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Nonlinear Ultrasonic Phased Array Imaging

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This Letter reports a technique for the imaging of acoustic nonlinearity. By contrasting the energy of the diffuse field produced through the focusing of an ultrasonic array by delayed parallel element transmission with that produced by postprocessing of sequential transmission data, acoustic nonlinearity local to the focal point is measured. Spatially isolated wave distortion is inferred without requiring interrogation of the wave at the inspection point, thereby allowing nonlinear imaging through depth.

Introduction.—Despite the sensitivity of acoustic nonlinearity to material microstructure [1–3], mechanical damage [4,5] and tissue pathology [6,7] having long been known, the lack of effective nonlinear acoustic imaging has held back the practical use of this important modality. The lack of progress in this field is primarily due to the difficulty in spatially isolating the source of wave distortion due to acoustic nonlinearity since the wave is not accessible by measurement systems at all points. Most measurements of acoustic nonlinearity have, therefore, been limited to spatially averaged values [8] or implicit localization through exhaustive tomographical methods [9–11]. Few explicit techniques have been proposed for the imaging of acoustic nonlinearity. Those that have require optical interrogation of the inspection point [12,13] and are consequently limited to surface measurements, which precludes their use in many applications.

Conversely, linear acoustic imaging has long been made tractable by the use of ultrasonic phased arrays. Through the application of relative delays to the parallel transmission of discrete elements, beam forming may be controlled, typically to achieve steering or focusing. More recently, an alternative approach to array imaging commonly referred to as full matrix capture has been employed [14]. Here, rather than transmitting on elements in parallel, time-domain responses from all transmitter-receiver pair combinations are acquired sequentially to yield the so-called full matrix capture. Delay laws are then applied in postprocessing to emulate the equivalent delayed parallel transmission. The work reported here identifies how differences in the wave propagation of these transmissions techniques can be used to image acoustic nonlinearity.

In the case of longitudinal wave propagation through an elastic solid with an isotropic bulk nonlinearity truncated to the second order, the amplitude of second-harmonic waves generated is proportional to $U^2$, and hence, the amount of energy lost from the fundamental wave is proportional to $U^4$. The acoustic energy lost from the fundamental wave for the single transmission cycle in parallel focusing is, therefore, $N^3$ greater than that lost through the summation of the $N$ transmission cycles necessary for the equivalent focal point.

Nonlinear array imaging method.—Consider, for example, the case of longitudinal wave propagation through an elastic solid with an isotropic bulk nonlinearity truncated to the second order. For the parallel and sequentially focused transmission of an $N$ element array, the absolute wave amplitude $U$ experienced at the focal point differs by a factor of $N$. The amplitude of second-harmonic waves generated is proportional to $U^2$, and hence, the amount of energy lost from the fundamental wave is proportional to $U^4$. The acoustic energy lost from the fundamental wave for the single transmission cycle in parallel focusing is, therefore, $N^3$ greater than that lost through the summation of the $N$ transmission cycles necessary for the equivalent focal point.

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The implementation requires only a single ultrasonic array and knowledge of acoustic velocity within a material. Since these requisites are the same as for linear imaging, the proposed nonlinear imaging modality is immediately available to many applications where linear ultrasonic array imaging is currently applied.

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sequential focusing. Given that ultrasonic arrays with order $N = 100$ elements are now commonplace, the difference in nonlinear energy loss from the fundamental between parallel and sequential focusing is significant and measurable. Furthermore, because the physical amplitude difference between parallel and sequential focusing is only large close to the focal point the difference in energy loss from the transmission frequency band is predominantly local to this point. This is the reason why the proposed technique is able to spatially map nonlinearity. It is important to note that this could not be achieved by simply applying parallel focusing at two different amplitudes. In that case, amplitude scaling between the two transmissions is experienced along the whole wave propagation path, rather than just at the focal point, resulting in much poorer nonlinear localization. Furthermore, the obtained measurements would not separate instrument and material nonlinearity.

More generally, even restricting consideration to low-order nonlinear dynamics, sonification of common nonlinear media will generate not only harmonics but also subharmonics and frequencies corresponding to sum and difference combinations of the transmission bandwidth [15–17]. The property common to all these nonlinear processes is a transfer of energy from the incident frequency band. Consequently, inspection of the fundamental frequency provides a metric sensitive to a wide range of acoustic nonlinearity.

The task here is to measure the relative loss in energy from the transmission frequency band at the focal point and focal time for parallel and sequential focusing. As the wave field in the system at the focal point is inaccessible to the measurement system, the loss in energy at the focal point must be inferred from the subsequent behavior of the system.

Linear propagation is the same for the two transmission types, meaning effects of interference on apparent acoustic energy are the same. Transmission energy is also the same for both fields, including effects of instrumentation nonlinearity and nonlinear contact-acoustic effects at the array-specimen interface. Furthermore, away from the focal point where there is no interference between element transmissions, losses of energy from the transmission frequency due to nonlinear self-interaction are the same. Differences in energy within the transmission band are only introduced due to relative amplitude scaling local to the focal point and time. The relative nonlinear energy loss at focal point may thereby be obtained by evaluating the difference in the total acoustic energy in the system for the two focusing methods at any instant after the focal time. To this end, the subsequent diffuse field is examined.

Within a theoretically ideal diffuse field, the expected root-mean-squared acoustic pressure will be the same at all points and the flux of acoustic energy in any direction is equally probable. In practice, diffuse fieldlike conditions are reached some time after an initial coherent excitation due to multiple scattering from grain and specimen boundaries. The acoustic energy at any point in the diffuse field is proportional to the total energy in the system at that time. Consequently, despite the relative nonlinear energy loss from the focal point being present only within subset of the propagating coherent field, it exists uniformly in the diffuse field. The difference in energy at any point in the diffuse field for parallel and sequential focusing, therefore, provides an approximation to the energy difference at the focal point and focal time, which in turn provides a measure of acoustic nonlinearity at that point.

The acoustic field will converge on the diffuse state as time increases. However, since propagation is dissipative, the signal to noise ratio will decrease with time. Furthermore, since received sequentially focused data are of lower amplitude, they will be degraded by incoherent noise to a greater extent than the parallel data, reducing the efficacy of diffuse energy comparisons as the delay between transmission and data acquisition increases. Consequently, the choice of this reception delay $t_r$ is a compromise between maximizing amplitude and diffuse field convergence. Since the diffuse field is only static in a statistical sense, the energy value used should be the integral of that within a captured window, such that energy is evaluated between times $t_r$ and $t_r + T$, where $T$ is the window length.

The final consideration is that of frequency bandwidth. Increasing bandwidth will increase the acoustic energy incident at the focal point in addition to broadening sensitivity to a wider range of acoustic nonlinearity. However, any internal movement of energy within the band will not contribute to the measured value of nonlinearity. The evaluated bandwidth should, therefore, be the maximum for which there is no internal movement of energy. Noting the possible energy transfer to harmonics, subharmonics, sum, and difference frequencies, this corresponds to a value of two-thirds of the center frequency $\omega_0$.

Let $f_{j,k}(t)$ be the time-domain signal received on element $k$ for the sequential transmission of element $j$. $\delta_j(\bar{r})$ is the delay applied to the $j$th element to focus at point $\bar{r}$ (illustrated graphically in Fig. 1). $h_k(\bar{r}, t)$ is the time-domain received data on element $k$ for the parallel transmission of all elements delayed by $\delta_j(\bar{r})$. The frequency $\omega$ spectra of $f_{j,k}(t)$ and $h_k(\bar{r}, t)$ within the time window $t_r$ to $t_r + T$ are evaluated as $F_{j,k}(\omega) = \int_{t_r}^{t_r+T} f_{j,k}(t)e^{-i\omega t} \, dt$ and $H_k(\omega) = \int_{t_r}^{t_r+T} h_k(t)e^{-i\omega t} \, dt$, respectively.

For an $N$ element array, assuming a received value proportional to displacement, the diffuse acoustic kinetic energy in the sequential focusing case $E_S$ for focal position $\bar{r}$ is calculated as follows,

$$E_S(\bar{r}) = \sum_{k=1}^{N} \left( \int_{\omega_0}^{\omega_0/2} |F_{j,k}(\omega)|^2 \sum_{j=1}^{N} F_{j,k}(\omega)e^{i\omega_0(\bar{r})} \, d\omega \right). \tag{1}$$
energies as follows, the difference between the sequential and parallel transmission given in the Supplemental Material [18]. Equivalent time-domain definitions of these parameters are the transmission frequency band due to the nonlinear effects, which represents the proportional energy loss from the parallel transmission in comparison to the sequential transmission case. The amplitude is largest towards the tip of the crack, which is consistent with imaging of contact acoustic nonlinearity where the interface is most completely closed. Note that the observed features of the absolute energy images do not appear in the linear image. Despite the absolute energy images changing with reception time, only the signal to noise ratio of the nonlinear image is dependent on reception time and not the characteristic shape.

The normalized difference between the absolute energy images is taken to produce the nonlinear image shown in Fig. 2(c). The crack becomes clearly visible in the nonlinear image. The amplitude of γ increases local to the crack, signifying a proportionately higher loss of energy from the parallel transmission in comparison to the sequential transmission case. The amplitude is largest towards the tip of the crack, which is consistent with imaging of contact acoustic nonlinearity where the interface is most completely closed. Note that the observed features of the absolute energy images do not appear in the nonlinear image. Despite the absolute energy images changing with reception time, only the signal to noise ratio of the nonlinear image is dependent on reception time and not the characteristic shape.

A comparative linear image is shown in Fig. 2(d), plotted on a dB scale, where 0 dB is defined as the peak amplitude in the image, to better resolve the crack. This was produced using the total focusing method (TFM) [14], which applies focusing in both transmission and reception to each pixel in the image and represents close to the theoretical maximum linear imaging performance.

The crack is visible as a consequence of weak scattering from contact points along the crack interface. Although the presence of the crack is detectable linearly, accurate imaging of its geometry is not possible. Significantly, the nonlinear image more closely matches the location of the crack tip as measured at the surface (shown by a cross in Fig. 2).

Another significant feature that should be noted is the absence of signal from the back wall of the sample in the nonlinear image, which appears as the most dominant feature in the linear image. This highlights one of the most important properties of the proposed technique, the full...
separation of nonlinear and linear acoustic modalities. Commonly, an acoustic feature of interest will be in close proximity to a strong linear scatterer such as the case of crack growth initiated by stress concentration at some geometric feature. Whereas a linear image will be dominated by scattering from the local geometry, potentially masking the presence of a crack, nonlinear imaging will only show the feature of interest.

To demonstrate this, another test was conducted in which a 5 mm diameter hole was machined behind the crack tip of a similar fatigue specimen, such that the crack extends approximately 2.5 mm beyond the boundary of the hole, as shown in Fig. 1(b). This emulates early stage fatigue crack growth from a geometric feature such as a fixing hole. As a reference, an identical hole was positioned 15 mm away at the same depth in the specimen.

The generated nonlinear image is shown in Fig. 3(a), in which the hole locations are highlighted with circles. Since the technique is not dependent on scattering, neither of the holes are imaged and only the nonlinear crack is visible. This is compared to a linear TFM image in Fig. 3(b). Here, the fatigue crack cannot be resolved, with linear scattering from both holes dominating the image.

The results presented here clearly demonstrate the capability of the proposed technique to locally image acoustic nonlinearity. This method advances the measurement of acoustic nonlinearity to an easily acquirable and effective imaging modality with the potential to both enhance existing imaging and provide sensitivity to previously undetectable physical features.

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\[ \text{[2] P. B. Nagy, Ultrasonics} \text{ 36, 375 (1998).} \]