Shear-mode viscoelastic damage formulation interface element

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Abstract. In this paper, a viscoelastic-damage cohesive zone model is formulated and discussed. The interface element constitutive law has two elastic and damage regimes. Viscoelastic behaviour has been assumed for the shear stress in the elastic regime. Three element Voigt model has been used for the formulation of relaxation modulus of the material. Shear Stress has been evaluated in the elastic regime of the interface with integration over the history of the applied strain at the interface. Damage evolution proceeds according to the bilinear cohesive constitutive law up to the complete decohesion. Numerical examples for one element model has been presented to see the effect of parameters on cohesive constitutive law.

Introduction

Delamination is a major failure mode in various composite laminates under different loading and environmental conditions. Modeling of deformation behavior and damage evolution of delaminated composite materials is usually a complicated task. Time dependency of interlaminar fracture due to the material response and loading rate is also one important issue in the field of composite material. Experimental study by Wagner et al. [1] on unidirectional glass-reinforced epoxy revealed that fracture toughness (K_{IC}) increases with increasing the loading rate. Hashemi et. al. [2] performed a detailed study on the interlaminar failure of the Mode-I, Mode-II and Mixed-Mode I/II for Carbon-Fibre/Poly ether-ether ketone composite. From optical and electron microscopy studies, it was shown that in Mode-I the increasing "R-curve" behaviour mainly arises from the degree of fibre-bridging increasing as the interlaminar crack grows, whilst in Mode-II it appears to mainly arise from the increasing degree of microcracking and plastic deformation damage which develops around the tip of the advancing crack.

Continuum damage allows the possibility of considering rate dependency in failure process of delamination. Corigliano and Ricci [3] developed two rate-dependent interface models for the simulation of rate-dependent delamination in polymer matrix composites. The first one is viscoplastic (Perzyna kind viscoplastic law) and the second one is time-dependent elastic damage.Musto and Alfano [4] developed a novel rate-depend cohesive-zone model combining damage and visco-elasticity. They made assumptions of existence of a rate independent fracture energy. The underlying idea is that the energy of the bonds at the micro-level is rate-independent and that the rate-dependence of the overall dissipated energy during crack propagation is a natural by-product of the visco-elastic dissipation lumped on the zero-thickness interface. To validate the concept, they presented a comparative analysis of numerical and experimental results.

In the present model rate dependency of interlaminar fracture have been assumed to be originated from viscoelastic nature of matrix material. In the resin rich region of the interface under shear stress, cohesive constitutive law follows the matrix modulus. Three element Voigt model has been used for the formulation of relaxation modulus of the material in the resin rich region. Numerical integration has been usedover the history of the applied strain in the elastic zone of the interface to calculate the shear stress. Damage evolution proceeds according to the bilinear cohesive constitutive law up to the complete decohesion.

Interface Element Formulation

In the present study, interface element has been developed with cohesive constitutive law considering viscoelastic-damage behavior. It has been assumed that the shear stress in the first part of cohesive constitutive law follows the viscoelastic properties of the matrix. After the strain reaches the critical value of damage initiation, damage evolution proceeds according to the cohesive constitutive law in combination with coulomb friction up to the complete decohesion of the cohesive zone. Interface element with very small thickness has been used in the finite element modeling of the cohesive zone. The formulation of the cohesive constitutive law is in the form of stress-strain relation.

Computation of viscoelastic stress. In the viscoelastic regime of cohesive constitutive law, it has been assumed that shear stress at the interface follows the matrix material behavior. Three-elements Voigt model has been used for the formulation of shear relaxation modulus of the material. Shear stress has been evaluated in the elastic zone of the interface with the integration over the history of the applied strain at the interface.

For a viscoelastic material under a constant applied strain of γ, the Relaxation Modulus obtains by the following equation:

$$G(t) = \frac{\tau(t)}{\gamma}. \tag{1}$$

Where $\tau(t)$ is the applied stress as a function of time. Under an arbitrary applied strain of $\gamma(t)$, for the same material it follows:

$$d\tau = G(t)d\gamma = G(t)\dot{\gamma}dt. \tag{2}$$

Applying Boltzman integration to this equation, elastic shear stress,τ^{el},can be obtained from the strain rate history, $\dot{\gamma}$,by the following equation:

$$\tau^{el} = \int_{-\infty}^{t} G(t-t')\dot{\gamma}(t')dt'. \tag{3}$$

With the assumption of zero history of strain before zero time, it follows:

$$\tau^{el} = \int_{0}^{t} G(t-t')\dot{\gamma}(t')dt'. \tag{4}$$

Which is equivalent to the following equation:

$$\tau^{el} = \gamma(t)G(0) - \int_{0}^{t} \frac{dG(t-t')}{dt'}\gamma(t')dt'. \tag{5}$$

For three-element Voigt model, the Relaxation Modulus equals to [5]:

$$G(t) = G_e + G_1 e^{-\frac{t}{\tau_\sigma}}. \tag{6}$$

Where τ_σ (relaxation time), G_e and G_1 are material parameters. The schematic of the three-element Voigt model with parameters a_1, b_1 and m is depicted in Fig. 1. From which the material parametersG_e, G_1 and $\tau_\sigma$are defined by the following equations [5]:

$$\tau_\sigma = a_1,\ G_e = m,\ G_1 = \frac{b_1}{a_1} - m. \tag{7}$$

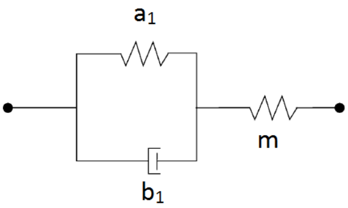

Fig.1. Schematic of the three-element Voigt model

Substituting Eq.6 in 5 gets:

$$\tau^{el} = \gamma(t)(G_e + G_1) - \frac{G_1}{\tau}\int_0^t e^{-\frac{t-t'}{\tau_\sigma}}\gamma(t')dt' \ . \tag{8}$$

Evolution of damage. When the strain reaches its critical damage value (γ^0), stress follows as:

$$\underline{\boldsymbol{\sigma}} = \begin{bmatrix}\sigma\\ \tau\end{bmatrix} = (1-d)\boldsymbol{K}\underline{\boldsymbol{\varepsilon}}. \tag{9}$$

Where $\boldsymbol{K}$ is a diagonal matrix containing the stiffness values in different modes:

$$\boldsymbol{K} = \begin{bmatrix}K_1 & 0\\ 0 & K_2\end{bmatrix}. \tag{10}$$

It has been also assumed that the normal interface stiffness,K_1, equals to the normal stiffness of bulk lamina,E_2, and K_2 equals to the shear relaxation modulus of the matrix material. The evolution of damage parameter in (9) drives the following equation:

$$d = \frac{\gamma^f(\alpha-\gamma^0)}{\alpha(\gamma^f-\gamma^0)}. \tag{11}$$

Where $\alpha$is the maximum applied γ in all previous iterations and$\gamma^f$is the complete de-cohesion strain in shear and defined by:

$$\gamma^f = \frac{2G_{IIc}}{h_0\tau_0} \ . \tag{12}$$

Where,G_{IIc} is the fracture toughness in mode-II, τ_0 is the shear strength of laminate, h_0 is the thickness of interface element and damage initiation strain (γ^0) defines as follows:

$$\gamma^0 = \frac{\tau^0}{G(0)}. \tag{13}$$

Results

The following results are for a model containing two lamina elements with one interface element between them. The Geometry of single interface element model has been illustrated in Fig. 2and the material properties are listed in Table 1.

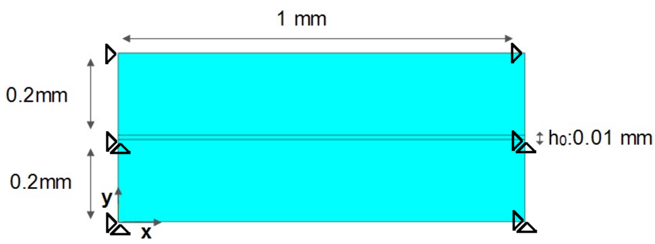

Fig.2. Geometry of single interface element model

Table 1. Mechanical properties of lamina and the interface

E_{11} (Gpa)	E_{22} (Gpa)	G_{12} (Gpa)	V_{12}	G_{IIc} (N/mm)	τ_0 (Mpa)	G_1 (Gpa)	G_e (Gpa)	τ_σ (s)
43.9	15.4	5.8	0.3	0.7	77.4	3.8	2.0	1

The lower lamina has been fixed in x and y directions and upper lamina has been extended for 0.001mm in x direction. This boundary condition results the interface to have 0.2 xy strain. Zero friction condition is also considered. To see the effect of applied strain rate, this simulation has been performed for two times. Once the 0.002mm displacement has been applied in 1s (strain rate of 0.2 /s) and in another time step in 100s (strain rate of 0.002 /s). Fig. 3. Shows the results for different applied strain rates. Increasing the strain rate results increasing the stress in the interface element.

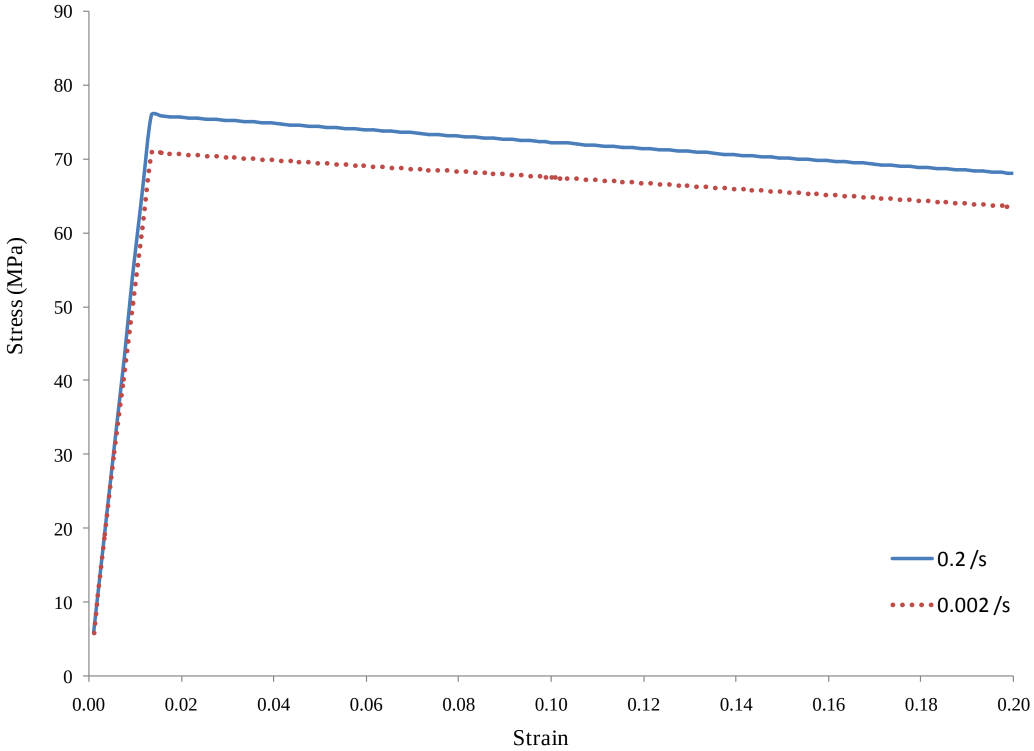

Fig. 3.Effect of applied strain rate on responce of single element model

Conclusion

In this paper formulation of shear mode viscoelastic damage interface element has been presented. Increasing the strain rate results increasing the stress in the one element model.This model can be used to predict the rate dependency of fracture toughness in mode-II shear.

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