation he has to make more payments to depositors. This in turn gives the banker a reason not to choose a diversified portfolio.

Next, we can derive the banker's benefit from choosing the diversified as opposed to the common asset:

$$\begin{aligned} &E[W_A^D] - E[W_A^C] = -\pi(1-\pi)(f(z_H) + f(z_L) - f(z_H - \tfrac{\triangle x}{2}) - f(z_L + \tfrac{\triangle x}{2})) \\ &-\pi(1-\pi)(R - \tfrac{1}{2})\Delta x(\Delta x - (z_H - z_L)) \\ &-\pi(\pi(1-p(z_H))p(z_H) + (1-\pi)((1-p(z_H - \Delta x)) - (1-p(z_H - \tfrac{\triangle x}{2}))^2))(z_H - x_H) \\ &-(1-\pi)((1-\pi)(1-p(z_L))p(z_L) + \pi((1-p(z_L + \Delta x)) - (1-p(z_L + \tfrac{\triangle x}{2}))^2))(z_L - x_L). \end{aligned} \tag{A13}$$

The first line is the cost of holding a diversified asset. It arises because when the policy is countercyclical, there will be more variations in scale when investing in the diversified asset. The other lines represent the lost value of the put-option for a diversified portfolio as a diversified portfolio leads to fewer defaults.

Proposition 10 *Under a flat capital requirement a banker chooses either the alternative or the diversified project. Under countercyclical capital requirements a banker chooses either the diversified or the common project.*

Proof. *Consider first flat capital requirements $\overline{d}_H = \overline{d}_L = \overline{d}$. We know that for such $\overline{d}$, the common project is dominated by the alternative project. We thus only have to compare the alternative and the diversified portfolio. The second and the third line in (A12) are positive when*

$$(1 - p(z_H - (1-\pi)\Delta x))p(z_H) > (1-\pi)\frac{\Delta x^2}{16}, \tag{A14}$$

$$(1 - p(z_L + \pi\Delta x))p(z_L) > \pi\frac{\Delta x^2}{16}. \tag{A15}$$

1. An example for the banker choosing the alternative project: When π becomes small, the first term in (A12) becomes negligible. Then, when conditions (A14) and (A15) are not fulfilled, we have $E[W_A^D] - E[W_A^U] < 0$ and the banker will choose the alternative asset.
2. An example for the banker choosing the diversified project: For $\overline{d}$ tending to zero, the second and third term in (A12) become zero. Since for flat capital requirements the first term in (A12) is positive (because of the diversification benefit there is a lower variance in scales at $t = 1$), we thus have that $E[W_A^D] - E[W_A^U] > 0$, and hence the banker diversifies.

Consider next countercyclical capital requirements: $\overline{d}_H < \overline{d}_L$. From the analysis in the previous section we know that the banker then strictly prefers the common to the alternative project. That is, the banker will either choose the common or the diversified project. The first two lines in (A13) are negative when

$$\frac{z_H - z_L}{\Delta x} < \frac{R-2}{2(R-1)}, \tag{A16}$$

which, if true, means that the whole expression is negative.

1. An example for the banker choosing the correlated project: Suppose the regulator fully stabilizes scale ($z_H = z_L$). The above inequality is then fulfilled for $R > 2$, and the banker will choose the correlated project. 2. An example for the banker choosing the alternative project: Suppose the regulator only uses a very small amount of countercyclicality (that is, $z_H - z_L \approx \Delta x$) and imposes very strict capital requirements ($\overline{d}_H$ and $\overline{d}_L$ close to zero). The left hand side of the condition then tends to one. It follows that the condition cannot be fulfilled, and the banker chooses the alternative project. ■

Finally, we show that the banker's private incentives create a similar bias as in the baseline model. Specifically, the welfare level of the benchmark case (where the project choice is observable) may no longer be obtainable because the banker deviates by choosing a more correlated portfolio.

Proposition 11 *The maximum level of welfare when the project choice is observable, $max[E[W^{C*}], E[W^{D*}]]$, may no longer be attainable because the banker deviates from the benchmark allocation by choosing a higher amount of correlation.*

Proof. *There are two cases to consider, based on the cases analyzed in the previous section: either diversification is optimal when the project choice is observable or correlation is optimal (Proposition 9 has shown that the uncorrelated economy is never optimal).*

Consider first that correlation is optimal in the benchmark case ($E[W^{C*}] \geq E[W^{D*}]$). The optimal benchmark policy is then to be fully countercyclical, that is, to impose a constant scale. For $z_H = z_L$, we can derive from (A13) that $E[W_A^D] - E[W_A^C] < 0$ and hence the banker chooses to be correlated. As a result, the same welfare level as in the benchmark can be achieved by setting z^{*C}.

Consider next that diversification is optimal in the benchmark case ($E[W^{C*}] < E[W^{D*}]$). Suppose that $E[W_A^D(z^{*D})] - E[W_A^C(z^{*D})] \geq 0$. The benchmark welfare level can then still be achieved by setting the same capital requirements as in the benchmark case (z_H^{*D} and

z_L^{*D}). Suppose next that $E[W_A^D(z^{*D})] - E[W_A^C(z^{*D})] < 0$. In this case, when the regulator implements z_H^{*D} and z_L^{*D}, the banker deviates by choosing the correlated outcome. Welfare is then necessarily lower as in the benchmark case since we know that $E[W^{C*}] < E[W^{D*}]$.

Summing up, we have shown that whenever the benchmark welfare is attainable, the banker chooses an amount of correlation that is identical to the socially optimal one. However, when benchmark welfare is not attainable, this is because the banker chooses a higher level of correlation. ■

Uninsured deposits

We now assume that deposits are uninsured. At date 1 a deposit market opens for each bank. In this market, a bank can raise funds in return for a promised unit payment of $1 + r$ at date 2. Consumers (of which we assume to be a continuum) are assumed to behave competitively in this market. Note that since deposit insurance does not affect the consumption available to consumers at date 2 (since deposit insurance is financed by taxation from consumers), its introduction does not modify the expression for welfare (in particular, it does not alter the expected costs from joint failures). Equations (9) and (10) hence still apply.

In an equilibrium, a consumer j will deposit a positive amount at both banks: $d_A^j, d_B^j > 0$ (if a consumer were to lend only to one bank, he would fall short of his consumption requirement already in the case of a single bank failure, which is not optimal). In the case of the correlated economy, the consumer's expected return given deposits d_A^j and d_B^j is given by

$$\omega - d_A^j - d_B^j + p(z_A)d_A^j(1 + r_A) + p(z_B)d_B^j(1 + r_B) - (1 - p_{R_A+R_B>0}^C(z_A, z_B))\gamma, \quad \text{(A17)}$$

where $p_{R_A+R_B>0}^C(z_A, z_B)$ is the likelihood of (at least) one of two correlated projects succeeding (as projects have different scales and hence different risk, correlation no longer necessarily means that projects always succeed at the same time). From differentiating with respect to d_A^j we can obtain that the interest rate which makes the consumer indifferent to more deposits to bank A is determined by $1 = p(z_A)(1 + r_A)$. For deposits at bank B we obtain in addition the condition: $1 = p(z_B)(1 + r_B)$. Similarly, the expected return for the consumer in the uncorrelated economy is given by

$$\omega - d_A^j - d_B^j + p(z_A)d_A^j(1 + r_A) + p(z_B)d_B^j(1 + r_B) - (1 - p^U_{R_A,R_B>0}(z_A, z_B))^2\gamma, \quad \text{(A18)}$$

where $p^U_{R_A+R_B>0}(z_A, z_B)$ is the probability of the two uncorrelated projects succeeding. Differentiating with respect to d_A^j and d_B^j gives again $1 = p(z_A)(1 + r_A)$ and $1 = p(z_B)(1 + r_B)$. The pricing of deposits is thus the same in both economies. Note that the likelihood of joint failure does not affect pricing. The reason is twofold. First, an individual depositor takes investment z_A and z_B as given, and hence the risk of failure. Second, the marginal value of a unit of deposits (once the consumer lends a positive amount) is not affected by the likelihood of joint failures. Hence the project choice (correlated or uncorrelated) does not change pricing.

Inserting equilibrium interest rates into bank equity we obtain that a banker gets $f(z_i) + x_i$, that is, the project surplus plus his endowment. Using this, we can next derive the expected returns for a banker in the correlated and uncorrelated economy:

$$E[W_A^C] = \pi f(z_H) + (1 - \pi)f(z_L) + \widehat{x}, \quad \text{(A19)}$$

$$E[W_A^U] =$$
$$\pi^2 f(z_H) + (1 - \pi)^2 f(z_L) + \pi(1 - \pi)(f(z_H - \Delta x) + \pi(1 - \pi)(f(z_L + \Delta x) + \widehat{x}. \quad \text{(A20)}$$

The first-order condition for investment – which does not differ between the correlated and the uncorrelated economy – is then

$$f'(z_i) = 0. \quad \text{(A21)}$$

Comparing to equations (6) and (7) in the paper we can see that the value of the put-option to the bank has disappeared; hence the banker simply maximizes the expected surplus on the project. However, comparing to equation (11) and (12) in the paper, we can see that at the socially optimal levels of z^* we still have that $f'(z^*) > 0$. Thus the banker continues to choose excessive levels of investment. In the baseline model there were two distortions in the investment choice: deposit insurance and the systemic costs. The first distortion is no longer present, but the second still is (we can see that the difference between the social and private optimality condition arises entirely from $\gamma > 0$). Hence, there is still a role for capital requirements.

We have for the banker's expected gains from choosing correlation

$$\begin{aligned} E[W_A^C] - E[W_A^U] &= \pi f(z_H) + (1-\pi) f(z_L) - \pi^2 f(z_H) - (1-\pi)^2 f(z_L) \\ -\pi(1-\pi)(f(z_H - \Delta x) &- \pi(1-\pi)(f(z_L + \Delta x). \end{aligned} \tag{A22}$$

Using equation (17) we obtain

$$(E[W_A^C] - E[W_A^U]) - (E[W^C] - E[W^U]) =$$
$$-\pi(1-p(z_H))p(z_H)\gamma - (1-\pi)(1-p(z_L))p(z_L)\gamma - (1-\pi)\pi(p(z_L) - p(z_H))\gamma\frac{\triangle x}{2}. \tag{A23}$$

The latter expression is identical to the second line in equation (17), which has been shown to be negative in the paper (this line represents the cost from joint failures for the correlated relative to the uncorrelated economy). Hence, the banker still perceives lower costs from correlation, relative to the social optimum. The reason is that the cost of joint failure is not reflected in the individual values of bank deposits, hence causing the correlation bias. Corollary 3 thus still holds.

Thus, as in the baseline, there is a tendency for excessive investment and for excessive correlation. Capital requirements hence cause the same trade-off as in the baseline model. Since a banker's benefit from correlation is positive whenever the policy is countercyclical (this follows from $E[W_A^C] - E[W_A^U]$ above), Proposition 3 still holds. The regulatory problem is thus unchanged, and hence Proposition 4 continues to hold.

Strategic interaction among banks

In this section we analyze strategic interaction among banks, using the setting where banks can trade equity stakes among each other (Section 3.2). We start by deriving the intuition for how trade can create strategic interaction. Note first that a bank benefits from the possibility to trade regardless of whether it buys or sells equity. This can be seen from the fact that the equilibrium price for equity is above the pre-trade value for equity for the bank that has the high return (and hence will sell equity) and below the pre-trade value of equity for the bank that has the low return (and hence will buy equity): from (26) and (27) we have that

$$f'(x_H + \overline{d}) + (1 - p'(x_H + \overline{d}))\overline{d} < r_e < f'(x_L + \overline{d}) + (1 - p'(x_L + \overline{d}))\overline{d}. \tag{A24}$$

Suppose that bank B has chosen the alternative asset. If bank A chooses the alternative asset as well, the likelihood that it can use the equity market (and hence improve its profits) is equal to the likelihood of the two banks having asymmetric shocks. Given that the asset returns are independent of each other, this likelihood is $2\pi(1-\pi)$. If bank A chooses the common asset, asset returns are still uncorrelated and the likelihood of trade taking place is again $2\pi(1-\pi)$. Thus, bank A benefits from trading regardless of which asset it chooses.

Suppose now that bank B has chosen the common asset. If bank A chooses the alternative asset, returns are still independent and the likelihood of trade is $2\pi(1-\pi)$. However, if the bank chooses the correlated asset as well, trade can never take place and the bank cannot benefit from the cross-sectional smoothing of shocks. Thus, when bank B chooses the common asset, it reduces the benefits for bank A to choose the common asset. The asset choices of bank A and bank B are hence strategic substitutes.

Proposition 12 *Banks' project choices are substitutes when banks can trade equity, that is, bank A's benefit from choosing the common asset declines when bank B chooses the common asset as well.*

Proof. *Suppose that bank B has chosen the common asset. Consider first that bank A chooses the common asset as well. In this case trade does not take place and pay-offs are the same as in the baseline model (equation (19)). Consider next that bank A chooses the alternative asset. In cases where bank A's interim return is the same as for the common asset, trade is again not taking place and the pay-offs are as in the baseline model. With probability $\pi(1-\pi)$, however, the alternative asset of bank A has a high return, while the common asset (the investment of bank B) has a low return. Capital requirements will then be $\overline{d}_L$ and the bank will sell $\frac{\triangle x}{2}$ of equity. The expected return for the bank is then*

$$f(z_L+\frac{\triangle x}{2})+(1-p(z_L+\frac{\triangle x}{2}))\overline{d}_L+\frac{\triangle x}{2}r_e(\overline{d}_L)\ +x_L. \tag{A25}$$

In the opposite case (low return on the alternative asset of A, high return on the common asset), bank A will buy equity and have an expected return of

$$f(z_H-\frac{\triangle x}{2})+(1-p(z_H-\frac{\triangle x}{2}))\overline{d}_H-\frac{\triangle x}{2}r_e(\overline{d}_H)\ +x_H. \tag{A26}$$

We can hence obtain for the expected profit of bank A:

$$\begin{aligned} E[W_A^U]_{B=\{C\}} &= \pi^2(f(z_H)+(1-p(z_H))(z_H-x_H))+(1-\pi)^2(f(z_L)+(1-p(z_L))(z_L-x_L)) \\ &+\pi(1-\pi)(f(z_L+\frac{\triangle x}{2})+(1-p(z_L+\frac{\triangle x}{2}))(z_L-x_L)+\frac{\triangle x}{2}r_e(z_L-x_L)) \\ &+\pi(1-\pi)(f(z_H-\frac{\triangle x}{2})+(1-p(z_H-\frac{\triangle x}{2}))(z_H-x_H)-\frac{\triangle x}{2}r_e(z_H-x_H)\,) \\ &+\omega+\widehat{x}. \end{aligned} \tag{A27}$$

Suppose next that bank B has chosen the alternative asset. Consider first that bank A invests in the common asset. This case is similar to the one considered last as assets are again uncorrelated. However, a difference arises because it is now the state of bank A that determines capital requirements. Conditional on the common state, this now means that bank A will be selling equity if it was buying it before, and vice versa. We can thus write *the expected benefits:*

$$
\begin{aligned}
E[W_A^C]_{B=\{U\}} &= \pi^2(f(z_H) + (1-p(z_H))(z_H - x_H)) + (1-\pi)^2(f(z_L) + (1-p(z_L))(z_L - x_L)) \\
&+ \pi(1-\pi)(f(z_L + \frac{\triangle x}{2}) + (1-p(z_L + \frac{\triangle x}{2}))(z_L - x_L) - \frac{\triangle x}{2} r_e (z_L - x_L)) \\
&+ \pi(1-\pi)(f(z_H - \frac{\triangle x}{2}) + (1-p(z_H - \frac{\triangle x}{2}))(z_H - x_H) + \frac{\triangle x}{2} r_e (z_H - x_H)) \\
&+ \omega + \widehat{x}. \qquad \text{(A28)}
\end{aligned}
$$

The final case is the one where bank B and bank A both choose the alternative asset. In this case, the common state is independent of the returns of either bank. Thus, for each pair of return realizations for bank A and B there are two cases (high and low) for the common asset. We thus obtain for the expected benefits

$$
\begin{aligned}
E[W_A^U]_{B=\{U\}} &= \pi^2(\pi(f(z_H) + (1-p(z_H))(z_H - x_H)) \\
&+ (1-\pi)(f(z_L + \Delta x) + (1-p(z_L + \Delta x))(z_L - x_L))) \\
&+ (1-\pi)^2(\pi(f(z_H - \Delta x) + (1-p(z_H - \Delta x))(z_H - x_H)) + (1-\pi)(f(z_L) + (1-p(z_L))(z_L - x_L))) \\
&+ 2\pi(1-\pi)(\pi(f(z_H - \frac{\triangle x}{2}) + (1-p(z_H - \frac{\triangle x}{2})))(z_H - x_H) \\
&+ (1-\pi)(f(z_L + \frac{\triangle x}{2}) + (1-p(z_L + \frac{\triangle x}{2}))(z_L - x_L))) + \omega + \widehat{x}. \qquad \text{(A29)}
\end{aligned}
$$

From (19), (A27), (A28) and (A29), and recalling that $E[W_A^C] = E[W_A^C]_{B=\{C\}}$*, we can see that* $E[W_A^C]_{B=\{C\}} - E[W_A^U]_{B=\{C\}} < E[W_A^C]_{B=\{U\}} - E[W_A^U]_{B=\{U\}}$*, that is, A's gain from choosing correlation are lower when B has chosen correlation as well.* ■

The reason why the choices are strategic substitutes is that banks benefit from being different from each other as this allows them to smooth shocks bilaterally. The argument is akin to the one arising from the "last-bank-standing effect" (Perotti and Suarez (2002)), through which a surviving bank can benefit from the failure of its rival. In particular, Perotti and Suarez (2002) show that bank leverage decisions become substitutes in the presence of the possibility of gaining market share in the event of failures of other banks.

The incentives to be different also alleviate the incentive compatibility constraint of the regulator. From the foregoing discussion it is apparent that a bank's benefit from choosing the correlated asset is lower than in the baseline model. This provides the regulator with more freedom to impose welfare-improving capital requirements, while retaining the uncorrelated economy. A policy implication of this section is thus that it may be beneficial to promote trading among banks, as this lowers banks' incentives to take on systemic risk.

Monopolistic behavior

We consider two types of monopolistic behavior, first in the project choice at date 0 and, second, in the capital reallocation at date 1.

Suppose first that at date 0 the two banks choose the project (of bank A) to maximize their joint surplus (this can be likened to a single monopolistic bank that has two subsidiaries). We maintain the assumption that banks can reallocate capital as in Section 3.2 (otherwise there is no interaction among the banks and its does not matter whether projects are chosen cooperatively or not). As discussed in Section 3.3, the project choices of a bank can only affect the other when it creates opportunities for trade at date 1. Specifically, bank B benefits when bank A implements alternative investment as only then there can be trade. It follows that when projects are chosen to maximize joint utility, there is a higher tendency to choose the alternative investment as now the impact on bank A is taken into account. However, as banks will still not internalize the impact on the consumer, the correlation choice remains excessive from a social perspective.

Proposition 13 *The joint utility gain for bank A and bank B from a correlated economy is*

(i) lower than the utility gain for bank A only: $E[W_A^{C,Trade}]+E[W_B^{C,Trade}]-(E[W_A^{U,Trade}]+E[W_B^{U,Trade}]) < E[W_A^{C,Trade}] - E[W_A^{U,Trade}]$*;*

(ii) higher than the social gain: $E[W_A^{C,Trade}]+E[W_B^{C,Trade}]-(E[W_A^{U,Trade}]+E[W_B^{U,Trade}]) >$

$E[W^{C,Trade}] - E[W^{U,Trade}]$.

Proof. *Part (i). The joint gain will be higher iff* $E[W_B^{C,Trade}] < E[W_B^{U,Trade}]$*. The left hand side of the inequality,* $E[W_B^{C,Trade}]$*, is the utility for bank B when bank A invests in the common asset. In this case, both invest in the same asset (the common one) and there is no trade. The utility is hence identical to the one of the baseline model, equation (19). The right hand side of the inequality,* $E[W_B^{U,Trade}]$*, is the utility for B when it has invested in the common asset but when bank A has invested in the alternative asset. Because of symmetry, this is identical to the utility of bank A when it has chosen the common asset, but bank B has chosen the alternative asset. This case was considered in Section (3.3), and the utility is given by equation (A28). Comparing (19) and (A28) we obtain* $E[W_A^C] > E[W_A^C]_{B=\{U\}}$ *and hence* $E[W_B^{C,Trade}] > E[W_B^{U,Trade}]$.

Part (ii). Using equation (19) for $E[W_A^{C,Trade}] = E[W_A^C]$ *and* $E[W_B^{C,Trade}] = E[W_A^{C,Trade}] = E[W_A^C]$*, equation (A27) for* $E[W_A^{U,Trade}] = E[W_A^{U,Trade}]_{B=\{C\}}$*, equation (A28) for* $E[W_B^{U,Trade}] = E[W_A^C]_{B=\{U\}}$*, equation (9) for* $E[W^{C,Trade}] = E[W^C]$ *and equation (28) for* $E[W^{U,Trade}]$ *, we obtain* $E[W_A^{C,Trade}]+E[W_B^{C,Trade}]-(E[W_A^{U,Trade}]+E[W_B^{U,Trade}]) > E[W^{C,Trade}]-E[W^{U,Trade}]$.

■

Suppose next that – instead of a common project choice at date 0 – bank A has a strategic advantage in the market for equity at date 1. In particular, assume that the bank can make a *take-it-or-leave-it* offer to bank B. It will then capture the entire surplus from trade. This in turn will make trade more valuable for bank A, and hence lower its incentives to choose the common asset. However, there is still the effect on the consumer, and hence there is a tendency for its correlation choice to be excessive.

Proposition 14 *Bank A's utility gain from correlation when it can make a take-it-or-leave-it offer is*

i) lower than without market power: $E[W_A^{C,T}] - E[W_A^{U,T}] < E[W_A^C] - E[W_A^U]$*;*

ii) higher than the social gain from correlation: $E[W_A^{C,T}] - E[W_A^{U,T}] > E[W^{C,T}] - E[W^{U,T}]$.

Proof. *We first solve for interest rates r and equity transactions q at date 1. Let us denote with $q > 0$ equity issuance by bank B ($q < 0$ is consequently equity issuance by bank A). The take-it-or-leave-it offer by bank A to bank B which consists of an equity stake q and an associated rate of return $r_e(q)$. The take-it-or-leave-it offer will make bank B indifferent to autarchy, that is, the situation without trade ($q = 0$). This condition gives*

$$x_B + f(x_B + q + \overline{d}) + (1 - p(x_B + q + \overline{d}))\overline{d} - r_e q = x_B + f(x_B + \overline{d}) + (1 - p(x_i + \overline{d}))\overline{d} \quad \text{(A30)}$$

For given q, equation (A30) defines $r_e(q)$. Bank A then chooses q to maximize profits

$$\max_q + f(x_A - q + \overline{d}) + (1 - p(x_A - q + \overline{d}))\overline{d} + r_e(q)q \quad \text{(A31)}$$

Using (A30) to substitute $r_e(q)q$ into (A31), and taking derivative with respect to q we obtain the first-order condition for bank A:

$$f'(z_A) - p'(z_A))\overline{d} = f'(z_B) - p'(z_B))\overline{d}, \quad \text{(A32)}$$

This implies that investment levels are equalized across banks: $z_A = z_B$. It follows that the capital allocation is exactly the same as in Section 3.2. Thus, the joint surplus for both banks will also be the same as in Section 3.2:

$$E[W_A^{C,T}] + E[W_B^{C,T}] = E[W_A^{C,Trade}] + E[W_B^{C,Trade}] \quad \textit{(A33)}$$

$$E[W_A^{U,T}] + E[W_B^{U,T}] = E[W_A^{U,Trade}] + E[W_B^{U,Trade}] \quad \textit{(A34)}$$

Since bank A extracts the entire surplus from bank B, the utility for bank B is the same as in the absence of trade (equation (A30). We hence have $E[W_B^{C,T}]_{A=\{U\}} = E[W_B^{C}]_{A=\{U\}}$, which because of symmetry is identical to $E[W_A^U]$, hence $E[W_B^{C,T}]_{A=\{U\}} = E[W_A^U]$. For similar reasons we have $E[W_B^{C,T}]_{A=\{C\}} = E[W_A^C]$.

Part i) From (A33) we have that

$$\begin{aligned} E[W_A^{C,T}] - E[W_A^{U,T}] &= \\ & E[W_A^{C,Trade}] + E[W_B^{C,Trade}] - E[W_B^{C,T}] - E[W_A^{U,Trade}] - E[W_B^{U,Trade}] + E[W_B^{U,T}] \\ &< E[W_A^{C,Trade}] - E[W_B^{C,T}] - E[W_B^{U,Trade}] + E[W_B^{U,T}] < E[W_A^C] - E[W_A^U], \quad \textit{(A35)} \end{aligned}$$

where we have used twice Property i) of Proposition 13.

Part ii) Using (A33) we have that

$$\begin{aligned} E[W_A^{C,T}] - E[W_A^{U,T}] &= \\ & E[W_A^{C,Trade}] + E[W_B^{C,Trade}] - E[W_B^{C,T}] - E[W_A^{U,Trade}] - E[W_B^{U,Trade}] + E[W_B^{U,T}] \\ &> E[W^{C,Trade}] - E[W^{U,Trade}] = E[W^{C,T}] + E[W^{U,T}], \qquad (A36) \end{aligned}$$

where the first inequality follows from Property ii) of Proposition 13. ■

This result is reminiscent of Acharya et al. (2011), where it is shown that allowing banks to buy distressed rivals at below fundamental prices lowers ex-ante risk-taking.

Taken together, the general message of this section is that providing monopoly power to a bank limits the distortion arising from the ex-ante correlation choice, but does not eliminate it. Thus while monopoly power is known to have negative effects because of inefficient pricing vis-a-vis consumers, in a world with systemic concerns it can also provide benefits.

Bank-specific capital requirements

In the following, we analyze the incentive problem that arises when banks can misreport the return on their assets. We modify the baseline model by assuming that there is now a continuum (of measure one) of type A banks, and a continuum of type B banks (also of measure one). As in the baseline model, type A banks can invest in the common asset or in an alternative asset, whereas type-B banks can only invest in the common asset. The alternative assets are specific to each type-A bank and are uncorrelated across banks.

We also assume that at date 1, each bank observes the return on the asset it has invested in. The regulator, however, cannot observe asset returns in the economy. In order to implement state-contingent capital requirements, the regulator hence has to rely on banks truthfully reporting their state. To simplify matters, let us assume that banks have a weak preference for correctly reporting the state, that is, when their expected profit does not depend on truth-telling, they will tell the truth.

We show first that the regulator can still implement capital requirements based on the aggregate state. For this consider that the regulator implements $\overline{d}_s$ whenever at least 75% of the banks report state s. Since an individual bank is negligible in the continuum, it can hence not affect capital requirements. Given the weak preference for truthful reporting, each bank will hence correctly announce the state of its project. Suppose that a fraction α ($\alpha \in [0,1]$) of type-A banks has chosen the common asset. All type-B banks and a fraction α of the type-A banks will then report the true common shock. Of the fraction $1-\alpha$ of the type-A banks that have chosen the alternative asset, half will also be in the same state as the common asset, and hence report s. Thus, a total fraction of $\frac{1}{2}(1+\alpha+\frac{(1-\alpha)}{2}) = \frac{3}{4}+\frac{\alpha}{4}$ ($\geq \frac{3}{4}$) will report s. Hence, by basing capital requirements on the 75% rule, the regulator can implement capital requirements based on the aggregate state.

Consider next whether the regulator can also implement capital requirements based on the return of the alternative project of an individual bank. Since alternative projects are uncorrelated across banks, the regulator has to rely solely on the state reported by the bank in question, and not on states reported by other banks. Consider in particular arbitrary countercyclical capital requirements based on the state reported by an individual bank, denoted with $\overline{d}_H$ and $\overline{d}_L$ ($\overline{d}_H < \overline{d}_L$). Suppose first that the true state of the bank's project is high. If the bank reports a high state to the regulator at date 1, it will get the same return as in the baseline model (equation (19), conditional on the state being high):

$$E_1[W_A]_{x_A=H,\widehat{x}_A=H} = x_H + f(x_H+\overline{d}_H) + (1-p(x_H+\overline{d}_H))\overline{d}_H. \tag{A37}$$

If, on the other hand, the bank reports the low state, its expected return becomes

$$E_1[W_A]_{x_A=H,\widehat{x}_A=L} = x_H + f(x_H+\overline{d}_L) + (1-p(x_H+\overline{d}_L))\overline{d}_L. \tag{A38}$$

The bank's gain from misreporting is hence

$$E_1[W_A]_{x_A=H,\widehat{x}_A=L} - E_1[W_A]_{x_A=H,\widehat{x}_A=H} =$$
$$f(x_H + \overline{d}_L) + (1 - p(x_H + \overline{d}_L))\overline{d}_L - f(x_H + \overline{d}_H) - (1 - p(x_H + \overline{d}_H))\overline{d}_H. \qquad \text{(A39)}$$

Let us define with $h(x, d) := f(x + d) + (1 - p(x + d))d$ the bank's expected profit when its state is x and capital requirements are d. By our assumption that capital requirements are always binding (equation (8), showing that bankers would benefit from expanding investment), we have that $\frac{\partial h(x_H,d)}{\partial d}_{d=d_L} > 0$. Since $\overline{d}_L > \overline{d}_H$ and h is a continuous function in the relevant interval we thus have that $E_1[W_A]_{x_S=H,x_A=L} - E_1[W_A]_{x_S=H,x_A=H} > 0$, and the bank will misreport.

Suppose next that the true state of the world is low. The gains from misreporting (that is, reporting the high state) are given by

$$E_1[W_A]_{x_A=L,\widehat{x}_A=H} - E_1[W_A]_{x_S=L,x_A=L} = h(x_L, \overline{d}_H) - h(x_L, \overline{d}_L). \qquad \text{(A40)}$$

Since $\frac{\partial h(x_L,d)}{\partial d}_{d=d_H} > 0$, we have that $E_1[W_A]_{x_A=L,\widehat{x}_A=H} - E_1[W_A]_{x_A=L,\widehat{x}_A=L} < 0$. The bank will hence (correctly) report the low state.

In sum, when capital requirements are countercyclical (in terms of reported states), the bank will always report the low state. The reason is that countercyclical requirements are more lenient in the low state, allowing the bank to expand its investment. The true state will thus not be revealed to the regulator. The consequence is that actual capital requirements cannot be made dependent on the state of a bank's alternative project.